Relativistic mean-field models of hadron and quark matter in neutron stars

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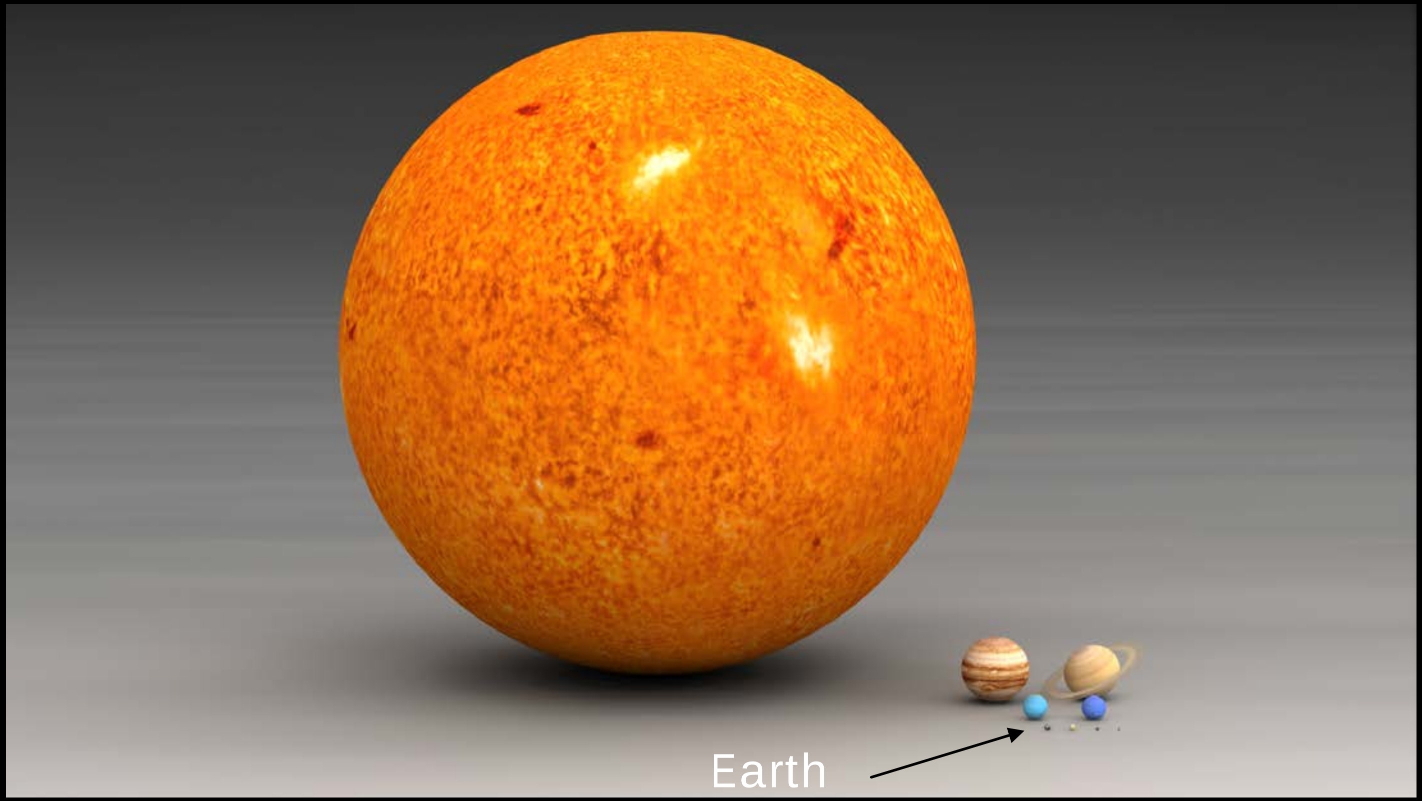
I. Introduction. Neutron stars

Normal stars

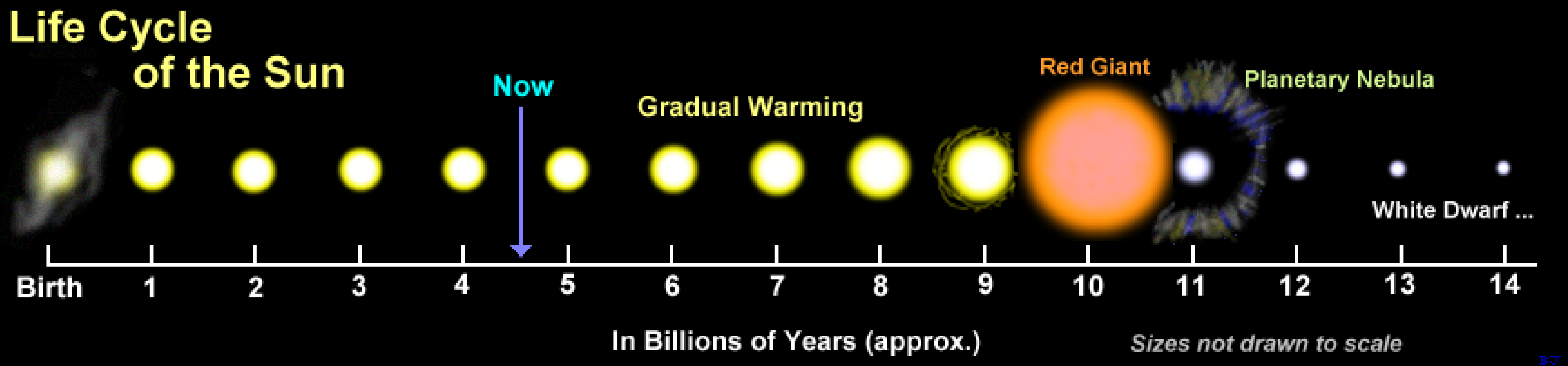
The Sun: $M_{\odot} = 2 \times 10^{33}\, g = 3.3 \times 10^{6}\, M_{\oplus}$

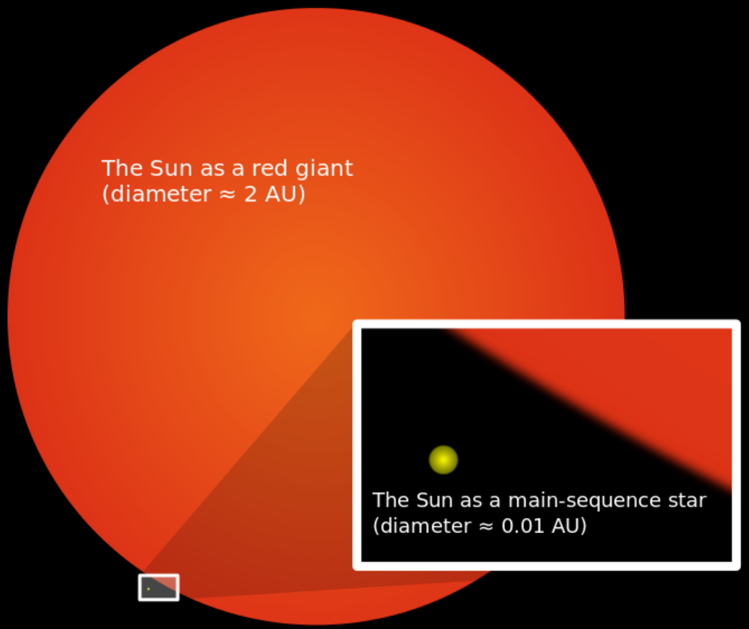
$R_{\odot} \sim 10^{6} km, \quad R_{\oplus} = 6400\ km$

