Phase Structure of Strongly Interacting Matter beyond Mean Field

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We discuss chiral symmetry, isospin symmetry and $U_A(1)$ symmetry beyond mean field approximation in effective models at finite $T$ and $B$.

Spontaneous Symmetry Breaking and QCD Phase Diagram

condensate $\langle \bar{q}q \rangle \rightarrow$ chiral symmetry breaking

condensate $\langle qq \rangle \rightarrow$ color symmetry breaking

condensate $\langle \pi \rangle \rightarrow$ isospin symmetry breaking
Quantum Fluctuations above Mean Field

- **loop summation**
  - mean field (classical) approximation
  - Gaussing fluctuations
  - loop summation (hard thermal loop resummation, hard dense loop resummation, RPA, DSE, CJT, ……)

\[
\langle \ldots \rangle \approx X + \lambda X + \lambda^2 X + \ldots = \frac{X}{1 - \lambda X}
\]

- **renormalization group (FRG)**
  - based on symmetry and space-time dimension
  - model independent critical phenomena

- **lattice simulations**
Chiral Symmetry Restoration, Pion Superfluid and $U_A(1)$ Symmetry at Finite Temperature with FRG
Flow Equation for Effective Potential
in SU(2) Quark-Meson Model

\[ \Gamma = \int_x \left[ \bar{\psi} S \psi + \left( (\partial_\mu + 2\delta_{\mu 0}\mu I)\pi_- \right) \left( (\partial_\mu - 2\delta_{\mu 0}\mu I)\pi_+ \right) + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi_0)^2 + U(\phi^2) - c\sigma \right] \]

\[ S = \begin{pmatrix} M_+ & -\sqrt{2}g\gamma_5\pi_- \\ -\sqrt{2}g\gamma_5\pi_+ & M_- \end{pmatrix} \]

\[ M_\pm = i(\phi \pm \gamma_0\mu I) + ig(\sigma \pm i\gamma_5\pi_0) \]

For uniform field configuration

\[ \Gamma_k = \beta V U_k \]

\[ \partial_k U_k = \text{Tr} \int_p \left[ \frac{1}{2} G_{\phi,k}(p) R_{\phi,k}(p) - G_{\psi,k}(p) R_{\psi,k}(p) \right] \]

FRG modified quark and meson propagators

\[ G_{\phi,k}(q) = \left[ \Gamma_k^{(2)}[\phi] + R_{\phi,k}(q) \right]^{-1} \]

\[ G_{\psi,k}(q) = \left[ \Gamma_k^{(2)}[\psi] + R_{\psi,k}(q) \right]^{-1} \]

Regulators

\[ R_{\phi,k}(q) = (k^2 - \tilde{q}^2) \Theta(\tilde{q}^2 - k^2) \]

\[ R_{\psi,k}(q) = \vec{\gamma} \cdot \frac{\tilde{q}}{|\tilde{q}|}(k - |\tilde{q}|) \Theta(|\tilde{q}| - k) \]

Expanding U around mean field

\[ \phi \rightarrow \langle \phi \rangle + \phi \quad \text{chiral condensate} \ \langle \sigma \rangle \text{ and pion condensate} \ \langle \pi \rangle \]
Truncating the flow equation by neglecting the $p$-dependence of higher order vertices $\Gamma^{(3)}$ and $\Gamma^{(4)}$. 

\begin{align*}
\partial_{k} \Gamma_{k,p}^{(2)}[\phi_i] &= \tilde{\partial}_{k} \text{Tr} \int_q \left[ \frac{1}{2} G_{\phi,k}(q) \Gamma_{k}^{(4)}[\phi,\phi_i] ight. \\
& \quad - \frac{1}{2} G_{\phi,k}(q) \Gamma_{k}^{(3)}[\phi,\phi_i] G_{\phi,k}(q+p) \Gamma_{k}^{(3)}[\phi,\phi_i] \\
& \quad + \left. G_{\psi,k}(q) \Gamma_{k}^{(3)}[\psi,\phi_i] G_{\psi,k}(q+p) \Gamma_{k}^{(3)}[\psi,\phi_i] \right] \\
\end{align*}
Chiral Symmetry Restoration and Pion Superfluid

Beyond Potential Approximation

Potential Level
Meson Spectral Functions and BCS-BEC Crossover

\[ m_D = 2m \]

\[ \rho[\pi^+] / \Lambda^2 \]

\[ \omega (\text{MeV}) \]

\[ m_{\pi^+} = 2m_q \]

BEC

BCS

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Meson Spectral Functions and BCS-BEC Crossover

\[ \rho(\pi^+) / \Lambda^2 \]

\[ \omega \text{ (MeV)} \]

\[ (\mu, T) \text{ MeV} \]

- (65,10)
- (110,173.2)
- (190,194.2)

\[ T \text{ (MeV)} \]

\[ \mu_I \text{ (MeV)} \]

