Dense quark matter and compact stars

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Publication: PRD 84,105023; 91,105002

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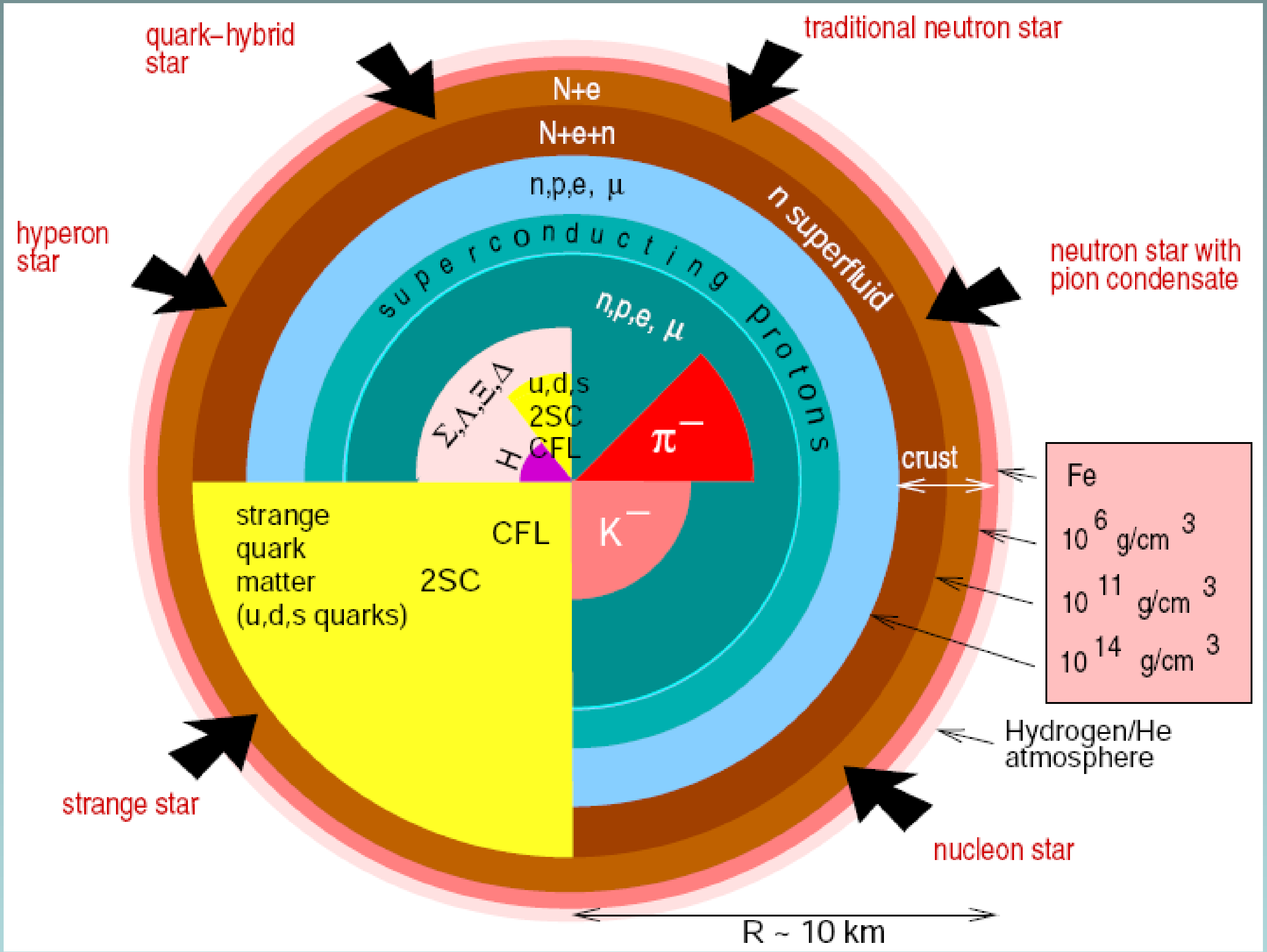
Outline

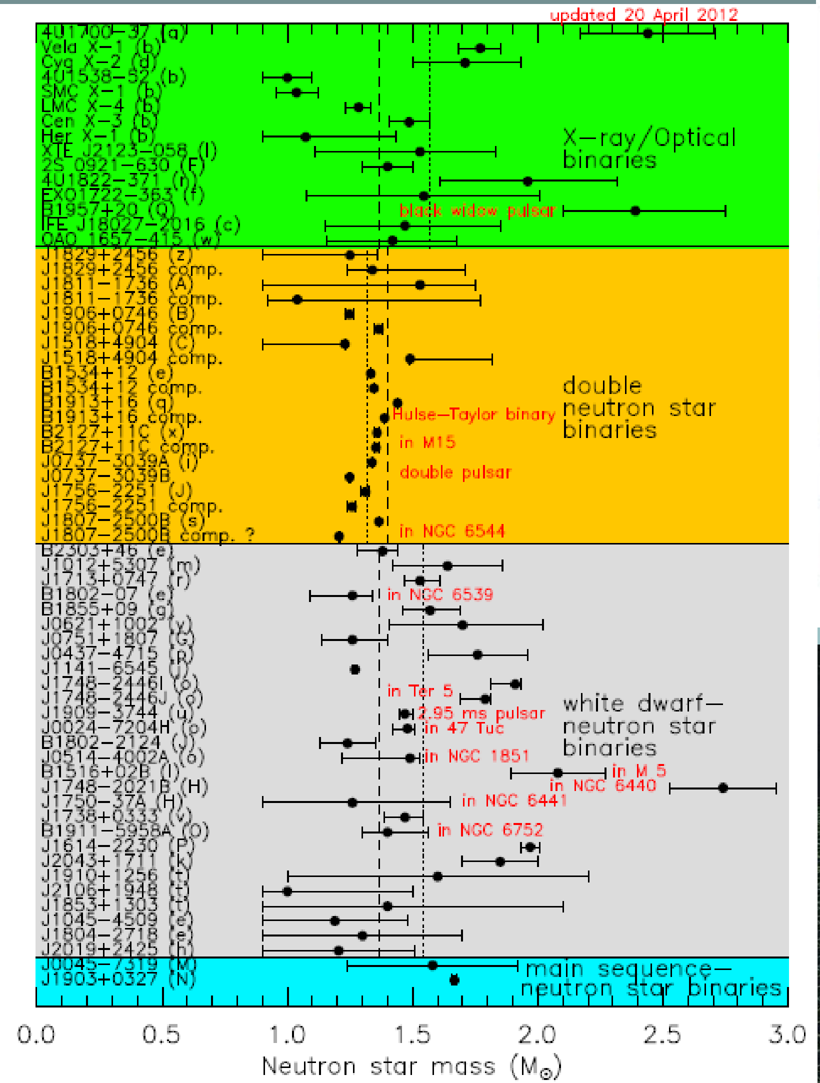
- Compact stars
- EOS of dense matter
- Hybrid quark stars
- Strange quark matter and quark stars
- Summary

