Topological nature of deconfinement transition in QCD

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Introduction: Phase diagram

Schematic QCD phase diagram

- Hadron Phase
- Quark-Gluon Plasma Phase
- Color Superconducting Phase
- Real Quark chemical potential
In pure gauge theory, the **Polyakov-loop** becomes the order parameter of the deconfinement transition. (Direct relation between deconfinement transition and $\mathbb{Z}_3$ symmetry)

If there are dynamical quarks, the Polyakov-loop is no longer an order-parameter.
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If there are dynamical quarks, Polyakov-loop is no longer order-parameter

Usually, phase transitions are induced by the spontaneous symmetry breaking, but there are phase transitions without the spontaneous symmetry breaking

Topological order


In this study, we investigate deconfinement transition from the topological viewpoint
Three operations


Aharonov-Bohm effect

Integer charge  \rightarrow  Commutable

Fractional charge  \rightarrow  Non-commutable

Ground-state degeneracy is necessary
We can clarify the confined and deconfined states from the ground-state degeneracy by modifying the topology at zero temperature.
Introduction: Imaginary chemical potential

Response against the imaginary chemical potential

Symbols
- $\mu_i$: Imaginary chemical potential
- $T$: Temperature
- $N_c$: Number of color

Roberge-Weiss (RW) periodicity

$2\pi/N_c$ periodicity for $\theta \equiv \mu_i/T$

RW transition

First-order transition at $\theta = (2k-1)\pi/N_c$

RW endpoint

Endpoint of RW transition line

Details of imaginary chemical potential region; see A. Roberge and N. Weiss, Nucl. Phys. B275 (1986) 734
Result 1: Nontrivial free-energy degeneracy

For example, strong coupling limit

**Confined phase**

No nontrivial free-energy degeneracy

Infinite quark mass region:

Result 1: Nontrivial free-energy degeneracy

**Confined phase**

No nontrivial free-energy degeneracy

**Deconfined phase**

Nontrivial free-energy degeneracy

Visualization of topological differences

**Confined phase**

Realized states for each $\theta$

$\theta = \mu_l / T = 0^{\circ} \sim 2\pi$

Map of thermodynamic quantities to circle $S^1$

**Deconfined phase**

Three states exit at each $\theta$

Lowest free-energy states is realized

$\theta = 0^{\circ} \sim 2\pi$

RW transition occurs

Map to 2 dim.

Topological deference between two phases
Result 2: Quantum order-parameter

**Definition**

Quark number susceptibility

\[ \Psi = \left[ \int_{0}^{2\pi} \left\{ \text{Im} \left( \frac{d\tilde{n}_q}{d\theta} \bigg|_T \right) \right\} d\theta \right] \]

Dimensionless quark number density

\[ \tilde{n}_q \equiv \tilde{C} n_q \]

\[ C = T^3 \]

Definition

Quark number susceptibility

\[ \Psi = \left[ -\int_{0}^{2\pi} \text{Im} \left( \frac{d\tilde{n}_q}{d\theta} \bigg|_T \right) \right] d\theta \]

Dimensionless quark number density

\[ \tilde{n}_q \equiv \tilde{C} n_q \]

\[ \tilde{C} = T^3 \]

Quark number density:

- **High T**: Singular periodic function
  \[ \rightarrow \text{Deconfined phase} \]

- **Low T**: Smooth periodic function
  \[ \rightarrow \text{Confined phase} \]

Result 2: Quantum order-parameter

Expected phase diagram at finite $\mu_1$

**Second-order point**

Lattice QCD predict this scenario;


**Triple point**
Expected behavior of $\Psi$

**Result 2:** Quantum order-parameter

To discuss the density effect to the deconfinement transition, we introduce the isospin chemical potential ($\mu_{\text{iso}}$)

Sign problem free (even with $\mu_I$)

$$\tau_2 \gamma_5 \mathcal{D} \gamma_5 \tau_2 = \mathcal{D}^\dagger \implies \det(\mathcal{D}) \geq 0$$

Orbifold equivalence

Outside of pion condensed phase

Phase diagram at finite $\mu_R$ $\leftrightarrow$ Phase diagram at finite $\mu_{\text{iso}}$

Correspondence

($N_c \to \infty$ limit)
To investigate the phase structure, we here use the Polyakov-loop extended Nambu–Jona-Lasinio (PNJL) model.

\[ \mathcal{L} = \bar{q}(\mathcal{D} + m_0) - G[(\bar{q}q)^2 + (\bar{i}q\gamma_5\bar{q})^2] + V_g(\Phi, \bar{\Phi}) \]

Polyakov-loop and its conjugate

In our study, we thin that the Polyakov-loop is just introduced to model QCD properties at finite imaginary chemical potential.

The PNJL model can well reproduce QCD properties at finite $\mu_I$.

Result 3: Isospin chemical potential

**QCD phase diagram**

Effective model computation (PNJL model)

[Graph showing QCD phase diagram with various temperatures and chemical potentials.]
Summary

We investigate the deconfinement transition from topological viewpoints.

1. To discuss the deconfinement transition at finite temperature, we here use the nontrivial free-energy degeneracy.

2. We determine the new order-parameter of deconfinement transition

\[ \Psi = \left[ \int_0^{2\pi} \left\{ \text{Im} \left( \frac{d\tilde{n}_q}{d\theta} \right) \bigg|_{T} \right\} \right] d\theta \]

3. The density-dependence of the deconfinement transition is shown by introducing the isospin chemical potential to the PNJL model.