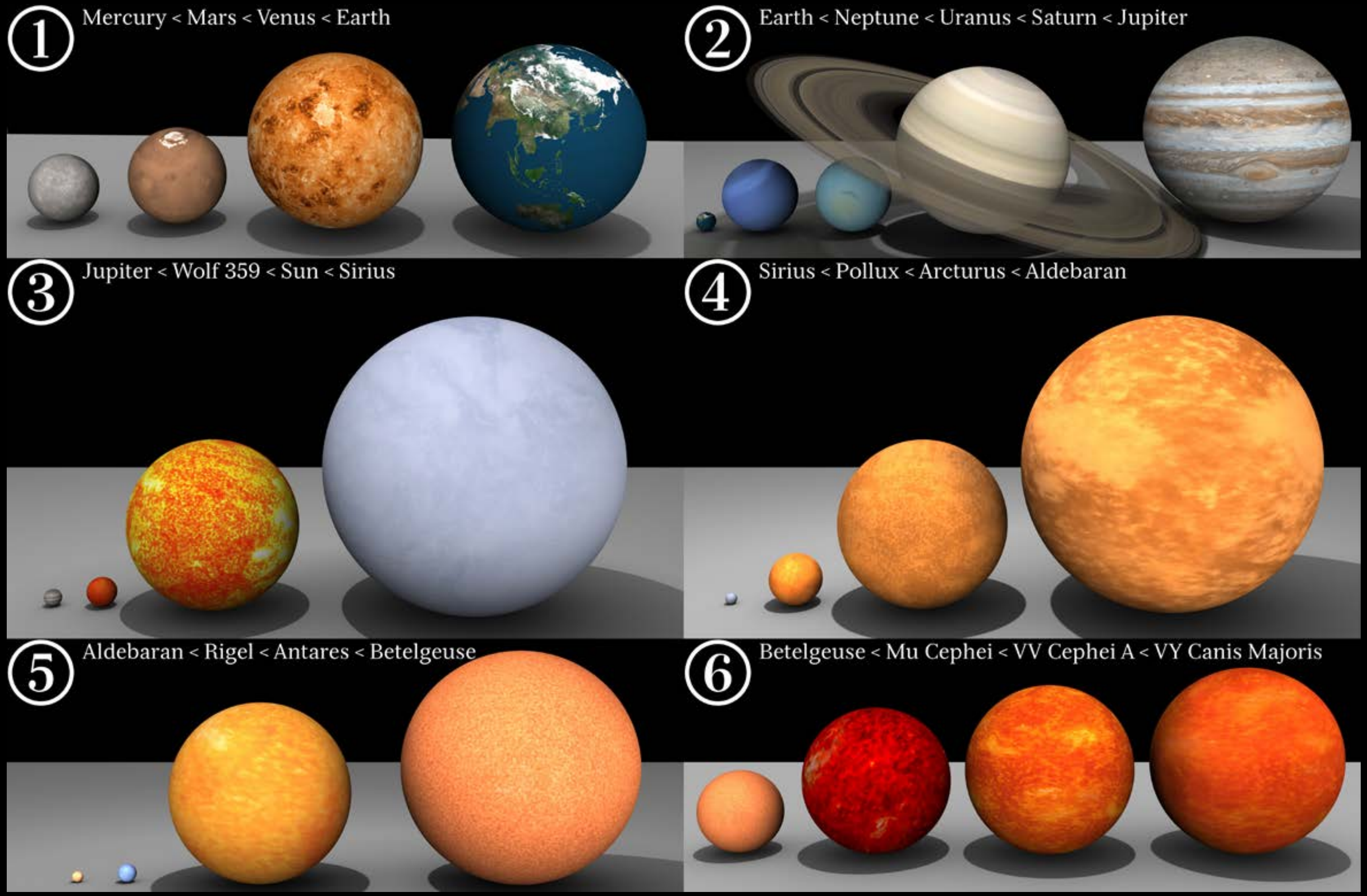
$\odot$ – Sun
$\oplus$ – Earth



Life of a star has the beginning and the end…

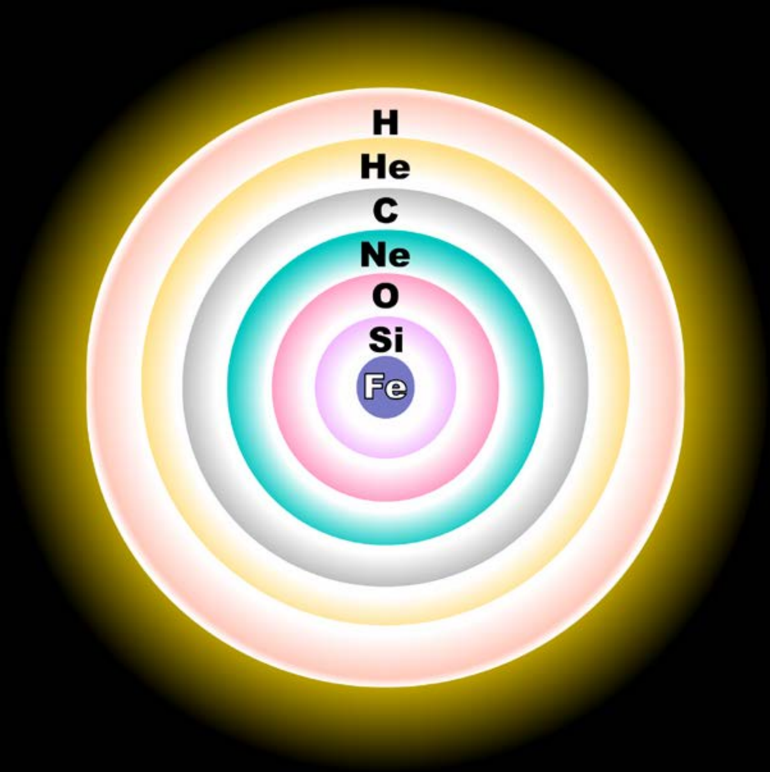




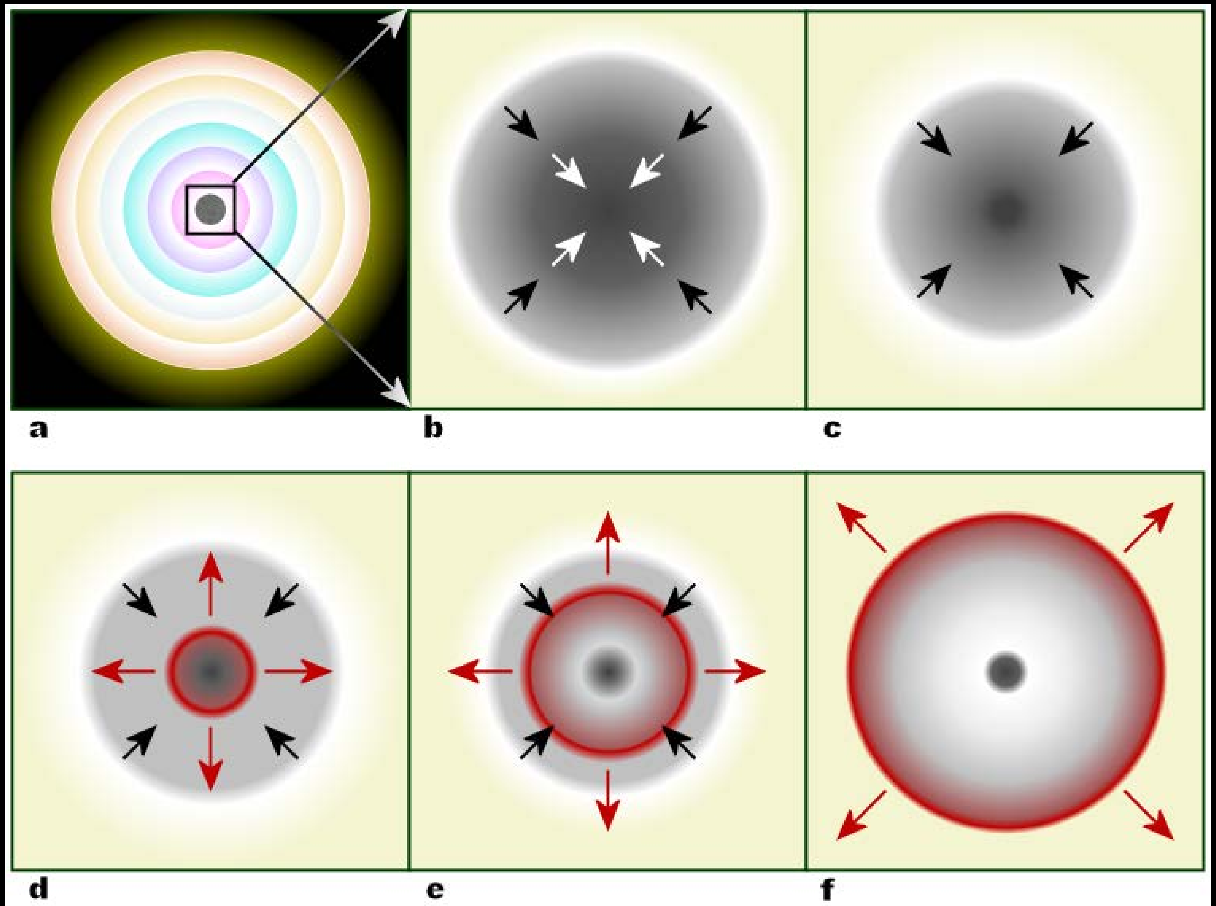

1 Mercury < Mars < Venus < Earth
2 Earth < Neptune < Uranus < Saturn < Jupiter
3 Jupiter < Wolf 359 < Sun < Sirius
4 Sirius < Pollux < Arcturus < Aldebaran
5 Aldebaran < Rigel < Antares < Betelgeuse
6 Betelgeuse < Mu Cephei < VV Cephei A < VY Canis Majoris

Massive star schematic

before the explosion



- White dwarf-like core cannot resist gravitation ⇒ instability (core-collapse)

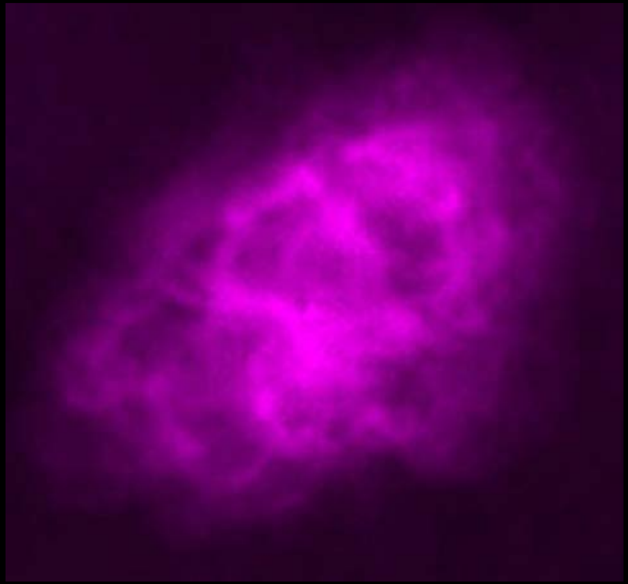


Space telescope Chandra (1999)

SN 1054
Crab nebula

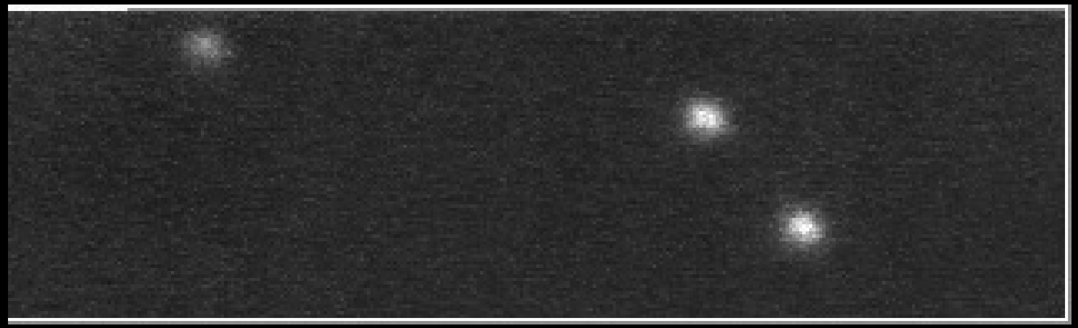
Described by Chinese astronomers (1054)
Recognized also as a "guest star" in "Diary of the Clear Moon" by Fujiwara no Teika

Optical



Radio

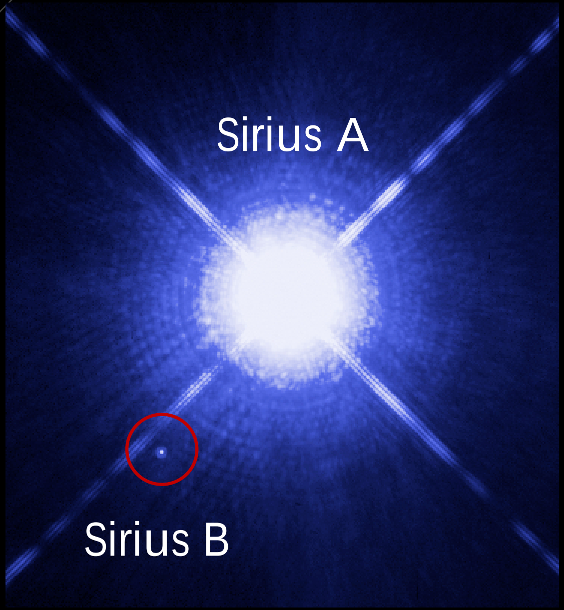
X-rays



Pulsar was discovered in1968 году
Period of 33 ms

Compact objects

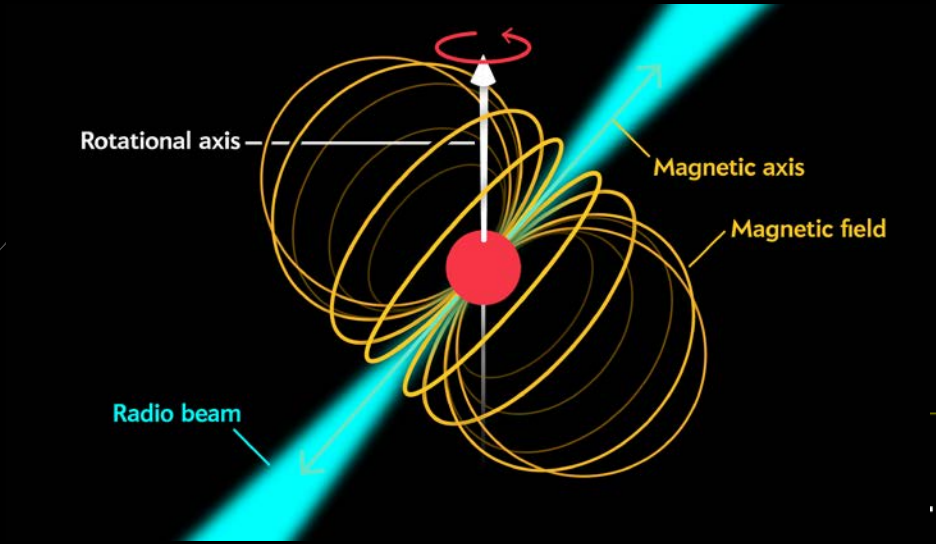
White dwarfs

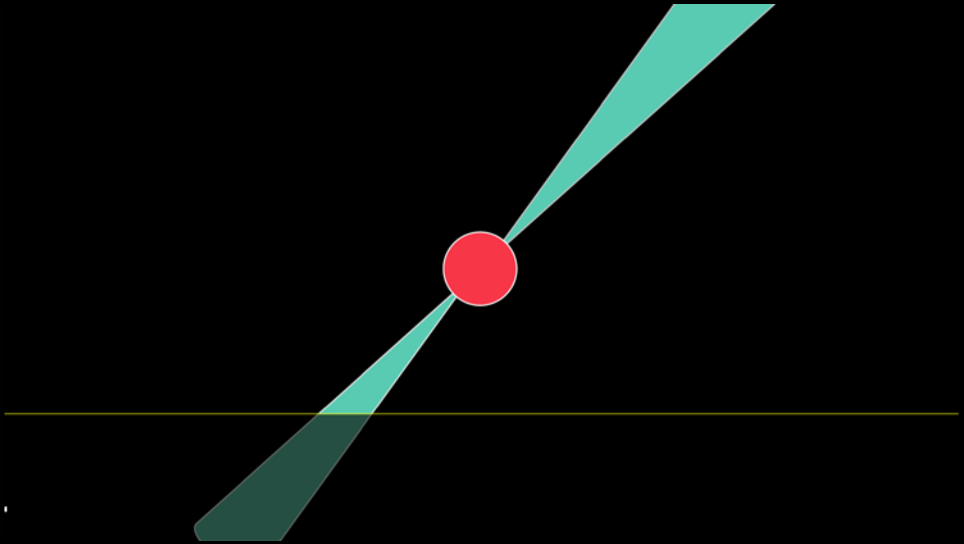


$M \sim M_{\odot}, \qquad R \sim R_{\oplus}$

Neutron stars (NSs)