BEC

BCS
Comparison with $O(2)$ Model in Continuous Dimension

Dimension reduction at finite temperature

$$\int_0^\infty d^d x \to \int_0^{T^{-1}} dt \int_0^\infty d^{d-1} x$$

$3 < d < 4$

$$\mathcal{L}_N = \frac{1}{2} \partial_\mu \phi_i \partial^\mu \phi_i + U(\phi^2)$$

$$U(\phi^2) = \frac{1}{2} a \phi_i \phi_i + \frac{1}{4} b (\phi_i \phi_i)^2.$$
**$U_A(1)$ Anomaly**

$U_A(1)$ anomaly in QCD leads to unconserved axial current

$$\partial_{\mu} J_{5}^{\mu} = 2N_f Q(x) + 2im_0 \bar{\psi} \gamma_5 \psi$$

Topological charge $Q(x) = \frac{g^2}{32\pi^2} F_{\mu\nu}^{a} F_{\mu\nu}^{a}$  
Topological susceptibility $\chi = \int d^4x \langle T(Q(x)Q(0)) \rangle$

What is the behavior of $U_A(1)$ anomaly at finite temperature?

JLQCD group claimed $UA(1)$ restoration at $1.2T_c$, but HotQCD group observed the opposite result!

In SU(3) quark-meson model

$$\mathcal{L} = \mathcal{L}_m + \mathcal{L}_q.$$  

$$\mathcal{L}_m = \text{Tr}[\partial_{\mu} \Phi \partial^{\mu} \Phi^\dagger] - (m^2 \rho_1 + \lambda_1 \rho_1^2 + \lambda_2 \rho_2^2) + c_\xi + \text{Tr}[H(\Phi + \Phi^\dagger)],$$  

$$\mathcal{L}_q = \bar{\psi} (i\gamma^{\mu} \partial_{\mu} - m_0 + \mu \gamma^0 - g \phi_5) \psi$$

$$Q = \frac{c}{2} \left[ \frac{2}{27} \pi_0^3 - \frac{1}{\sqrt{27}} \pi_3 - \frac{1}{\sqrt{6}} \pi_0 \left( \sum_{a=1}^{8} (\pi_a^2 - \sigma_a^2) + 2\sigma_0^2 \right) + \frac{1}{2} \pi_3 \left( \sum_{a=4}^{5} (\pi_a^2 - \sigma_a^2) - \sum_{a=6}^{7} (\pi_a^2 - \sigma_a^2) \right) 
+ \frac{1}{\sqrt{3}} \pi_8 \left( \sum_{a=1}^{3} (\pi_a^2 - \sigma_a^2) - \sum_{a=4}^{7} (\pi_a^2 - \sigma_a^2) + \pi_1 \left( \sum_{a=4}^{5} (\pi_a \pi_{a+2} - \sigma_a \sigma_{a+2}) + \sqrt{2} \sigma_0 \sigma_1 - \frac{2}{\sqrt{3}} \sigma_1 \sigma_8 \right) 
+ \pi_2 \left( \pi_5 \pi_6 - \pi_4 \pi_7 + \sqrt{2} \sigma_0 \sigma_2 - \sigma_5 \sigma_6 - \sigma_4 \sigma_7 - \frac{2}{\sqrt{3}} \sigma_2 \sigma_8 \right) + \sqrt{2} \pi_3 (\sigma_0 \sigma_3 - \sqrt{2} \sigma_3 \sigma_8) 
+ \pi_4 \left( \sqrt{2} \sigma_0 \sigma_4 - \sigma_3 \sigma_5 - \sigma_1 \sigma_6 + \sigma_2 \sigma_7 + \frac{1}{\sqrt{3}} \sigma_4 \sigma_8 \right) + \pi_5 \left( \sqrt{2} \sigma_0 \sigma_5 - \sigma_3 \sigma_5 - \sigma_2 \sigma_6 - \sigma_1 \sigma_7 + \frac{1}{\sqrt{3}} \sigma_5 \sigma_8 \right) 
- \pi_6 \left( \sigma_1 \sigma_4 - \frac{2}{3} \sigma_0 \sigma_6 - \sigma_3 \sigma_6 + \sigma_2 \sigma_5 - \frac{1}{\sqrt{3}} \sigma_6 \sigma_8 \right) + \pi_7 \left( \sigma_2 \sigma_4 + \frac{2}{3} \sigma_0 \sigma_7 - \sigma_1 \sigma_5 + \sigma_3 \sigma_7 + \frac{1}{\sqrt{3}} \sigma_7 \sigma_8 \right) 
+ \sqrt{2} \pi_8 \sigma_0 \sigma_8 \right].$$
**$U_A(1)$ Restoration?**

$$
\chi = \left( \frac{c}{12\sqrt{6}} \right)^2 \sum_{i,j,k,l,m,n=\sigma_\infty} \int d^4x \left[ a_{ijklmn} \langle \varphi_i \rangle \langle \varphi_j \rangle G_{kl}(x,0) \langle \varphi_m \rangle \langle \varphi_n \rangle + b_{ijklmn} \langle \varphi_i \rangle \langle \varphi_j \rangle G_{kl}(x,0) G_{mn}(0,0) \\
+ c_{ijklmn} G_{ij}(x,x) G_{kl}(x,0) \langle \varphi_m \rangle \langle \varphi_n \rangle + d_{ijklmn} \langle \varphi_i \rangle \langle \varphi_j \rangle G_{jm}(x,0) G_{kl}(x,0) \langle \varphi_n \rangle \\
+ e_{ijklmn} G_{ij}(x,x) G_{kl}(x,0) G_{mn}(0,0) + f_{ijklmn} G_{in}(x,0) G_{jm}(x,0) G_{kl}(x,0) \right],
$$

**FRG calculated condensates and propagators**

1. $U_A(1)$ symmetry cannot be restored even in chiral symmetry restored phase.
2. Fluctuations play an important role at high $T$. 