Structure of Compact stars

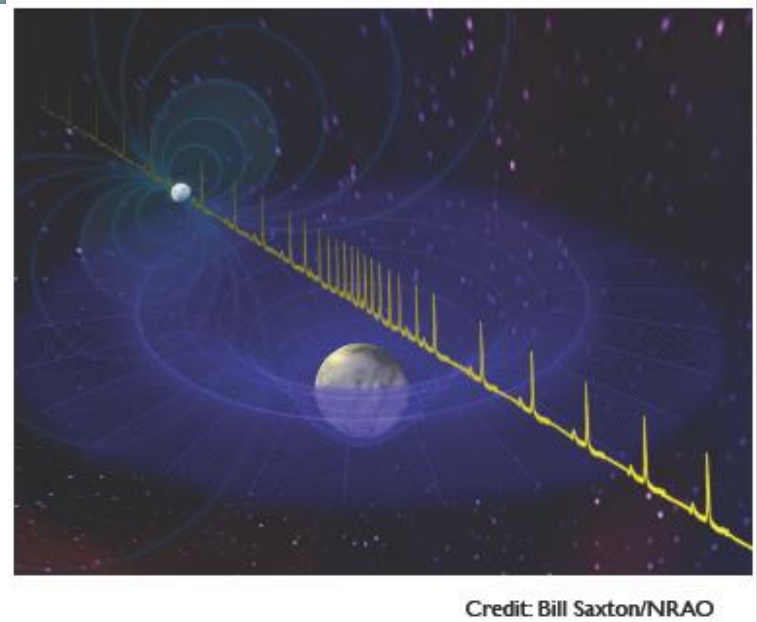


F.Weber, et.al. astro-ph/0705.2708

Observation of Mass

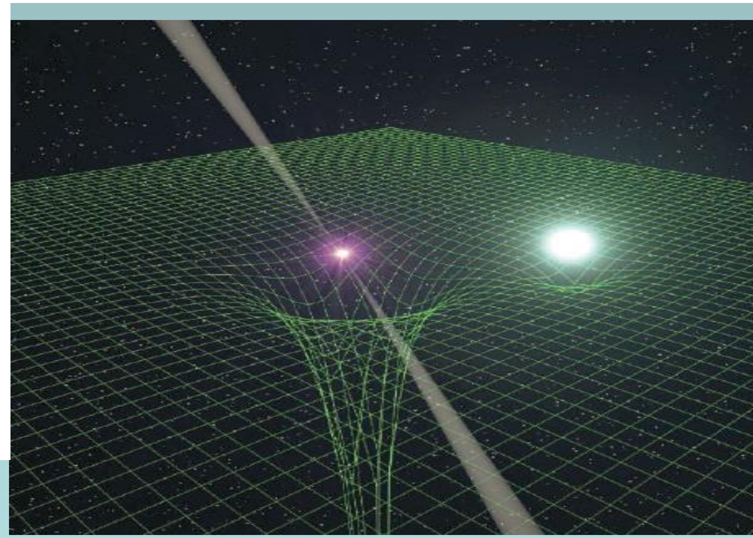


Lattimer 2012



J1614-2230
1.97±0.04
太阳质量

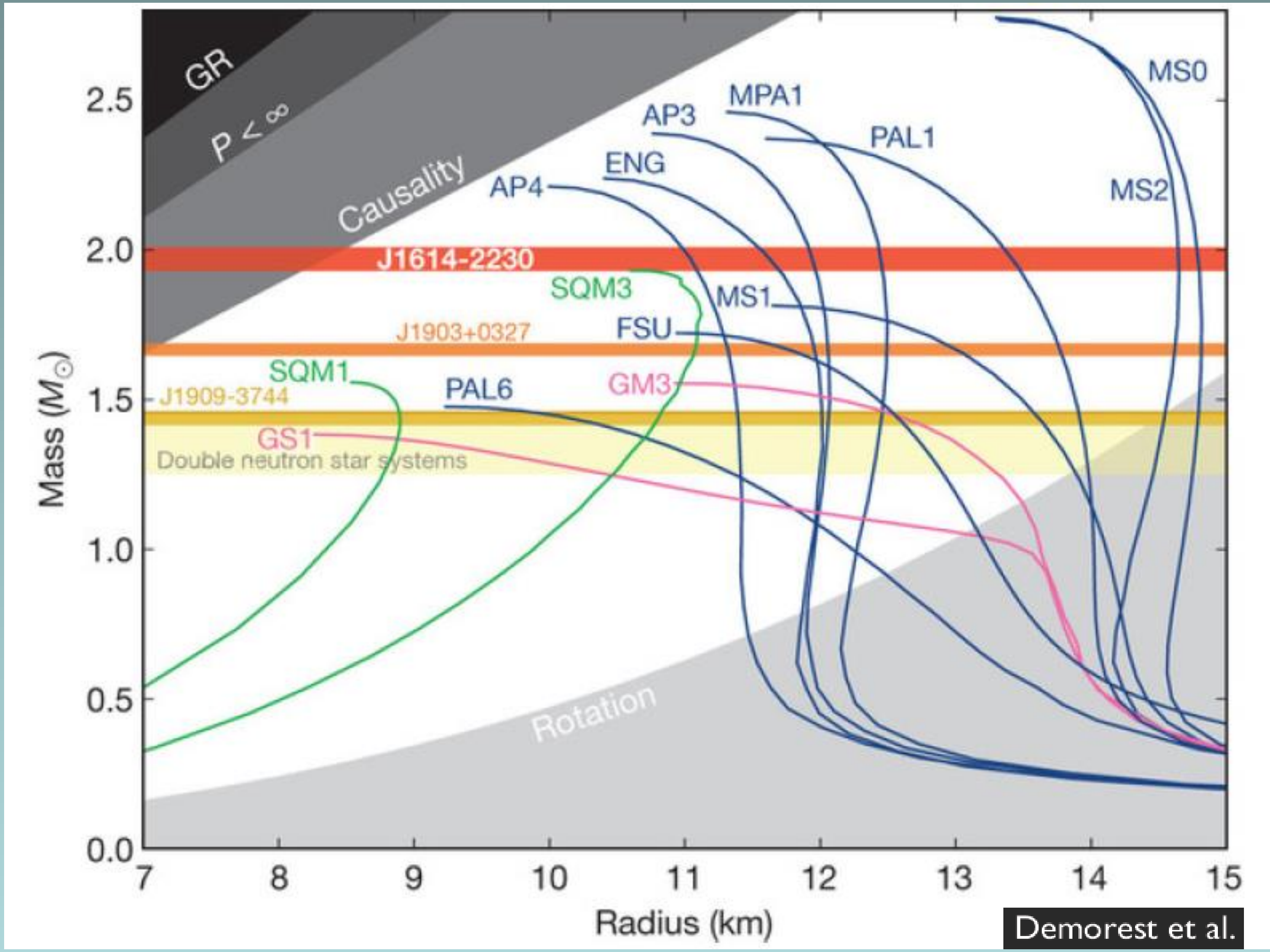
Nature 467, 1081(2010)



J0348+0432
2.01±0.04
太阳质量

Science 340, 1233232(2013)