$M \sim M_{\odot}, \qquad R \sim 10\ \mathrm{km}$





Estimate for the density

For the crab pulsar: $\bar{\rho} \geq 1.6 \times 10^{11} \frac{g}{cm^3}$

For a period of 1 ms: $\bar{\rho} \geq 1.4 \times 10^{14} \frac{g}{cm^3}$

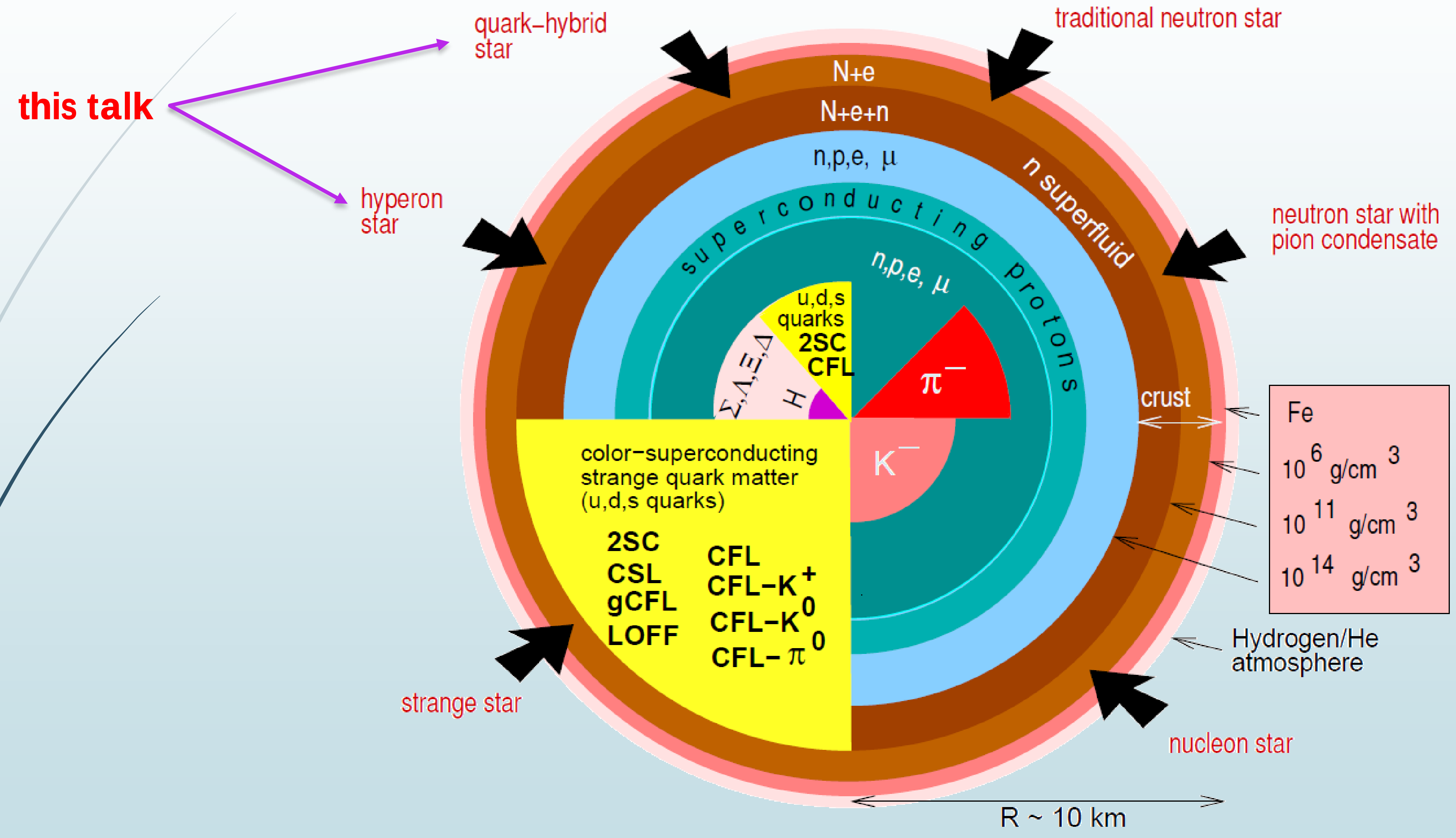
Minimum density is about the nuclear density

Why is it interesting?

We can study matter at the conditions, unreachable on the Earth

Strong,
gravitational,
weak and
electromagnetic
interactions
together!

Canonical possibilities for NS structure



II. A method of stiffening the relativistic mean-field (RMF) equation of state and its application to the description of neutron stars.

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In collaboration with D. N. Voskresensky and E. E. Kolomeitsev

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Chiba Institute of Technology, 2017

Introduction

- Description of the neutron star (NS) requires an equation of state (EoS) of cold ($T = 0$) dense ($n = 1$–$10\,n_0$, where n_0–nuclear saturation density) strongly interacting baryonic matter.
  For a given EoS the NS mass and radius can be obtained for a given central density.
- Any EoS is characterized by a maximum supported NS mass.
  A viable EoS should pass the observed maximum NS mass constraint $M > 2.01 \pm 0.04\,M_\odot$ and many others.
- Any new degree of freedom softens the EoS and lowers the maximum NS mass.
- There exist realistic phenomenological EoSs well tuned to describe finite nuclei and low-density nuclear matter properties, but yielding a low maximum NS mass
- We want to develop a method of stiffening of an EoS at high densities without altering it at low densities.

Obtaining the EoS

QCD does not allow for quantitative results for the hadronic energy scale
There are many of EoSs, built in different ways:

Microscopic

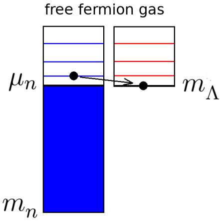
- Based on many-body theories, starting from the assumed properties of 2- (or more) body interactions in the vacuum
- Robust for low densities n, but large uncertainties already for $n \simeq n_0$
- Many of them are non-relativistic $\Rightarrow$ causality violation at large densities (speed of sound $v_S > c$)

Phenomenological approach

- Models with interaction strengths adjusted to reproduce the observables in many-body systems.
- Relatively simple and relativistic $\Rightarrow$ assure causality

Phenomenological relativistic mean-field (RMF) models are successfully applied to description of finite nuclei, heavy-ion collisions and neutron stars

Hyperon/Δ puzzle



With an increase of the density already at $n \gtrsim 2 \div 3\, n_0$ the conversion $n \to B + Q_B e^-$ becomes energetically favorable. Chemical equilibrium condition:

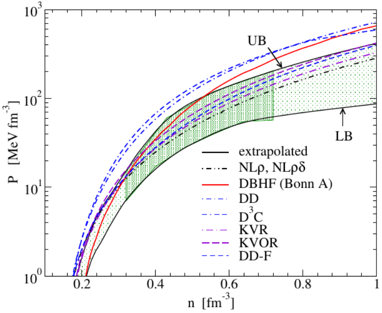
$$\mu_B = \mu_N - Q_B \mu_e$$

In standard realistic models the maximum NS mass decreases below the observed values.

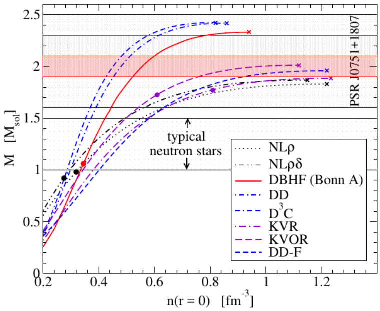
Problem can be resolved in relativistic mean-field (RMF) models by taking into account a hadron mass and couplings in-medium modifications [K. A. Maslov, E. E. Kolomeitsev and D. N. Voskresensky, Phys. Lett. B **748**, 369 (2015)]

Contradicting constraints

Constraint for the pressure, obtained from analyses of transverse and elliptic flows in heavy-ion collisions
Passed by rather soft EoSs



The maximum NS mass constraint favors stiff EoS



figures from [T. Klahn et al. PRC74 (2006)]

Traditional RMF models

H.-P. Dürr PR103 1956, J. D. Walecka 1974, J. Boguta & A. R. Bodmer 1977

Nonlinear Walecka (NLW) model

$$\begin{aligned}\mathcal{L} = & \bar{\Psi}_N \Big[(i\partial_\mu - g_\omega \omega_\mu - g_\rho \vec{t}\vec{\rho}_\mu)\gamma^\mu - m_N + g_\sigma \sigma \Big] \Psi_N \quad \text{nucleons} \\ & + \frac{1}{2}\Big[(\partial_\mu \sigma)^2 - m_\sigma^2 \sigma^2 \Big] - \Big(\frac{b}{3} m_N (g_\sigma \sigma)^3 + \frac{c}{4} (g_\sigma \sigma)^4 \Big) \quad \text{scalar field} \\ & - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu^2 - \frac{1}{4}\vec{\rho}_{\mu\nu}\vec{\rho}^{\mu\nu} + \frac{1}{2} m_\rho^2 (\vec{\rho}_\mu)^2 \quad \text{vector fields} \\ & + \sum_{l=e,\mu} \bar{\psi}_l (i\partial_\mu - m_l)\psi_l \quad \text{leptons}\end{aligned}$$

Mean-field approximation

Static homogeneous meson fields:

$$\sigma \to \langle \sigma \rangle, \quad \omega^\mu \to \langle \omega^\mu \rangle \equiv (\omega_0, \vec{0}), \quad \rho_i^\mu \to \langle \rho_i^\mu \rangle \equiv \delta_{i3}(\rho_0, \vec{0}).$$

Eqs. of motion for vector fields:

$$\Big\langle \frac{\partial \mathcal{L}}{\partial \omega^0} \Big\rangle = 0 \Rightarrow \omega_0 = \frac{g_\omega (n_n + n_p)}{m_\omega^2}$$

$$\Big\langle \frac{\partial \mathcal{L}}{\partial \rho_3^0} \Big\rangle = 0 \Rightarrow \rho_0 = \frac{g_\rho (n_n - n_p)}{2 m_\rho^2}$$

Energy density

Nucleon effective mass $m_N^* = m_N - g_\sigma \sigma$. In terms of $f \equiv \dfrac{g_\sigma \sigma}{m_N}$:

$$E = \frac{m_\sigma^4 f^2}{2C_\sigma^2} + U(f) + \frac{C_\omega^2 (n_n + n_p)^2}{2m_N^2} + \frac{C_\rho^2 (n_n - n_p)^2}{8m_N^2}$$

$$+ \sum_{i=n,p} \int_0^{p_{\mathrm{F},i}} \frac{p^2\, dp}{\pi^2} \sqrt{p^2 + m_N^{*2}} + \sum_{l=e,\mu} \int_0^{p_{\mathrm{F},l}} \frac{p^2 dp}{\pi^2} \sqrt{p^2 + m_l^2}\,,$$

Free parameters: $C_i = \dfrac{g_{iN} m_N}{m_i}, \quad i = \sigma, \omega, \rho +$ parameters of $U(\sigma)$:

$U(\sigma) \equiv m_N^4 (\frac{b}{3} f^3 + \frac{c}{4} f^4)$

- Equation of motion for the scalar field:

$$\frac{\partial E}{\partial f} = 0 \Rightarrow \frac{m_N^4 f}{C_\sigma^2} + U'(f) = g_\sigma (n_{S,n} + n_{S,p}),$$

$$n_{S,i} = \int_0^{p_{\mathrm{F,i}}} \frac{p^2 dp}{\pi^2} \frac{m_N^*}{2\sqrt{p^2 + m_N^{*2}}}$$

- Electrical neutrality condition: $n_p = n_e + n_\mu$
- Beta-equilibrium conditions: $\mu_e = \mu_n - \mu_p, \quad \mu_i = \frac{\partial E}{\partial n_i}$

Input parameters

Energy per particle expansion:

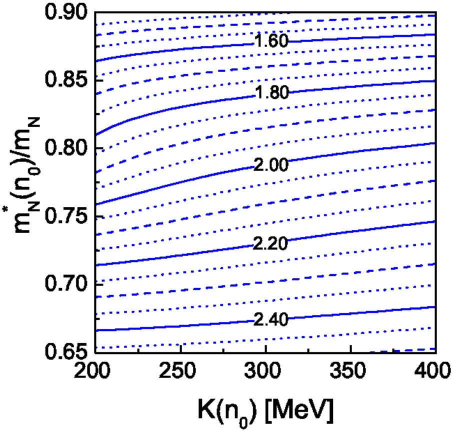
$$\mathcal{E} = \mathcal{E}_0 + \frac{K}{18}\epsilon^2 - \frac{K^{'}}{162}\epsilon^3 + ... + \beta^2\left(\mathcal{E}_{\rm sym} + \frac{L}{3}\epsilon + \frac{K_{\rm sym}}{18}\epsilon^2 ...\right),$$
$$\epsilon = (n - n_0)/n_0, \quad \beta = [(n_n - n_p)/n_0]_{n_0}$$

$$n_0 = 0.16 \text{ fm}^{-3}, \quad \mathcal{E}_0 = -16 \text{ MeV}, \quad K = 250 \text{ MeV},$$
$$\mathcal{E}_{\rm sym} = 30 \text{ MeV}, \quad m_N^*(n_0)/m_N = 0.8$$

Gives $M_{\rm max} = 1.92\ M_\odot$

Maximum mass for NLW model

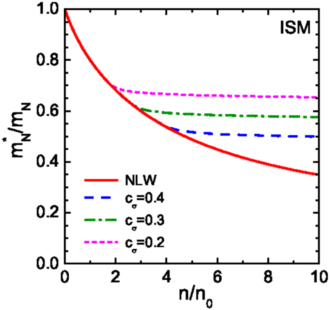
$M_{\max}$ contours for NLW model:



Additional terms $\sim \omega^4, \omega^2\vec{\rho}^2$ for better description of the finite nuclei $\Rightarrow$ maximum mass decrease
[FSUgold Todd-Rutel, Piekariewicz 2005]

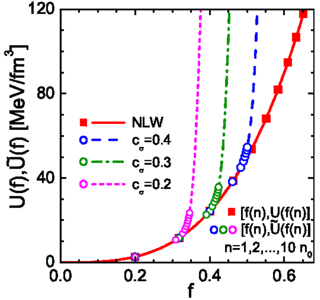
Can we stiffen the EoS by playing with the scalar field potential?