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Chiral Symmetry Restoration in Magnetic Field with RPA
NJL Model in Mean Field

\[ \mathcal{L} = \bar{\psi} \left( i \gamma_\nu D^\nu - m_0 \right) \psi + \frac{G}{2} \left[ (\bar{\psi} \gamma^5 \psi)^2 + (\bar{\psi} i \gamma_5 \vec{\tau} \psi)^2 \right] \]

\[ D^\nu = \partial^\nu + i Q A^\nu \quad \mathbf{B} = (0, 0, B) = \nabla \times \mathbf{A} \]

At mean field level

\[ \Omega_{mf} = \frac{m^2}{2G} + \Omega_q, \]

\[ \Omega_q = -3 \sum_{f=u,d} \sum_n \alpha_n \int \frac{dp_z}{2\pi} \frac{|Q_f B|}{2\pi} \left[ \frac{E_f^+ + E_f^-}{2} + T \ln \left( \left( 1 + e^{-E_f^+ / T} \right) \left( 1 + e^{-E_f^- / T} \right) \right) \right] \]

Quark dimension reduction in magnetic field

\[ 2 \int \frac{d^3 \vec{p}}{(2\pi)^3} \rightarrow \sum_{n_f} |Q_f B| \sum_n \frac{\alpha_n}{2\pi} \int \frac{dp_z}{2\pi} \]

Gap equation for the order parameter (quark mass)

\[ \frac{\partial \Omega_{mf}}{\partial m} = 0 \]

Magnetic catalysis (MC): \( T_c \) increases with \( B \).

However, inverse magnetic catalysis (IMC) from lattice QCD!

NJL Model beyond Mean Field

Mesons may play important role in the realization of IMC.

Mesons as quantum fluctuations above mean field

\[ \neq \frac{1}{1-\chi} \]

\[ \rightarrow \text{meson mass } m_M \text{ and coupling constant } g_{Mq\bar{q}} \]

\[ \Omega = \Omega_{mf} + \sum_M \Omega_M \]

\[ \Omega_M = \int \frac{d^3k}{(2\pi)^3} \left[ \frac{E_M}{2} + T \ln \left(1 - e^{-E_M/T} \right) \right] \]

\[ \text{meson energy } E_M = \sqrt{m_M^2 + k_3^2 + v_{\perp}^2 (k_1^2 + k_2^2)} \]

\[ \text{meson transverse velocity } v_{\perp} = \left( g_0^0 \right)^2 / \left( g_1^0 \right)^2 \]

New gap equation

\[ \frac{\partial \Omega}{\partial m} = 0 \]

MC at T=0 and IMC at T_c!
Meson Mass Jump Induced by Quark Dimension Reduction

Quark dimension reduction in magnetic field

\[ 2 \int \frac{d^3 \vec{p}}{(2\pi)^3} F(p) = \int \frac{p^2 dp}{\pi^2} F(p) \rightarrow \sum_{n,f} \left| \frac{Q_f B}{2\pi} \right| \alpha_n \int \frac{dp_z}{2\pi} F(p) \]

possible infrared divergence in quark momentum integration!

A sudden mass jump for the Goldstone mode at the Mott transition point \((m_\pi = 2m_q)\), induced by the quark dimension reduction in magnetic field.

→ sudden enhancement of hadronization in heavy ion collisions?
Chiral and Deconfinement Phase Transitions in PNJL

PNJL model \((Fukushima, Weise et al.)\)

\[
\mathcal{L} = \bar{\psi} (i\gamma_{\mu} D^{\mu} - m_0) \psi + \frac{G}{2} \left[ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5 \tau \psi)^2 \right] - \mathcal{U}(\Phi, \bar{\Phi})
\]

\[
\frac{\mathcal{U}}{T^4} = -\frac{b_2(T)}{2} \bar{\Phi} \Phi - \frac{b_3}{6} (\bar{\Phi}^3 + \Phi^3) + \frac{b_4}{4} (\bar{\Phi} \Phi)^2
\]

**IMC effect on both chiral and deconfinement phase transitions.**
Summary

1) Significant fluctuations at both low (mesons) and high (quarks) temperature.

2) Fluctuations induced change in phase structure at finite temperature:
   a) $U_A(1)$ symmetry breaking even in chiral restoration phase,
   b) model independent critical exponents, and
   c) BCS-BEC crossover in pion superfluid.

3) Fluctuations induced change in phase structure in magnetic field:
   a) inverse magnetic catalysis for both chiral and deconfinement phase transitions,
   b) mass jump for the Goldstone mode at Mott transition point.