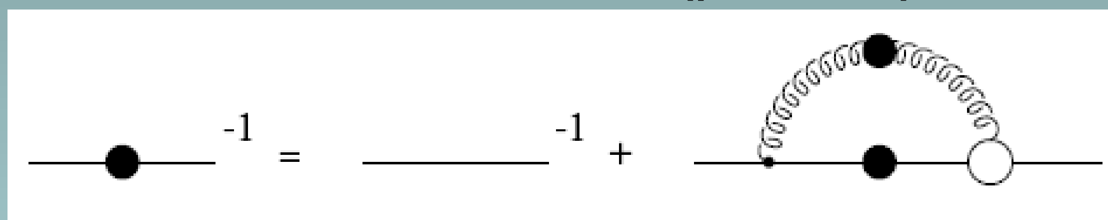
Star mass and EOS

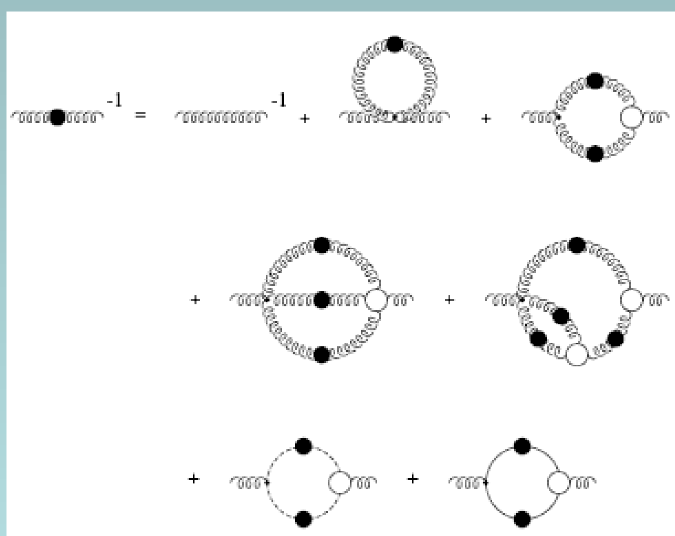


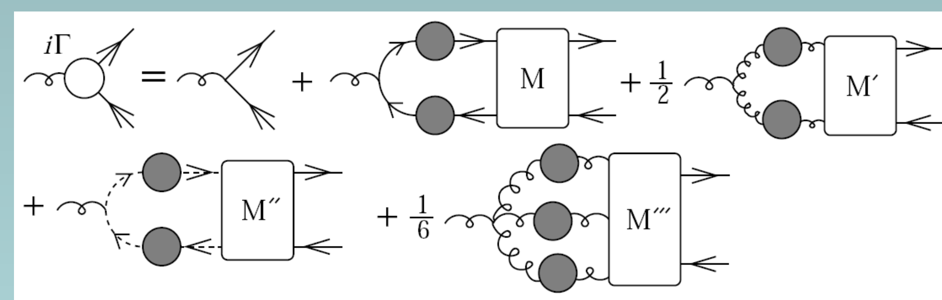
Dyson-Schwinger Equations

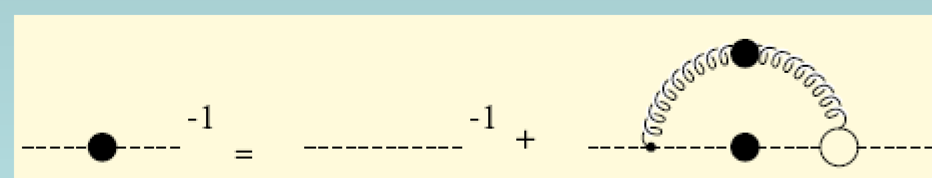
➢ Dyson-Schwinger Eqs (DSE)

- ➢ Equation of motion in QFT
- ➢ Infinite set of coupled equations => Truncation
- ➢ Nonlinear => multi-solution (phases)









Dyson-Schwinger Equations

➢ Model of gluon propagator：

$$g^2 D_{\mu\nu}(k;T,\mu) = (\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \frac{4\pi^2 d}{\omega^6} k^2 e^{-k^2/\omega^2} e^{-\alpha\mu^2/\omega^2}$$

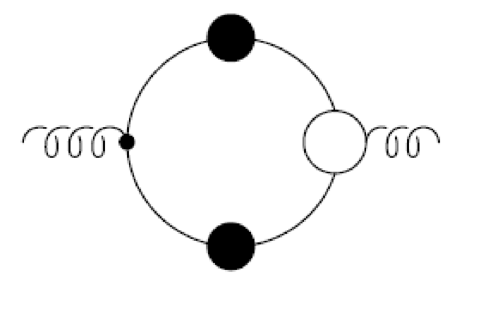
Param： ω, d **fitting，** α **free(adjusted)**

Maris-Tandy Model

$$\frac{\mathcal{G}(t)}{t} = \frac{4\pi^2}{\omega^6} D\, t e^{-t/\omega^2} + \frac{8\pi^2 \gamma_m}{\ln\left[\tau + \left(1 + t/\Lambda^2_{\rm QCD}\right)^2\right]} \frac{1 - \exp(-t/[4m^2_{\mathcal{F}}])}{t}$$

Medium effects

$$D_{\mu\nu}(k;T,\mu) = D^{\rm T}{}_{\mu\nu}(k;T,\mu) + D^{\rm L}{}_{\mu\nu}(k;T,\mu)$$



Dyson-Schwinger Equations

➢ Model of quark-gluon vertex：

- Rainbow (RB) approx. $\Gamma_\nu(\vec{q},\tilde{\omega}_l,\vec{p},\tilde{\omega}_n)=\gamma_\nu$
- 1BC $i\Gamma^{1BC}_\sigma(q,p;\mu) = i\Sigma_A(q,p;\mu)\gamma^\perp_\sigma + i\Sigma_C(q,p;\mu)\gamma^\parallel_\sigma$
- Ball-Chiu(BC)

$$i\Gamma^{BC}_\sigma(q,p;\mu) = i\Gamma^{1BC}_\sigma(q,p;\mu) + (\tilde{q}+\tilde{p})_\sigma\Big[\Delta_B(\tilde{q},\tilde{p};\mu) + \frac{i}{2}\gamma^\perp\cdot(\tilde{q}+\tilde{p})\Delta_A(\tilde{q},\tilde{p};\mu) + \frac{i}{2}\gamma^\parallel\cdot(\tilde{q}+\tilde{p})\Delta_C(\tilde{q},\tilde{p};\mu)\Big]$$

where $\tilde{q}=q+u$, $\tilde{p}=p+u$, and $\gamma^\parallel=\hat{u}\gamma\cdot\hat{u}$, $\gamma^\perp=\gamma-\gamma^\parallel$ with $\hat{u}^2=1$, $u=(\mathbf{0},i\mu)$ $F=A,B,C$

$$\Sigma_F(q,p;\mu) = \frac{1}{2}\left[F(\mathbf{q}^2,q_4;\mu)+F(\mathbf{p}^2,p_4;\mu)\right],$$

$$\Delta_F(\tilde{q},\tilde{p};\mu) = \frac{F(\mathbf{q}^2,q_4;\mu)-F(\mathbf{p}^2,p_4;\mu)}{\tilde{q}^2-\tilde{p}^2},$$

Dyson-Schwinger Equations

➢ DSE for quark propagator

$$S(p;\mu)^{-1} = Z_2[i\boldsymbol{\gamma p} + i\gamma_4(p_4 + i\mu) + m_q] + \Sigma(p;\mu),$$

Self-energy

$$\Sigma(p;\mu) = Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4} g^2(\mu) D_{\rho\sigma}(p-q;\mu) \times \frac{\lambda^a}{2}\gamma_\rho S(q;\mu)\Gamma^a_\sigma(q,p;\mu),$$

➢ Multi-solution：Chiral symmetry broken, confined vs. Chiral symmetric, deconfined

$$S(p;\mu)^{-1} = i\boldsymbol{\gamma p}A(p^2, pu, u^2) + B(p^2, pu, u^2) + i\gamma_4(p_4 + i\mu)C(p^2, pu, u^2),$$