Scalar potential modification



$$\frac{df}{dn} = \frac{2(\partial n_S/\partial n)}{m_N^3 C_\sigma^{-2} + \widetilde{U}''(f)/m_N - 2(\partial n_S/\partial f)}$$

$$\frac{\partial n_S}{\partial n} = \frac{m_N^*}{2\sqrt{p_F^2 + m_N^{*2}}}, \quad -\frac{\partial n_S}{\partial f} = \int_0^{p_F} \frac{m_N p^4 dp/\pi^2}{(p^2 + m_N^{*2})^{3/2}}$$



NLWcut models

[K.A.M., E.E.K. & D.N.V. PRD92 (2015)]

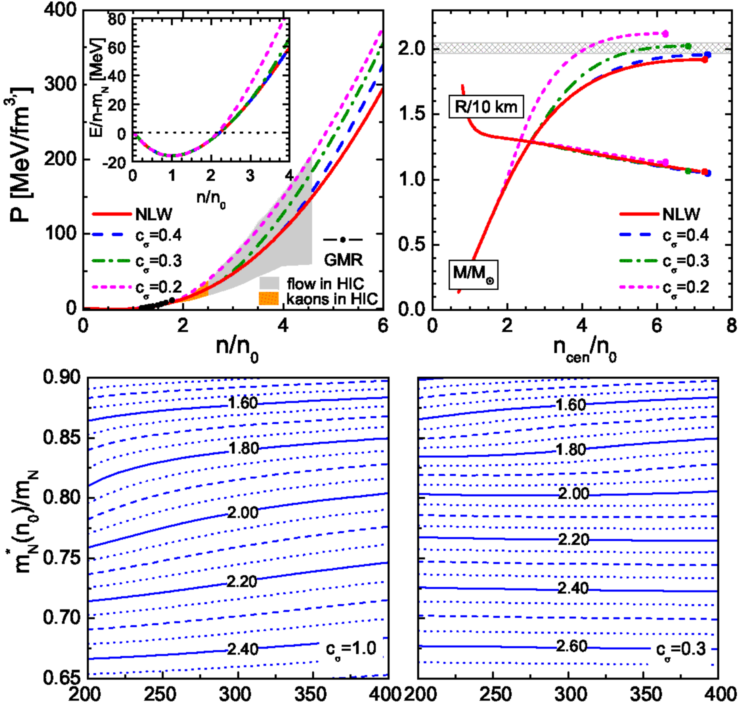
$$U(f) \to \widetilde{U}(f) = U(f) + \Delta U(f)$$

soft core: $\Delta U(f) = \alpha \ln[1 + \exp(\beta(f - f_{\rm s.core}))]$,

hard core: $\Delta U(f) = \alpha[\delta f/(f_{h.core} - f)]^{2\beta}$

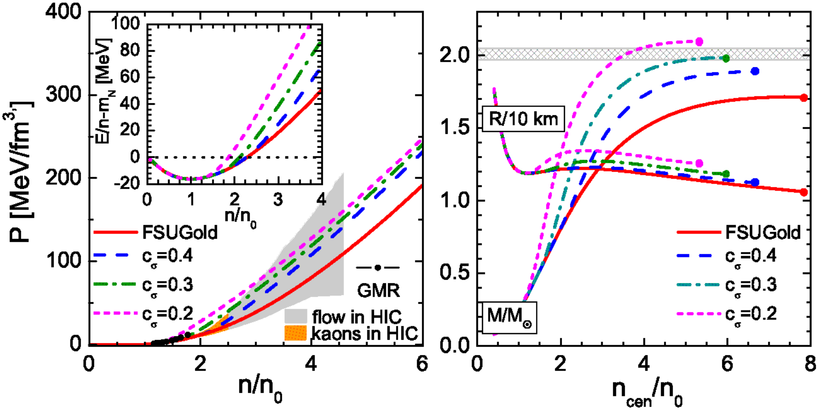
$f_{\rm s.core} = f_0 + c_\sigma(1 - f_0)$

$m_N^*(f) = m_N(1 - f)$



P [MeV/fm^3]
n/n_0
E/n-m_N [MeV]
NLW
c_σ=0.4
c_σ=0.3
c_σ=0.2
GMR
flow in HIC
kaons in HIC
R/10 km
M/M$_\odot$
n_{cen}/n_0
$m^*_N(n_0)/m_N$
1.60
1.80
2.00
2.20
2.40
2.60
c_σ=1.0
c_σ=0.3

Application to the FSUgold model



Can be applied to all RMF models

Generalized RMF model

E. E. Kolomeitsev and D. N. Voskresensky NPA 759 (2005) 373

- Model with the in-medium change of masses and coupling constants of all hadrons.
- Common decrease of hadron masses:

$$\frac{m_N^*}{m_N} \simeq \frac{m_\sigma^*}{m_\sigma} \simeq \frac{m_\omega^*}{m_\omega} \simeq \frac{m_\rho^*}{m_\rho}$$

- Sigma-field dependent masses and coupling constants

Model labelled KVOR was succesfully tested in Klaehn at al., PRC74 (2006) 035802.

Aim: Construct a better parametrisation (MKVOR) which satisfies new constraints on the nuclear EoS and incorporate more baryon species

Generalized relativistic mean-field model

E. E. Kolomeitsev, D.N. Voskresensky, NPA 759 (2005)
K. A. M, E. E. K. and D. N. V., Phys. Lett. B 748 (2015),
E. E. K., K. A. M. and D. N. V., arXiv:1610.09746, to be published in NPA 961 (2017)

$$\mathcal{L} = \mathcal{L}_{\rm bar} + \mathcal{L}_{\rm mes} + \mathcal{L}_l,$$

$$\mathcal{L}_{\rm bar} = \sum_{i=b\cup r} (\bar{\Psi}_i \left(iD_\mu^{(i)} \gamma^\mu - m_i \Phi_i(\sigma) \right) \Psi_i,$$

$$D_\mu^{(i)} = \partial_\mu + i g_{\omega i} \chi_{\omega i}(\sigma) \omega_\mu + i g_{\rho i} \chi_{\rho i}(\sigma) \vec{t} \vec{\rho}_\mu + i g_{\phi i} \chi_{\phi i}(\sigma) \phi_\mu,$$

$$\{b\} = (N, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}, \Delta^-, \Delta^0, \Delta^+, \Delta^{++})$$

$$\mathcal{L}_{\rm mes} = \frac{\partial_\mu \sigma \partial^\mu \sigma}{2} - \frac{m_\sigma^2 \Phi_\sigma^2(\sigma) \sigma^2}{2} - U(\sigma) +$$

$$+ \frac{m_\omega^2 \Phi_\omega^2(\sigma) \omega_\mu \omega^\mu}{2} - \frac{\omega_{\mu\nu} \omega^{\mu\nu}}{4} + \frac{m_\rho^2 \Phi_\rho^2(\sigma) \vec{\rho}_\mu \vec{\rho}^\mu}{2} - \frac{\rho_{\mu\nu} \rho^{\mu\nu}}{4} +$$

$$+ \frac{m_\phi^2 \Phi_\phi^2(\sigma) \phi_\mu \phi^\mu}{2} - \frac{\phi_{\mu\nu} \phi^{\mu\nu}}{4},$$

$$\omega_{\mu\nu} = \partial_\nu \omega_\mu - \partial_\mu \omega_\nu, \quad \vec{\rho}_{\mu\nu} = \partial_\nu \vec{\rho}_\mu - \partial_\mu \vec{\rho}_\nu,$$

$$\phi_{\mu\nu} = \partial_\nu \phi_\mu - \partial_\mu \phi_\nu,$$

$$\mathcal{L}_l = \sum_l \bar{\psi}_l (i \partial_\mu \gamma^\mu - m_l) \psi_l, \quad \{l\} = (e,\, \mu).$$

Energy density functional

$$E = \frac{m_N^4 f^2}{2C_\sigma^2}\eta_\sigma(f) + U(f) + \frac{C_\omega^2}{2m_N^2\eta_\omega(f)}\Big(\sum_b x_{\omega b} n_b\Big)^2 +$$

$$+\frac{C_\rho^2}{2m_N^2\eta_\rho(f)}\Big(\sum_b x_{\rho b} t_{3b} n_b\Big)^2 + \frac{C_\omega^2}{2m_N^2\eta_\phi(f)}\frac{m_\omega^2}{m_\phi^2}\Big(\sum_H x_{\phi H} n_H\Big)^2 +$$

$$+\sum_b \int\limits_0^{p_{\mathrm{F},b}} \frac{p^2\, dp}{\pi^2}\sqrt{p^2 + m_b^2 \Phi_b^2(f)} + E_l,$$

$$E_l = \sum_{l=e,\mu} \int\limits_0^{p_{\mathrm{F},l}} \frac{p^2 dp}{\pi^2}\sqrt{p^2 + m_l^2}\,, \quad C_i = \frac{g_{iN} m_N}{m_i}, \quad i = \sigma, \omega, \rho.$$

Scaling functions

In the homogeneous medium $\eta_M = \Phi_M^2(f)/\chi_{Mb}^2(f)\,,$

$\Phi_N(f) = \Phi_m(f) = 1 - f,$ universal scaling of hadron masses

$\Phi_H(f) = \Phi_N(g_{\sigma H}\chi_{\sigma H}(\sigma)\sigma/m_H) \equiv \Phi_N(x_{\sigma H}\xi_{\sigma H}(f) f m_N/m_H)\,,$

$\xi_{\sigma H}(f) = \chi_{\sigma H}(f)/\chi_{\sigma N}(f).$

Energy density functional

$$E = \frac{m_N^4 f^2}{2C_\sigma^2}\eta_\sigma(f) + U(f) + \frac{C_\omega^2}{2m_N^2\eta_\omega(f)}\Big(\sum_b x_{\omega b} n_b\Big)^2 +$$

$$+\frac{C_\rho^2}{2m_N^2\eta_\rho(f)}\Big(\sum_b x_{\rho b} t_{3b} n_b\Big)^2 + \frac{C_\omega^2}{2m_N^2\eta_\phi(f)}\frac{m_\omega^2}{m_\phi^2}\Big(\sum_H x_{\phi H} n_H\Big)^2 +$$

$$+\sum_b \int_0^{p_{\mathrm{F},b}} \frac{p^2\,dp}{\pi^2}\sqrt{p^2 + m_b^2 \Phi_b^2(f)} + E_l,$$

$$E_l = \sum_{l=e,\mu}\int_0^{p_{\mathrm{F},l}} \frac{p^2 dp}{\pi^2}\sqrt{p^2 + m_l^2}\,, \quad C_i = \frac{g_{iN} m_N}{m_i}, \quad i = \sigma, \omega, \rho.$$

⊕ Equation of motion: $\dfrac{\partial E}{\partial f} = 0 \Rightarrow f(\{n_B\})$.

Energy density functional

$$E = \frac{m_N^4 f^2}{2C_\sigma^2}\eta_\sigma(f) + U(f) + \frac{C_\omega^2}{2m_N^2\eta_\omega(f)}\Big(\sum_b x_{\omega b} n_b\Big)^2 +$$

$$+\frac{C_\rho^2}{2m_N^2\eta_\rho(f)}\Big(\sum_b x_{\rho b} t_{3b} n_b\Big)^2 + \frac{C_\omega^2}{2m_N^2\eta_\phi(f)}\frac{m_\omega^2}{m_\phi^2}\Big(\sum_H x_{\phi H} n_H\Big)^2 +$$

$$+\sum_b \int_0^{p_{\mathrm{F},b}} \frac{p^2\,dp}{\pi^2}\sqrt{p^2 + m_b^2\Phi_b^2(f)} + E_l,$$

$$E_l = \sum_{l=e,\mu} \int_0^{p_{\mathrm{F},l}} \frac{p^2 dp}{\pi^2}\sqrt{p^2+m_l^2}\,, \quad C_i = \frac{g_{iN} m_N}{m_i}, \quad i = \sigma, \omega, \rho.$$

- ⨁ Equation of motion: $\dfrac{\partial E}{\partial f} = 0 \Rightarrow f(\{n_B\})$.
- ⨁ Beta-equilibrium condition: $\mu_n = \mu_B - q_B\mu_e \Rightarrow \{n_B(n)\}$.

Energy density functional

$$E = \frac{m_N^4 f^2}{2C_\sigma^2}\eta_\sigma(f) + U(f) + \frac{C_\omega^2}{2m_N^2\eta_\omega(f)}\Big(\sum_b x_{\omega b} n_b\Big)^2 +$$

$$+\frac{C_\rho^2}{2m_N^2\eta_\rho(f)}\Big(\sum_b x_{\rho b} t_{3b} n_b\Big)^2 + \frac{C_\omega^2}{2m_N^2\eta_\phi(f)}\frac{m_\omega^2}{m_\phi^2}\Big(\sum_H x_{\phi H} n_H\Big)^2 +$$

$$+\sum_b \int_0^{p_{\mathrm{F},b}} \frac{p^2\,dp}{\pi^2}\sqrt{p^2 + m_b^2\Phi_b^2(f)} + E_l,$$

$$E_l = \sum_{l=e,\mu}\int_0^{p_{\mathrm{F},l}} \frac{p^2 dp}{\pi^2}\sqrt{p^2+m_l^2}\,, \quad C_i = \frac{g_{iN} m_N}{m_i}, \quad i = \sigma,\omega,\rho.$$

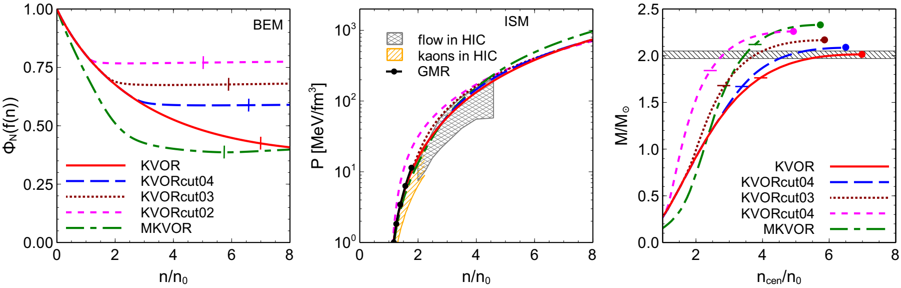
- $\oplus$ Equation of motion: $\dfrac{\partial E}{\partial f} = 0 \Rightarrow f(\{n_B\})$.
- $\oplus$ Beta-equilibrium condition: $\mu_n = \mu_B - q_B\mu_e \Rightarrow \{n_B(n)\}$.

Choice $\eta_i = 1$, $\Phi_N(f) = 1 - f$ reproduces the standard Walecka model

KVORcut models

The same procedure can be applied to the scaling functions $\eta_\omega(f)$:

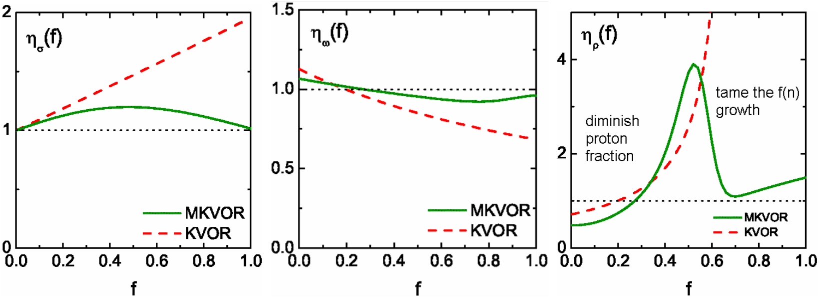
$$\eta_\omega(f)^{\mathrm{KVOR}}(f) \to \eta_\omega^{\mathrm{KVOR}}(f) + \frac{a_\omega}{2}[1+\tanh(b_\omega(f - f_{\mathrm{cut},\omega}))]$$



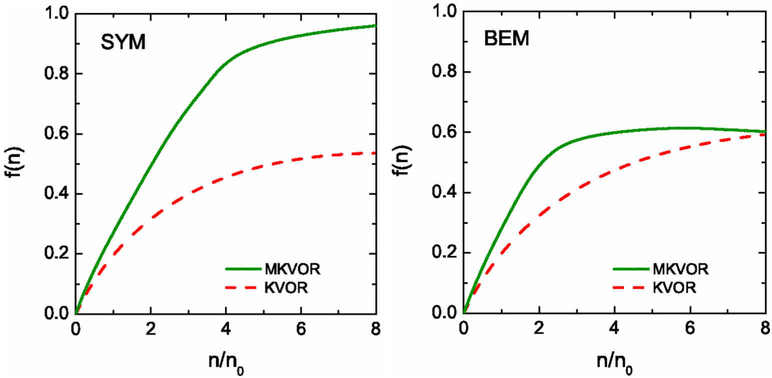
MKVOR model

The procedure can be applied to the isospin-asymmetric part ($\eta_\rho(f)$)
Does not change symmetric matter EoS, but stiffens the asymmetric part

Choice of the scaling functions

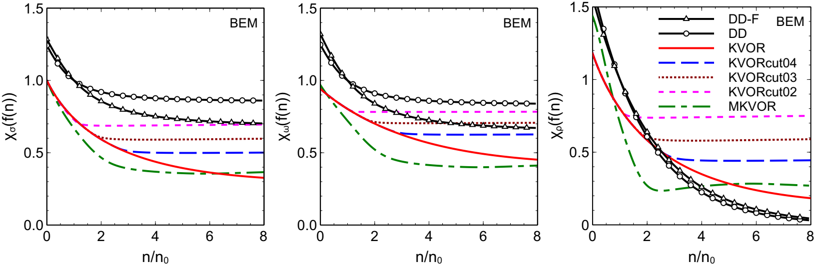


Density dependence of the mean scalar field



$\Phi_N(f) = 1 - f \Rightarrow$ effective mass decreases, then saturates at a constant value

Comparison with density-dependent couplings

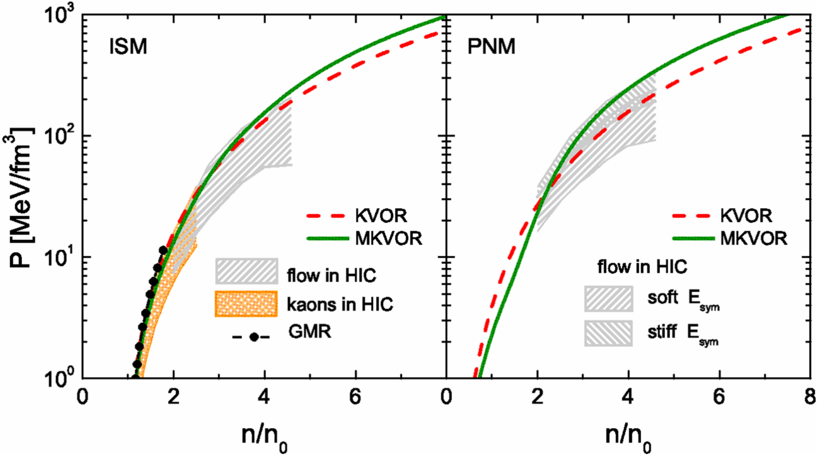


Constraints from HIC

Constraint on the pressure

- from the analyses of transverse and elliptic flows
- from the analyses of kaon production

  [W. G. Lynch et al. Prog. Part. Nucl. Phys. 62 (2009)]
- Cannot be passed by a typical EoS which yields a large maximum NS mass



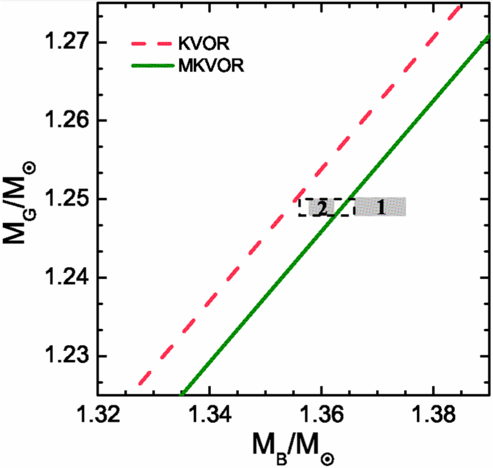
Gravitational vs baryon mass constraint

Pulsar J0737-3039B: (*May be*) electron capture in a O-Ne-Mg white dwarf [P. Podsiadlowski et al. (2005)]

$M_G = 1.249 \pm 0.001 M_\odot, \quad M_B = 1.366 - 1.375 M_\odot$

1, 2 – assuming no mass loss

dashed rectangle – assuming 1% mass loss



Inclusion of hyperons

Hyperons are included with the vector coupling constants from SU(6) symmetry:

$$g_{\omega\Lambda} = g_{\omega\Sigma} = 2g_{\omega\Xi} = \frac{2}{3} g_{\omega N}, \; g_{\rho\Sigma} = 2g_{\rho\Xi} = 2g_{\rho N},$$
$$2g_{\phi\Lambda} = 2g_{\phi\Sigma} = g_{\phi\Xi} = \frac{2\sqrt{2}}{\sqrt{3}} g_{\omega N}.$$

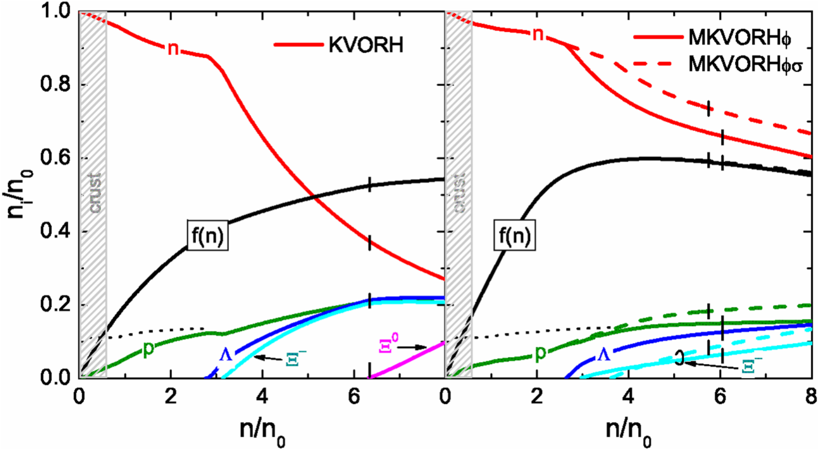
Scalar coupling constants are deduced from hyperon binding energies at $n = n_0$:

$$\mathcal{E}^{H}_{\rm bind}(n_0) = \frac{C_\omega^2}{m_N^2} x_{\omega H} n_0 - x_{\sigma H}\, \xi_{\sigma H}(\bar{f}_0)\, [m_N - m_N^*(n_0)],$$
$$\mathcal{E}^{\Lambda}_{\rm bind} = -28 \text{ MeV}, \quad \mathcal{E}^{\Sigma}_{\rm bind} = +30 \text{ MeV}, \quad \mathcal{E}^{\Xi}_{\rm bind} = -18 \text{ MeV}$$

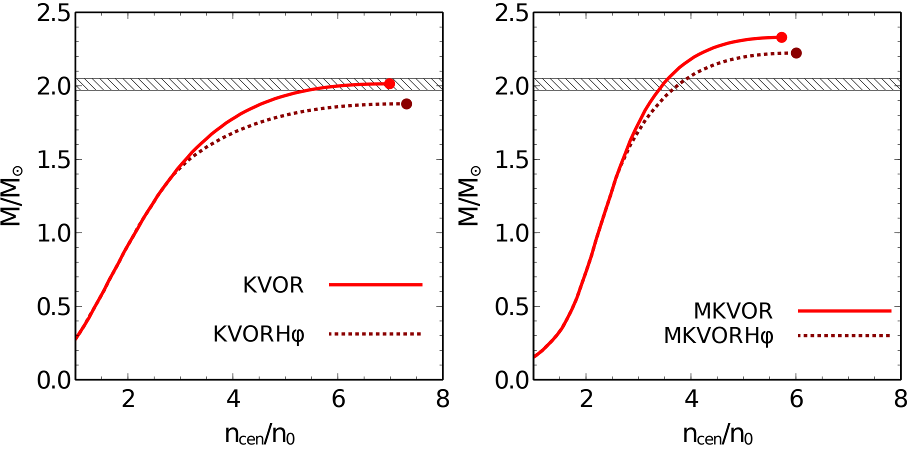
We assume the ϕ-meson universal *mass scaling*, but with *vacuum* coupling constants $(H\phi)$:

$\Phi_\phi(f) = 1 - f, \; \chi_\phi(f) = 1, \; \eta_\phi(f) = (1-f)^2.$

Hyperon concentrations

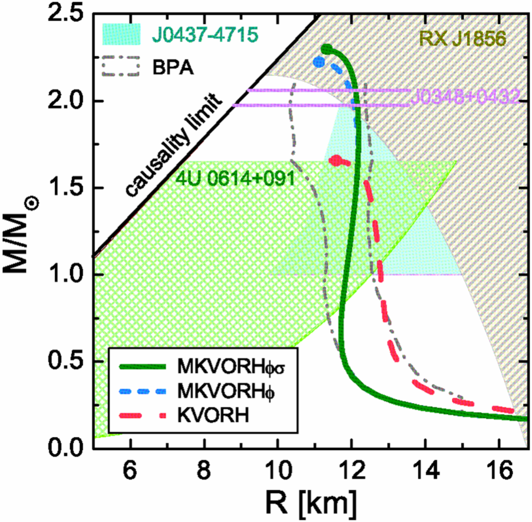


Maximum masses with strangeness



Maximum mass constraint

- The largest precisely measured NS mass $M[PSRJ0348+0432] = 2.01 \pm 0.04 M_\odot$ (Antoniadis et al., 2012).
- 4U 0614+091: QPO; RX J1856: isolated NS thermal radiation



Inclusion of Δ-isobars

Coupling constants

Coupling constants with vector mesons equal to nucleons' in the SU(6) symmetry assumption (quark counting):

$$g_{\omega\Delta} = g_{\omega N}\,, \quad g_{\rho\Delta} = g_{\rho N}\,, \quad g_{\phi\Delta} = 0$$

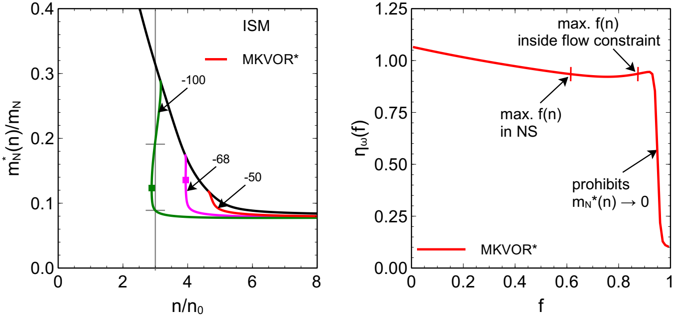
Δ coupling with the scalar meson is deduced from the Δ potential at the saturation density:

$$U_\Delta(n_0) = -x_{\sigma\Delta} m_N\, f_0 + x_{\omega\Delta} C_\omega^2 (n_0/m_N^2).$$

The estimate from the experimental data is $U_\Delta(n_0) \gtrsim -(30\text{—}50)\,\mathrm{MeV}$. In this work we explore $-50\,\mathrm{MeV} > U_\Delta > -100\,\mathrm{MeV}$ to estimate the maximum effect of Δ on the NS properties.