Neglect CSC

Mass function $M(p) = \frac{B(p)}{A(p)}$

Chiral condensate $<\bar{q}q> = \int_p tr[S(p)]$

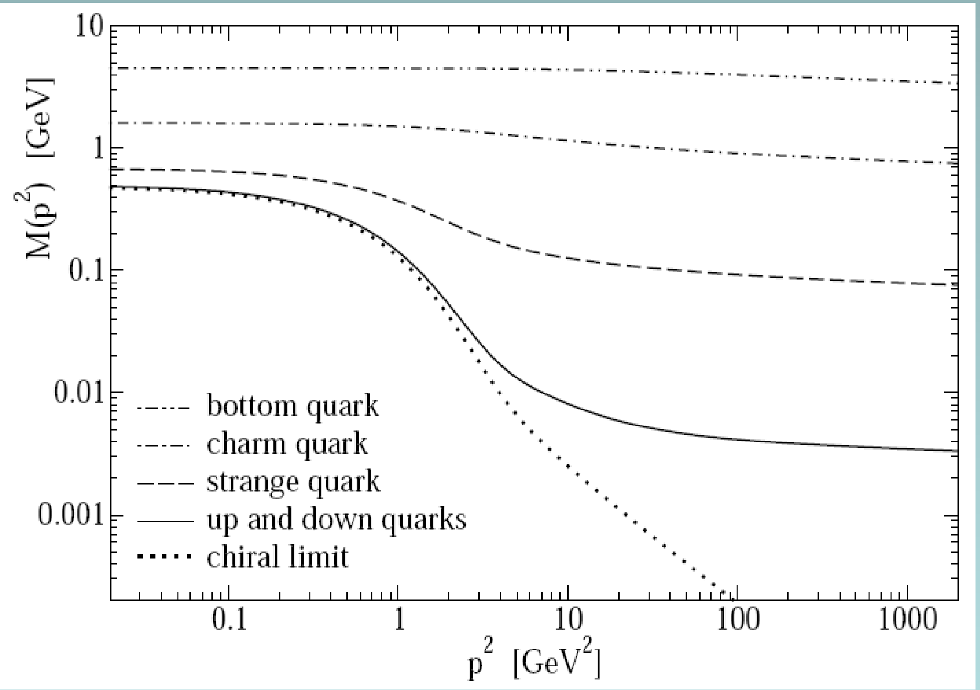
Dyson-Schwinger Equations

- **Numerical solution:**

$$B(p)=\frac{1}{4}Tr[S^{-1}(p)]=m_q+\int_q K_B(p,q;\cdots)\frac{B(q)}{\cdots+B^2(q)}$$

$$A(p)=\frac{1}{4\vec{p}^2}Tr[-i\vec{\gamma}\bullet\vec{p}S^{-1}(p)]=1+\int_q K_A(p,q;\cdots)\frac{A(q)}{\cdots+\cdots A^2(q)}+\cdots$$

Nambu solution



Vs. Wigner solution

$$B_{m_q=0}(p)\equiv 0$$

EOS of QM

➢ Thermodynamic of Quark Matter

$$\tilde{f}_q(|p|;\mu_q,T) = \frac{T}{2}\sum_{n=-\infty}^{\infty} \mathrm{tr_D}\left[-\gamma_4 S_q(p,\omega_n;\mu_q,T)\right]$$

$$\rho_q(\mu_q,T) = g\int\frac{d^3p}{(2\pi)^3}\tilde{f}_q(|p|;\mu_q,T)$$

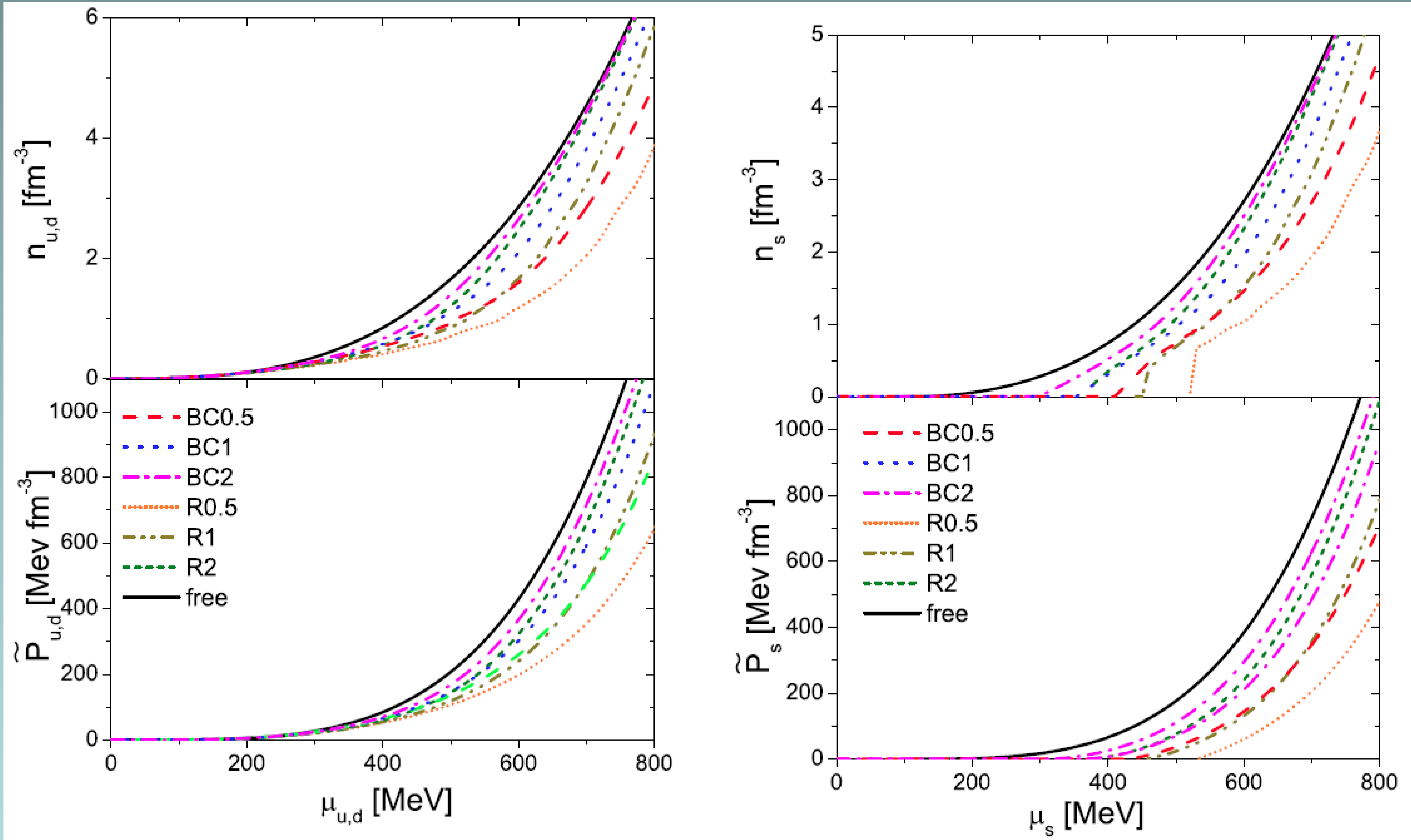
$$P_q(\mu_q,T) = P_q(\mu_{q,0},0)+\int_{\mu_{q,0}}^{\mu_{UV}}\mathrm{d}\mu'\rho_q(\mu',0) + \int_0^T dT' s_q^{\mathrm{free}}(\mu_{\mathrm{UV}},T')+\int_{\mu_{\mathrm{UV}}}^{\mu_q}\mathrm{d}\mu'\rho_q(\mu',T)$$