ISM: MKVOR* model

The fast decrease of the nucleon effective mass in MKVOR model in the ISM leads to early Δ appearance and at some point $m_N^* \to 0$.
Can be cured by introducing a sharp decrease into $\eta_\omega(f)$ at $f = f^*$. All the results for BEM and for ISM (for $n \le 5\, n_0$) remain unchanged.

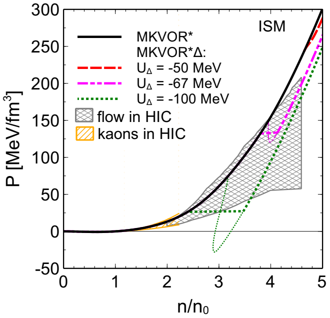


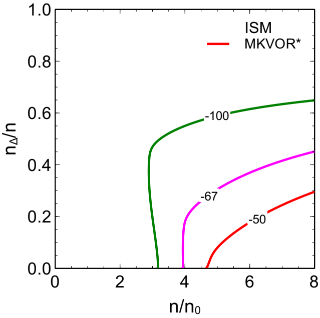
For $U_\Delta < -68$ MeV – multiple solutions for equilibrium n_Δ

$\Rightarrow$ $1^{\rm st}$ order phase transition!

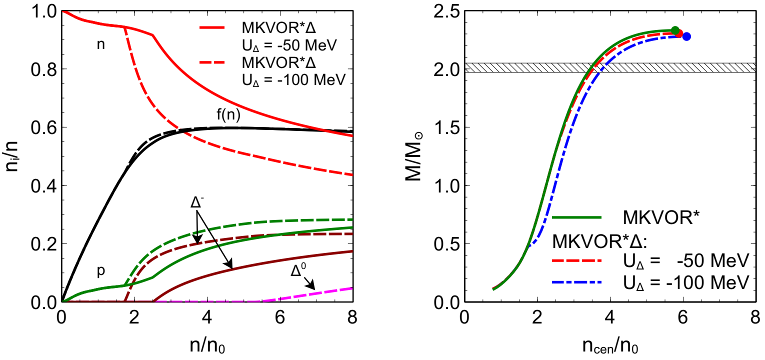
ISM: Δ concentrations and the pressure

For $U_\Delta < -65$ MeV the pressure curve lies within the constraint.



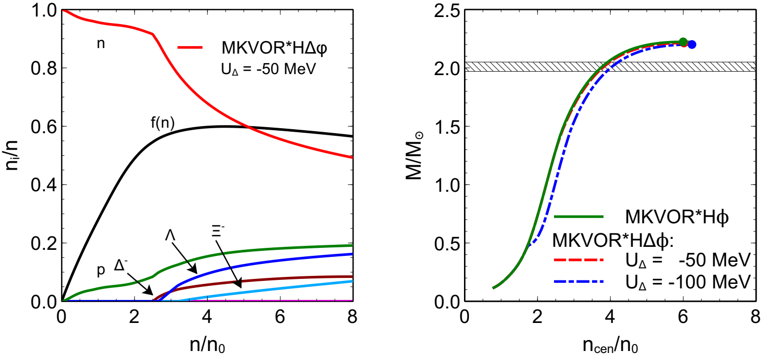


BEM: Δ and nucleons



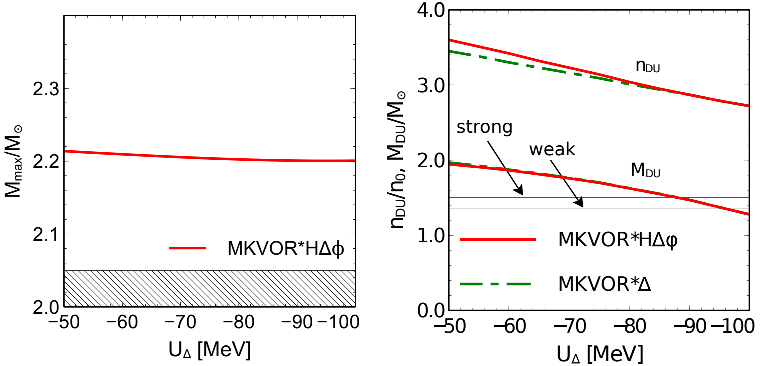
Δ appear at $1.7 - 2.5\, n_0$, but the maximum mass decrease is less than $0.06\, M_\odot$

BEM: H$\Delta\phi$



Hyperons suppress Δ concentrations

BEM: U_Δ dependence

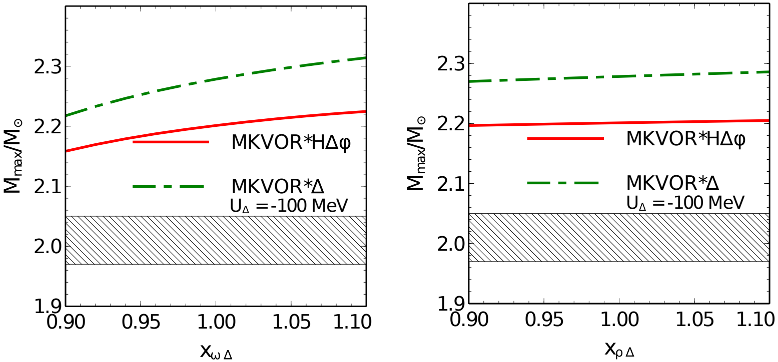


The DU constraint is passed for:

$U \gtrsim -88$ MeV – "strong" constraint $M_{DU} > 1.5\,M_\odot$

$U \gtrsim -96$ MeV – "weak" constraint $M_{DU} > 1.35\,M_\odot$

BEM: Additional parameters variation



Almost insensitive to $x_{\rho\Delta}$

Conclusion

- We have developed a simple procedure of stiffening an arbitrary RMF EoS, which can be applied in scalar (NLWcut), vector (KVORcut) and isovector (MKVOR) sectors.
- The RMF model with scaled hadron masses and couplings is flexible enough to satisfy many astrophysical constraints, constraints from HIC and miscroscopic calculations and resolve the hyperon puzzle.
- In the ISM Δs can appear by a I$^{\rm st}$ order phase transition, if U_Δ is sufficiently attractive
- Δ isobars do not spoil the description of 2 $M_\odot$ neutron star.

Further development

- Meson (ρ^- - done, π, K) condensation
- Calculation of the cooling
- Extension to the finite temperatures

III. String-flip model of deconfined quark matter: compact star perspective

K.A. Maslov

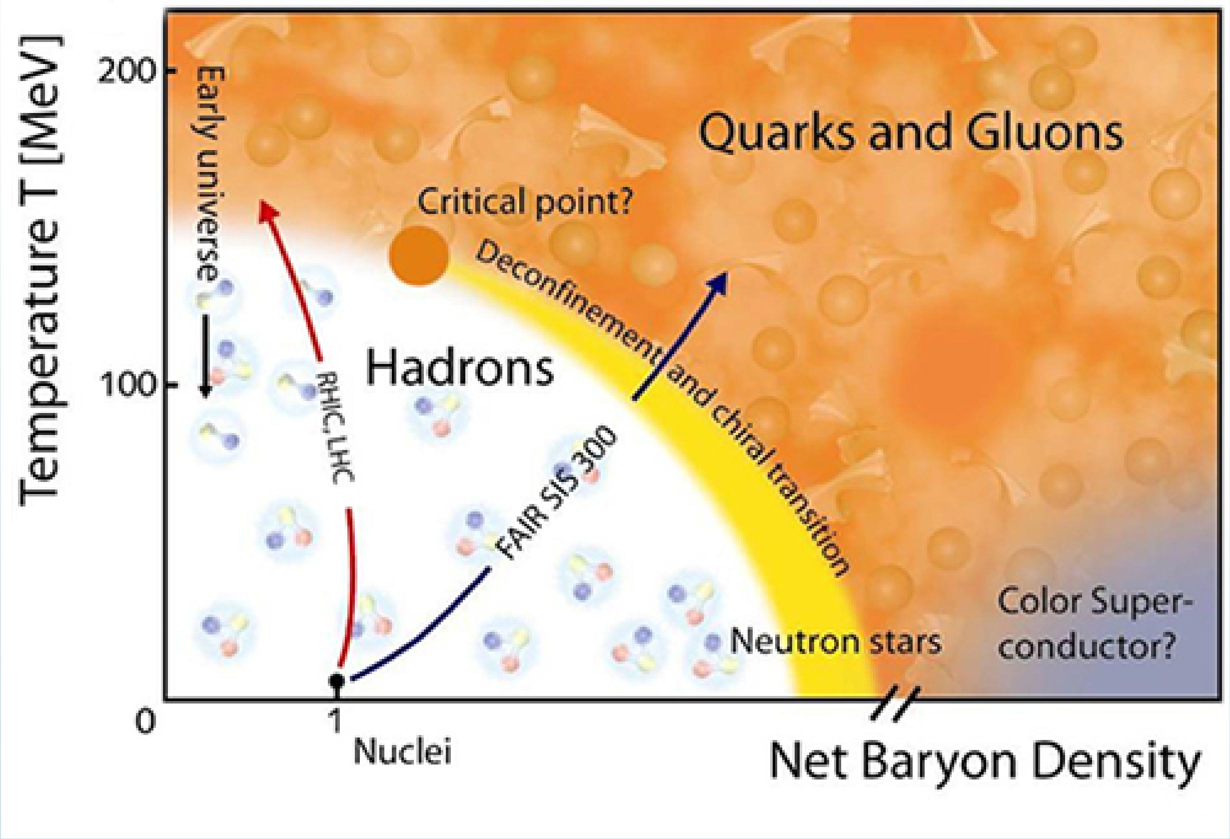
Based on

- N.-U. Bastian, M.A.R. Kaltenborn, D. Blaschke
  Phys.Rev. D96 (2017) no.5, 056024
- A. Ayriyan, N.-U. Bastian, D. Blaschke, H. Grigoryan, K.A.M., D.N. Voskresensky arXiv:1711.03926

QCD phase diagram

Strongly interacting hot and/or dense matter

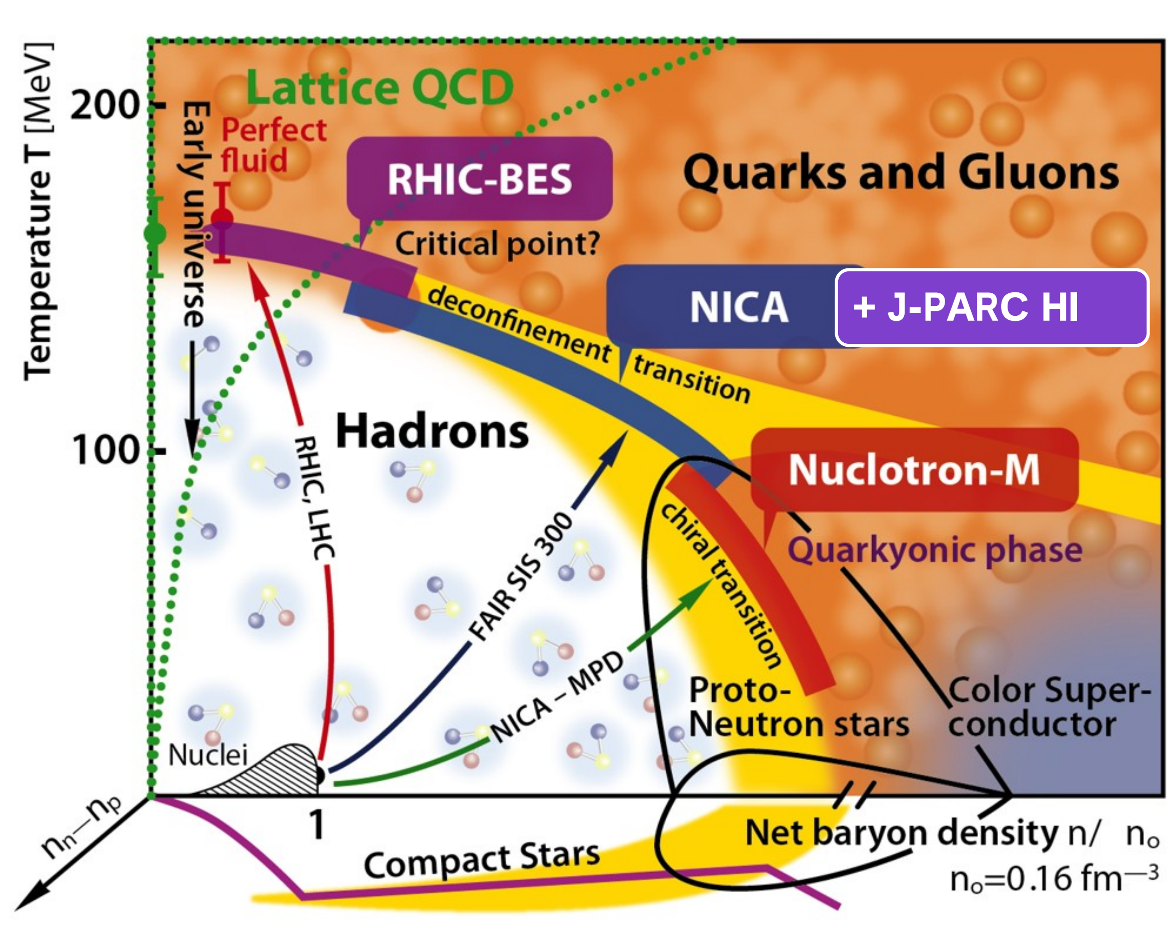
- Neutron stars
- Heavy-ion collisions
- Critical endpoint?