$$\tilde{P}_q(\mu_q,T)\equiv P_q(\mu_q,T)-P_q(\mu_{q,0},0)$$

$$B_{\mathrm{DS}} \equiv -\sum_{q=u,d,s}P_q(\mu_{q,0},0)$$

:Parameter

Single Flavor



Dense Hadron Matter

- **Hadron matter：Brueckner-Beth-Goldstone**

  N-N：Argonne V18, Bonn B(BOB),…

  TBF：(micro) density dependent two body force

  vs. phenomenology Urbana TBF

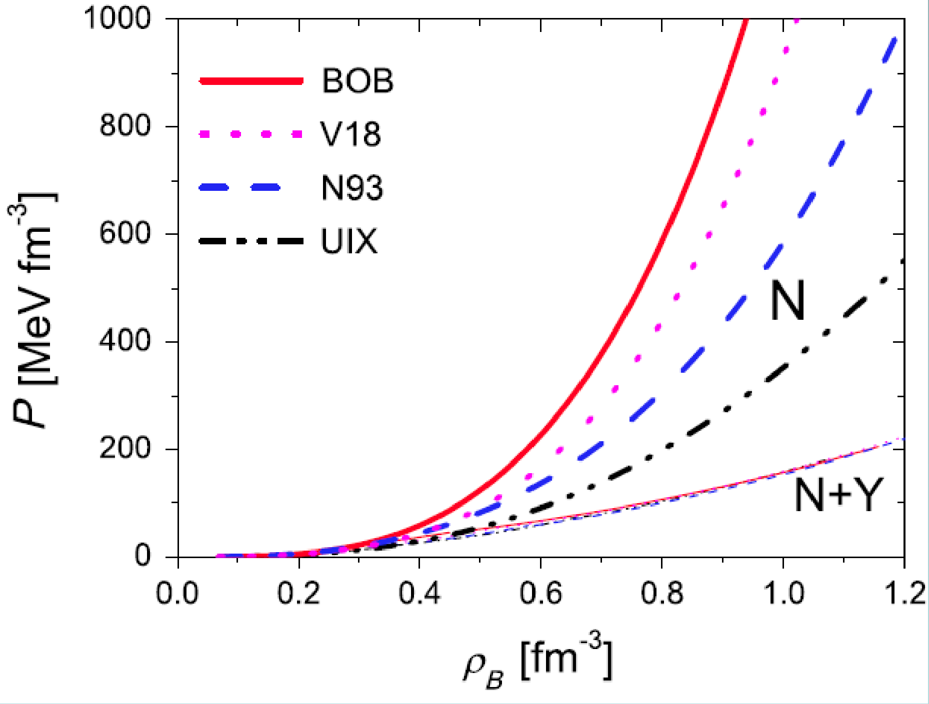
  N-Y：Nijmegen soft core potential

- **Thermal relation**

$$f = \omega + \sum_i \rho_i \tilde{\mu}_i \Rightarrow \begin{cases} \mu_i = \dfrac{\partial f}{\partial \rho_i}, \\ p = \sum_i \mu_i \rho_i - f, \\ s = -\dfrac{\partial f}{\partial T}, \\ \varepsilon = f + Ts. \end{cases}$$

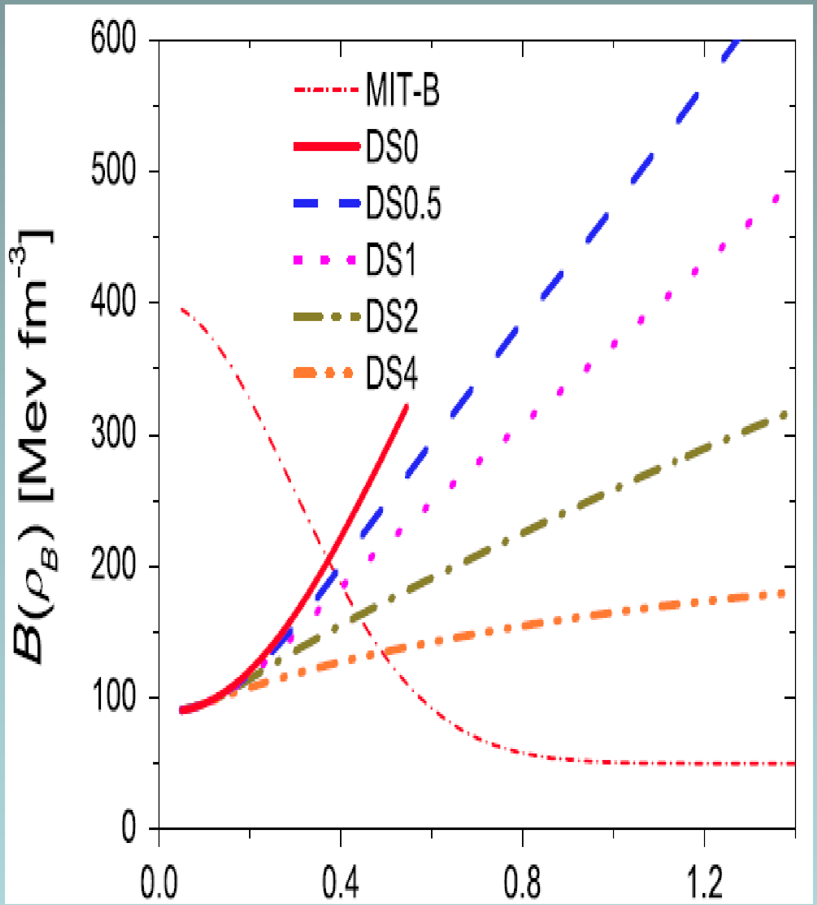
Dense Matter in Compact Stars

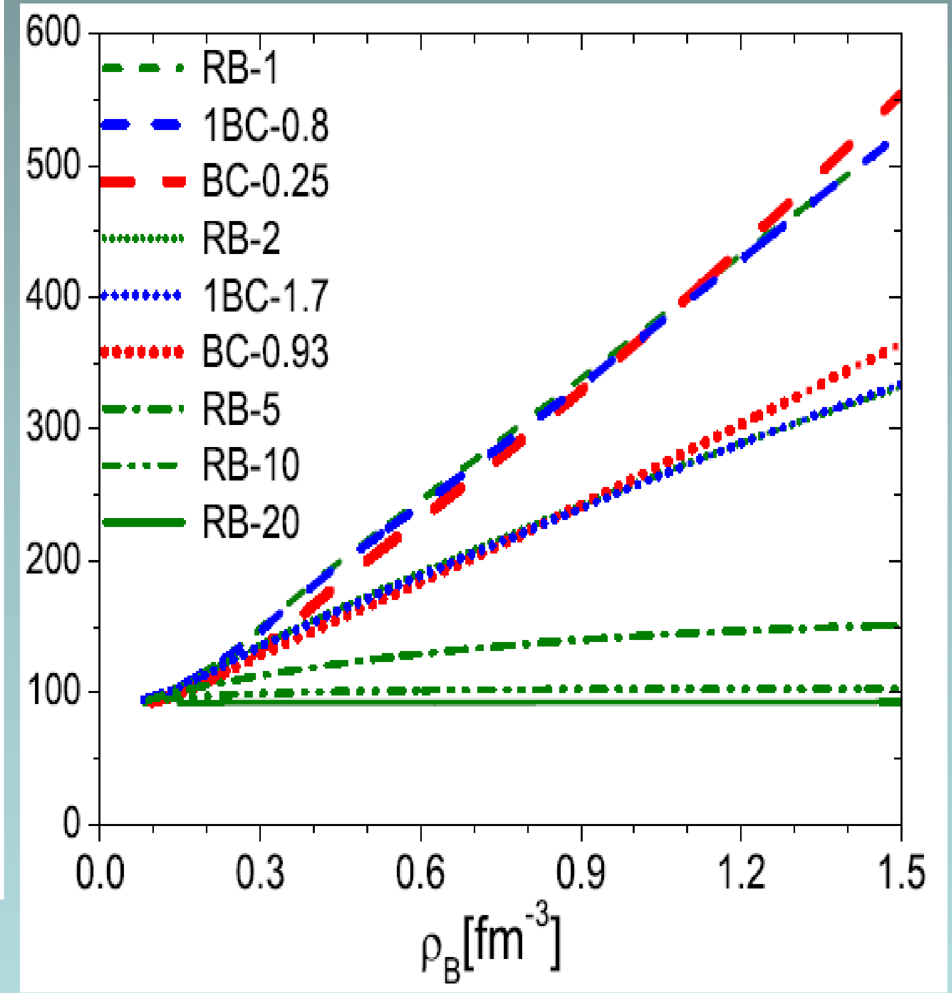
- Beta – equilibrium: $$\mu_i = B_i\mu_B + Q_i\mu_Q + L_i^{(e)}\mu_{L_e} + L_i^{(\mu)}\mu_{L_\mu}$$
- Charge – neutrality: $$\sum_i Q_i\rho_i = 0$$



EOS of QM

Effective bag constants: $B(\rho) \equiv \varepsilon(\rho) - \varepsilon_{\mathrm{free}}(\rho)$





$$B_{DS} = 90 MeV fm^{-3}$$

Hadron-Quark Phase Transition

- **Maxwell condition**
  - Thermal equilibrium: $T^H = T^Q$
  - Chemical equilibrium: $\mu_B^H = \mu_B^Q$
  - Mechanic equilibrium: $P^H = P^Q$
- **Gibbs condition（Glendenning）**