QCD phase diagram (updated)

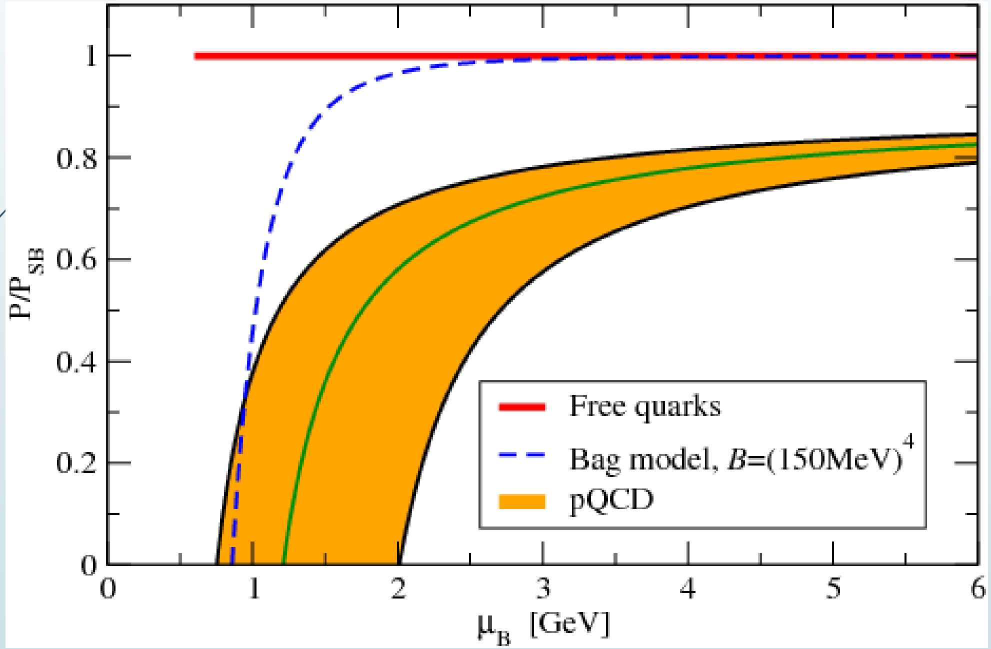
Rich phase structure

- Nucleonic liquid/gas phase transition (PT)
- Deconfinement to ideal (?) quark gas
- Chiral (χ) PT
- Color superconductivity
- Two conserving charges + e/m interaction: “pasta” structures



Quark matter EoS

Large densities $n \sim 100\, n_0$: perturbative QCD



Fraga, Kurkela, Vuorinen
Astrophys.J. 781 (2014) no.2, L25

Lower densities:

Non-perturbative methods & Modelling

- Build an effective theory (model), which represents main features of QCD:
  - Symmetries & symmetry breaking
  - Chiral condensates
  - Confinement

Effective relativistic models:

  - Nambu - Jona-Lasinio
  - Current version of SFM
  - …

Confinement potential approach

- Non-relativistic two-body confinement potential can be used to model hadronic states

$$V(\{\vec{r}_i\}) = \sum_{\{i<j\}} U^{\mathrm{conf}}(\vec{r}_i - \vec{r}_j)$$

E.g. $U^{\mathrm{conf}}(r) = \frac{m\omega^2 r^2}{2}$: exact solution of 2- and 3-body problem!

Successful description of hadron mass spectra, magnetic properties, deuteron form-factor, etc.

- Diverges at large distances - not suitable for many-body systems:
  - Hadrons are colorless, but residual forces aren't zero (like electric dipolar field):

QCD van der Waals forces growing with distance ⇒ infinite energy for infinite matter

Forces are spurious and need to be removed

Better way to build many-body potential?

String-flip model

Lenz et al. Annals of Physics 170 (1986)

- Saturation of confinement forces
  - When colorless clusters are separated, confine only nearest neighbors
- Simple way to model it in N-body colorless system

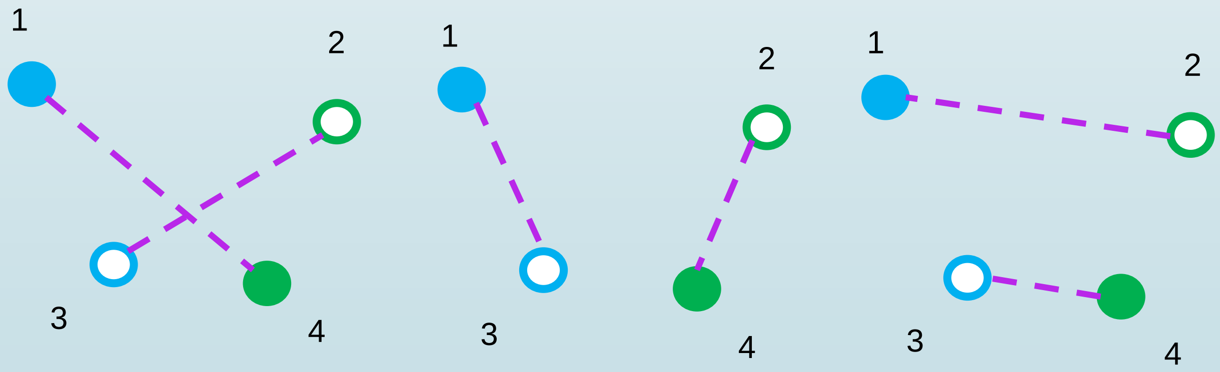
Define $V(\vec{r}_1,\ldots,\vec{r}_N;\text{string configuration}) = \sum_{\text{strings}} U^{\text{conf}}(\vec{r}_{ij})$

Then use the many-body potential

$$V(\vec{r}_1,\ldots,\vec{r}_N) = \min_{\text{string conf.}} V(\vec{r}_1,\ldots,\vec{r}_N;\text{string configuration})$$

E.g. for 4 particles

$$V(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4) = \min\Bigg(\qquad \Bigg)$$



Strings are allowed to ***flip*** from one configuration to another during the evolution (time, density change, ...)

String-flip model: estimates

- This formulation allows for statistical treatment

Details and exact formulas in G. Röpke, D. Blaschke, H. Schulz Phys. Rev. D 34 (1986) 11

- Distribution of lengths of the strings ~ probability $c(r)$ for a quark at the distance r to be the nearest neighbor to a quark at $r = 0$.

Solve $n(r) = n(r)c(r) + \int_{r'<r} d^3 r' \rho_2(\vec{r} - \vec{r}')$

For a uniform distribution with particle number density $n = N/\Omega$ (thermodynamic limit $N \to \infty, \Omega \to \infty$)

$$c(r) = \lim_{N\to\infty}\left(1 - \frac{4\pi r^3}{3\Omega}\right)^N = \lim_{N\to\infty}\left(1 - \frac{4\pi r^3 n}{3\mathrm{N}}\right)^N = e^{-\frac{4\pi r^3}{3}n}$$

Interaction is screened for high densities with such a scale factor

Three-body clusters: more complicated equations, but still

mean squared distance between interacting clusters is $\langle r_{12}^2 \rangle \sim n_B^{1/3}$

Many-body contribution estimates: quark matter

Model is now applicable for describing hadrons

- What about quark matter? Consider no bound states (not true for real QCD at low energies)
- Energy per particle estimate: $E_{\text{kin}} \sim n_B^{\frac{1}{3}}, \quad E_{\text{int}}^{\text{Hartree}} \sim m\omega^2 \langle r_{12}^2 \rangle \sim m\omega^2 n_B^{-\frac{2}{3}}$ (!)
- Energy per particle diverges for $n_B \to 0$ – effective interaction becomes less screened

Effective model with SFM-type interaction

- Linear confinement potential is suggested by the lattice QCD data

$$V^{\mathrm{conf}}(r) = \frac{\alpha}{r} + \sigma r + \mu + O(\frac{1}{r^3})$$

Leads to the contribution to the energy per particle $\sim n^{-\frac{1}{3}}$

- Phenomenologically can be reproduced via the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathrm{free}} + \mathcal{L}_{\mathrm{int}} = \bar{\psi}\left(i\partial_\mu\gamma^\mu - m\right)\psi - U\left(\bar{q}q, \bar{q}\gamma^0 q\right)$$

To use the quasiparticle picture, expand the potential the expectation values of the densities

$$U\left(\bar{q}q, \bar{q}\gamma^0 q\right) \simeq U(n_S, n_V) + (\bar{q}q - n_S)\Sigma_S + \left(\bar{q}\gamma^0 q - n_V\right)\Sigma_{\mathrm{V}} + \cdots, \text{ where}$$

$$n_S = \langle\bar{q}q\rangle, n_V = \langle\bar{q}\gamma^0 q\rangle$$

$$\Sigma_S = \left.\frac{\delta U\left(\bar{q}q,\bar{q}\gamma^0 q\right)}{\delta(\bar{q}q)}\right|_{\bar{q}q=n_S}, \Sigma_V = \left.\frac{\delta U\left(\bar{q}q,\bar{q}\gamma^0 q\right)}{\delta(\bar{q}\gamma^0 q)}\right|_{\bar{q}\gamma^0 q=n_V}$$

Quasiparticles and interaction

- Introduce the chemical potential into the partition function

$$Z = \int D\bar{q}Dq \exp\left\{\int_0^\beta d\tau \int d^3x \, [\mathcal{L} + \mu\bar{q}\gamma^0 q]\right\}$$

- The resulting equation of state is like for free gas but with effective chemical potential and masses (+ effective interaction energy)

$$P(\mu) = P_{FG}(\mu^*, m^*) + \Theta(n_S, n_V),$$

$$P_{FG} = \sum_{f=u,d} \int_0^{p_{F,f}} \frac{dp}{\pi^2} \frac{p^4}{E_f^*}, \qquad \Theta(n_S, n_V) = U(n_S, n_V) - \Sigma_S n_S - \Sigma_V n_V,$$

$$E_f^* = \sqrt{p^2 + m^{*2}}, \qquad p_{F,f} = \sqrt{\mu_f^{*2} - m^{*2}},$$

$$m^*(n_S, n_V) = m - \Sigma_S(n_S, n_V), \qquad \mu_f^*(n_S, n_V) = \mu_f + \Sigma_V(n_S, n_V)$$

Ensures thermodynamic consistency

Choice of the interaction

$$U(n_S, n_V) = D_0 \Phi(n_V) n^{2/3} + a n_V^2 + \frac{b n_V^4}{1 + c n_V^2}$$

- Modeling of the confinement:

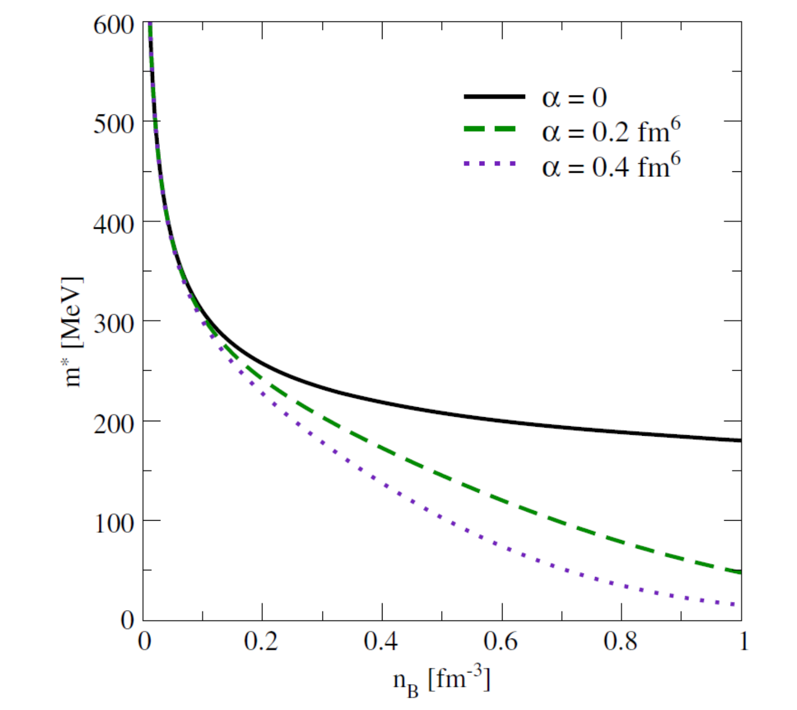
$$D_0 n_S^{1/3}$$

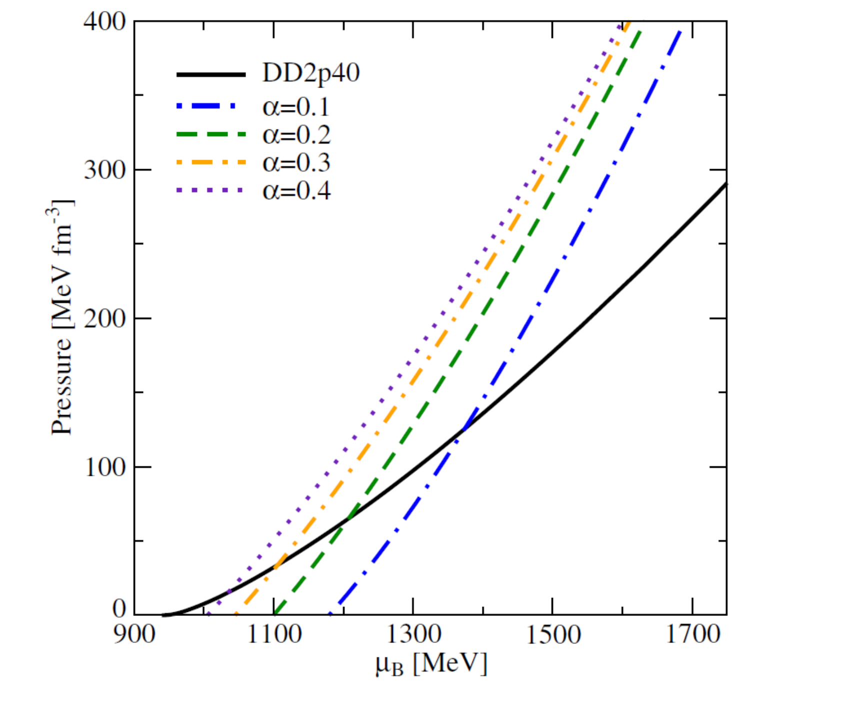
  - Scalar density corresponds to the chiral condensate ($n_S \simeq n_V$ for $n \to 0$)
  - Function $\Phi(n_V) = \exp\left[-\alpha n_V^2\right]$ models the effects of quark excluded volume
- Ordinary vector repulsion term $a n_V^2$