$$T^H = T^Q$$

$$\mu_{B,Q,L}^H = \mu_{B,Q,L}^Q$$ All Chemical potentials

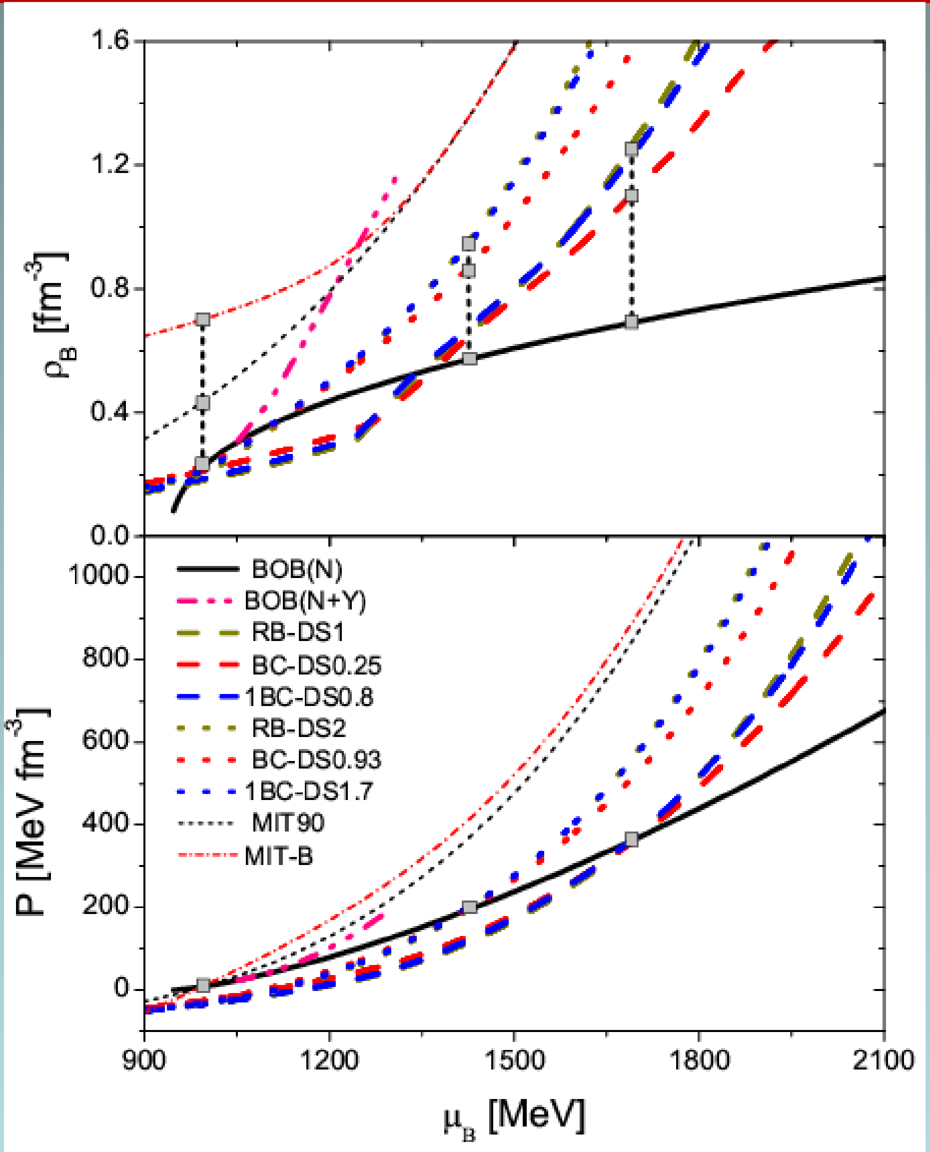
$$P^H = P^Q$$

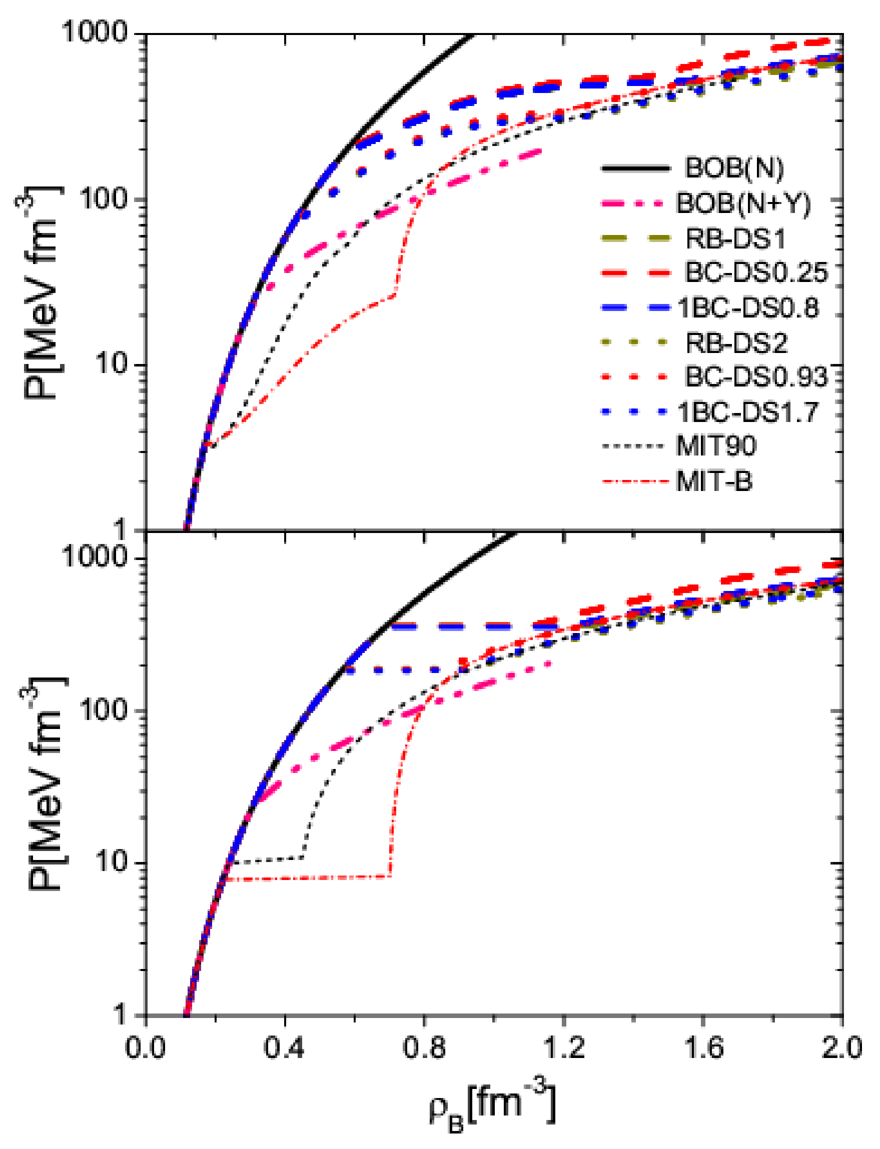
$$(1-x)\rho_Q^H + x\rho_Q^Q = 0$$ Global charge neutrality