- Higher-order repulsion $b n_V^4$ + multiplier $\frac{1}{1+c n_V^2}$ to restore causality

SFM model

- Variation of the excluded volume parameter

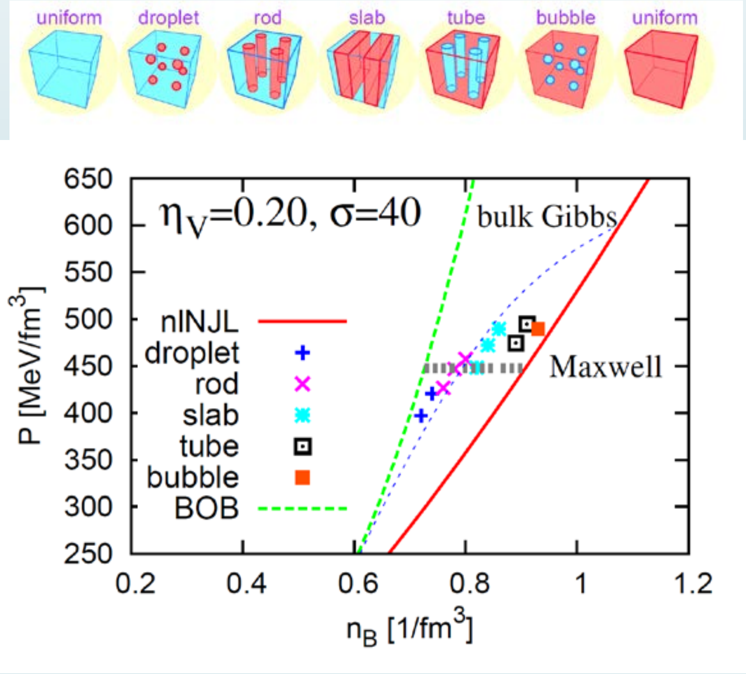




Quark-hadron mixed phase

Pasta calculations

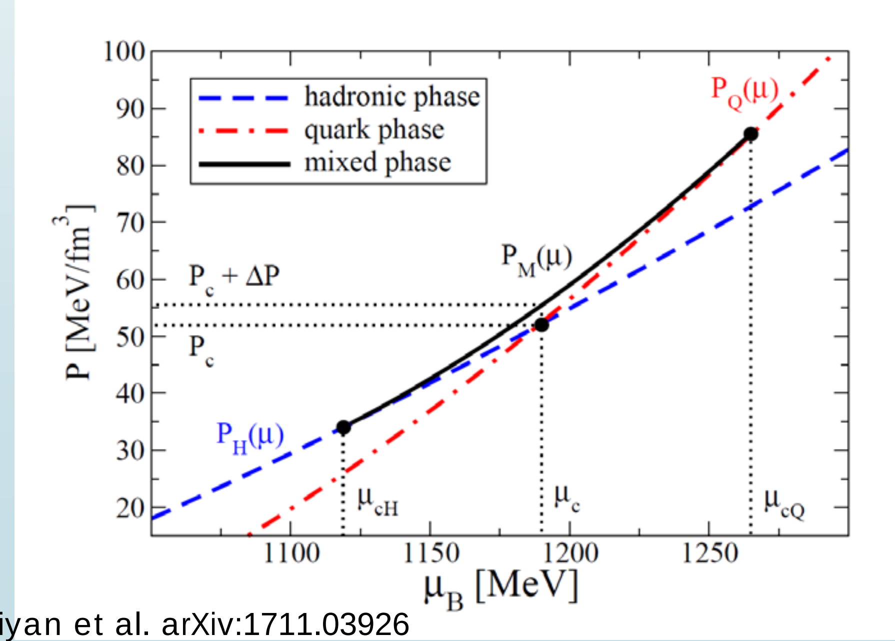
- Effects of the finite-size structures on the EoS
- 1 parameter – surface tension at the hadron-quark interface σ_C



N. Yasutake et.al. PRC89 (2014)

"Mimicking" of the pasta

- Interpolating formula for $P(\mu)$ between hadron and quark phases
- No need of complex calculations
- 1 parameter: pressure excess ΔP

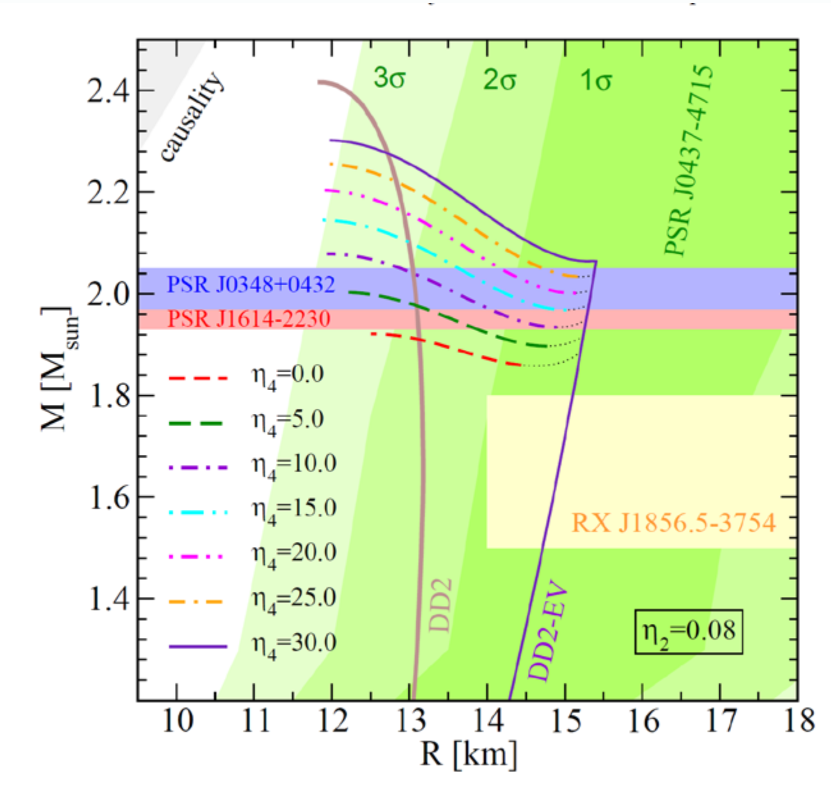


A. Ayriyan et al. arXiv:1711.03926

What can we learn from NSs?

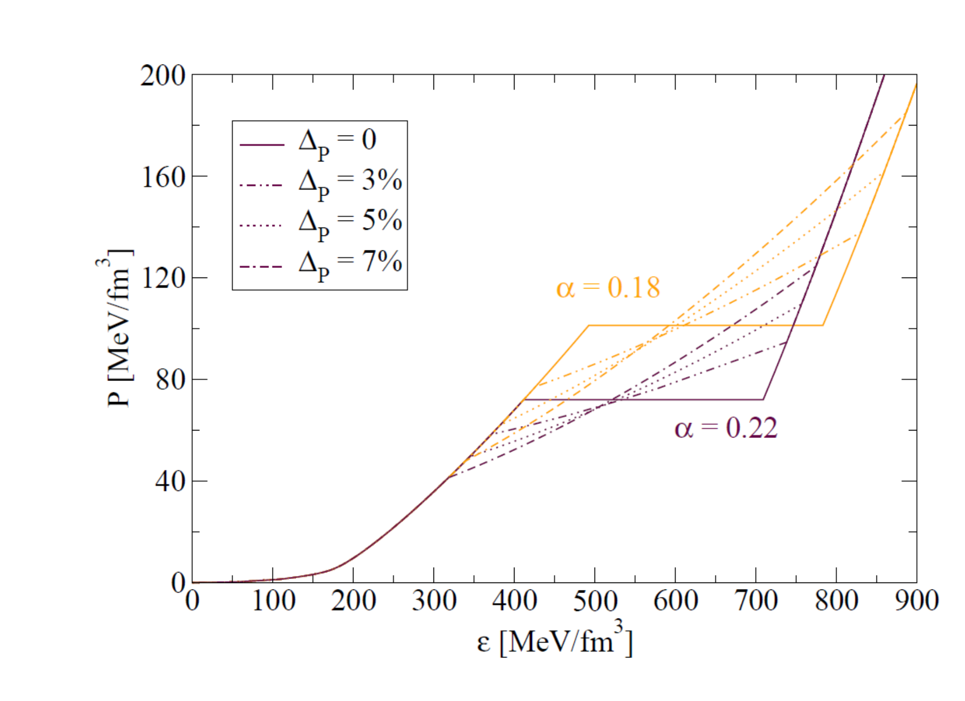
Existence of high-mass twins?

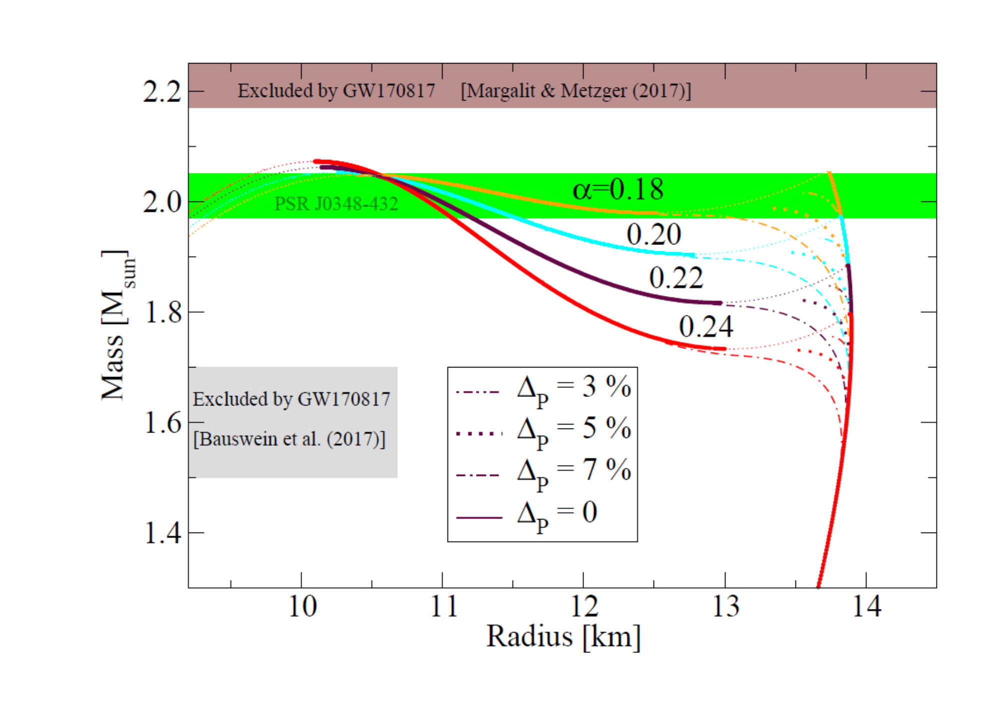
- 1^{st} order phase transition with a large energy jump ⇒ existence of third family of compact stars
- If the 2 stars with same mass and different radii are measured – CEP exists!
- Presence of pasta smoothes the phase transition ⇒ change of phase transition properties



Effect of the mixed phase

A. Ayriyan et al. arXiv:1711.03926





Conclusions

- We have constructed 2 effective relativistic models for hadron and quark matter
- Hadronic model allows for description of modern experimental data for $T = 0$ equation of state (including hyperons and Δs)
- Quark models can simulate confinement and can be adjusted to have twin configurations
- Pasta structures are important for MR-relation and existence of 3rd family; precision calculations needed

Prospective study:

- Inclusion of the isovector term into the SFM model
- Effects of the quark and hadron matter symmetry energy
- Effects of strangeness appearance

Effect on the high-mass twin configuration