$$(1-x)Y_L^H + xY_L^Q = Y_L$$

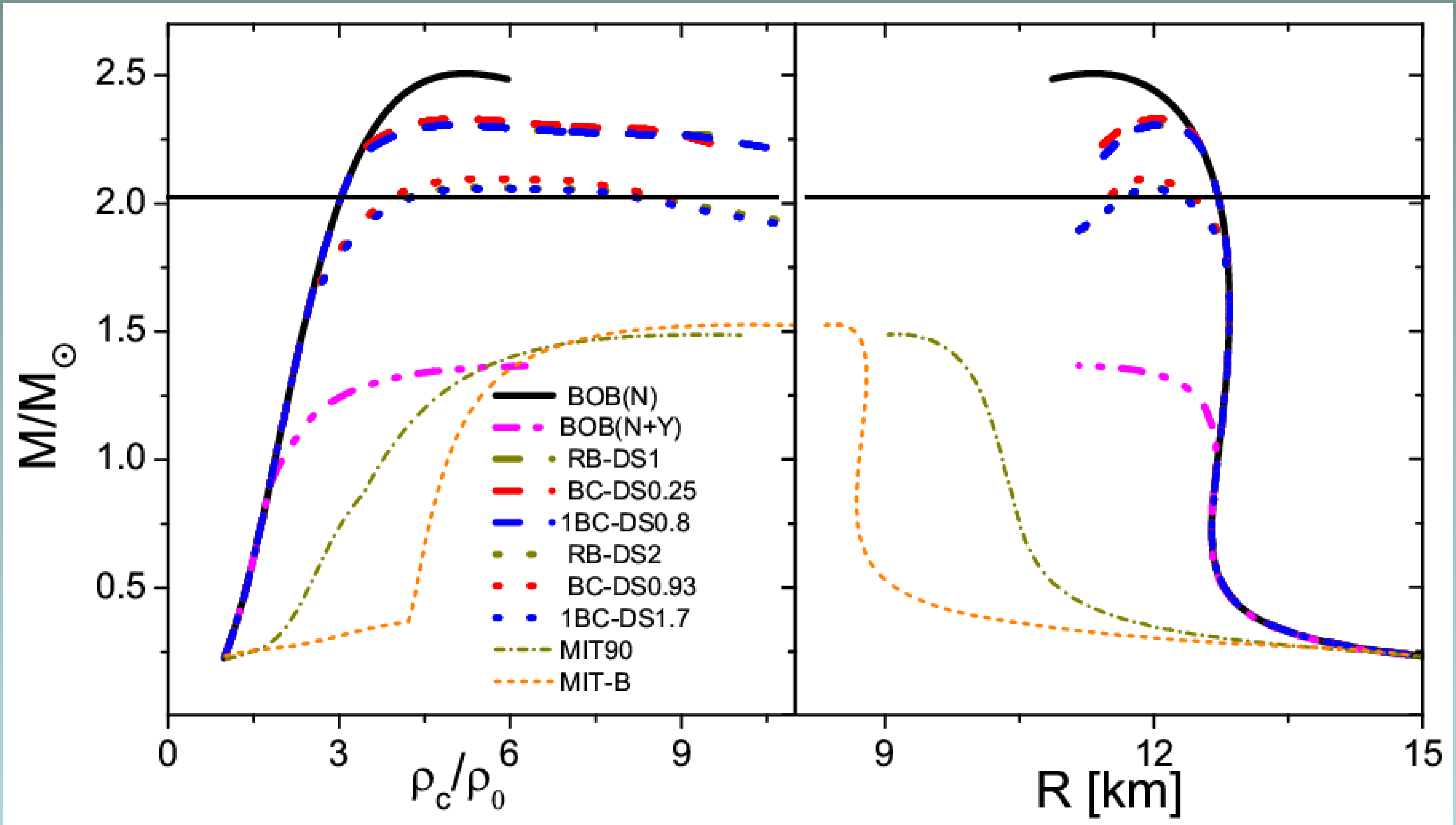
x: ratio of quark matter in mixed phase

Hadron-Quark Phase Transition

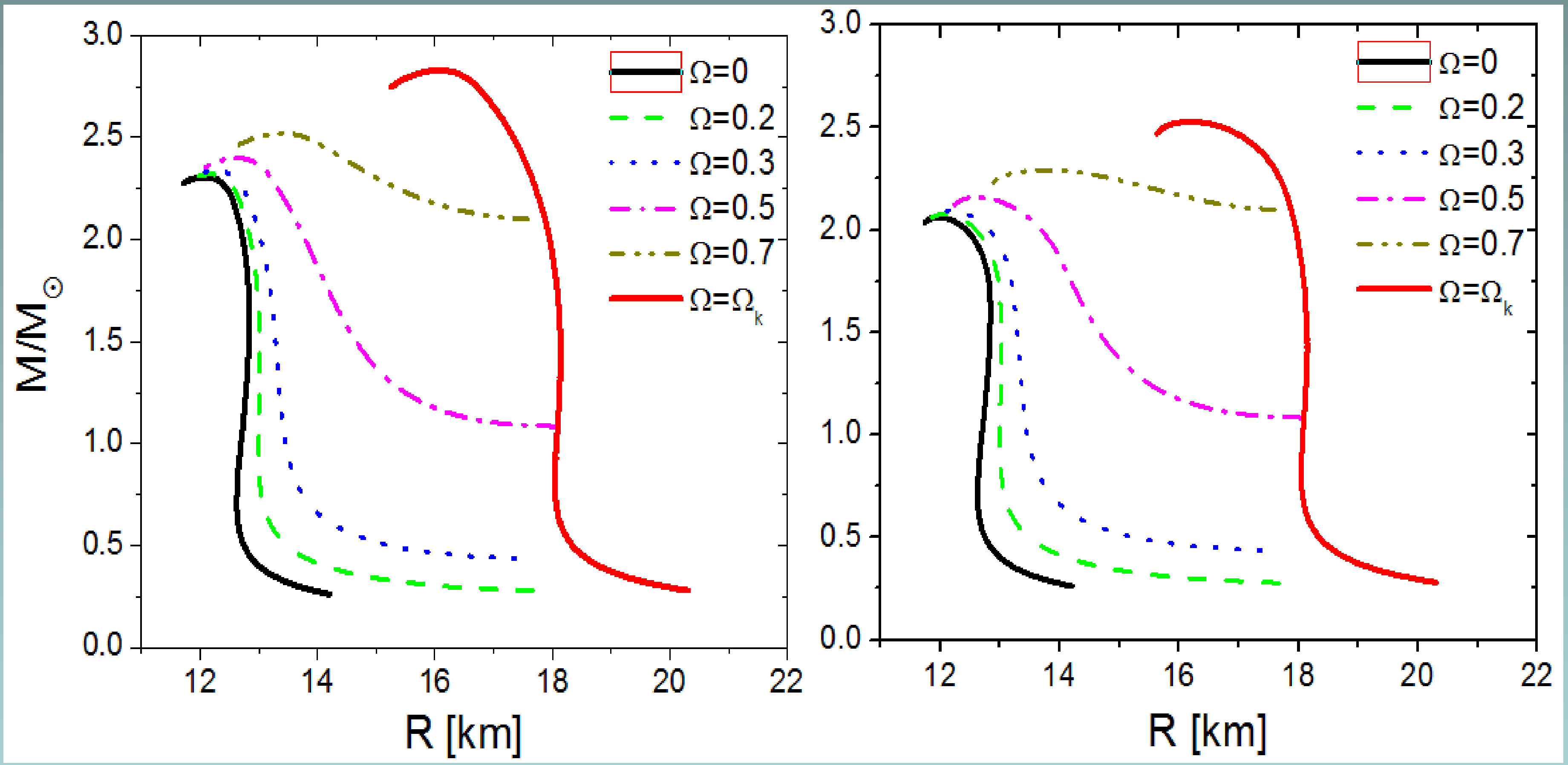




Hybrid Quark Stars



NS with Rotation



Left：BOB+RB1；Right：BOB+RB2. Ώ in unit of 10000rad/s

Code：http://www.gravity.phys.uwm.edu/rns/，**by N. Stergioulas**

XTE J1739-285：1122Hz/s=0.705*10000rad/s

Strange Quark Matter

Quark matter at p=0: $\mu_{B,N=3} < 930.4 MeV, \mu_{B,N=2} > 939.6 MeV,$

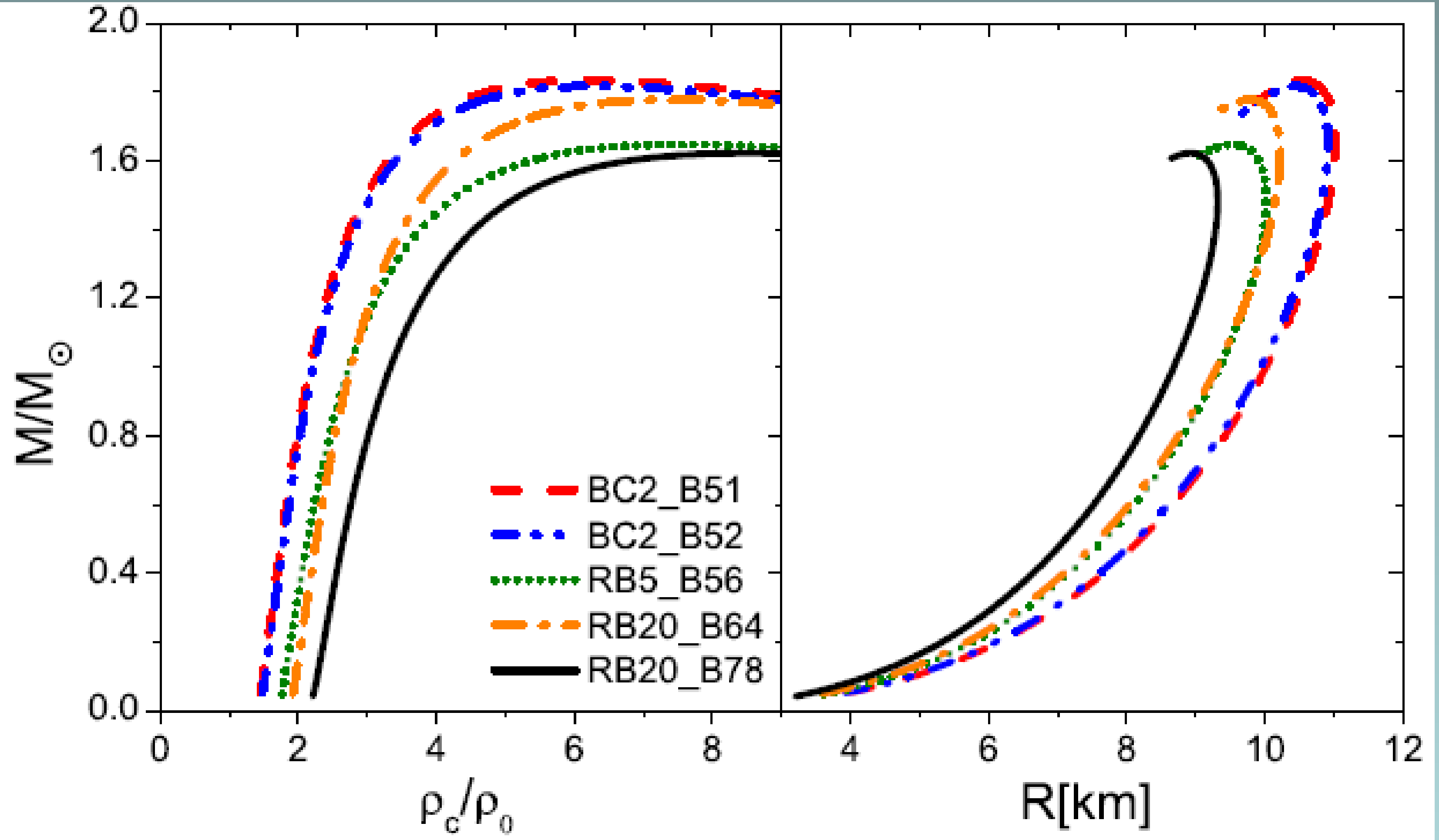
α	$B_{DS}[MeV\,fm^{-3}]$	$\mu_B(N_f=3)[GeV]$	$n_B[fm^{-3}]$	u[%]	s[%]	$\mu_B(N_f=2)[GeV]$
0.25	50	0.928	0.182	0.339	0	0.906
	70	1.926	0.227	0.339	0	1.002
	90	1.107	0.268	0.34	0	1.080
1	5	0.929	0.188	0.338	0	0.906
	70	1.021	0.26	0.336	0.082	1.002
	90	1.088	0.344	0.334	0.174	1.077
2	5	0.919	0.232	0.334	0.159	0.939
	7	0.992	0.316	0.334	0.226	1.032
	9	1.05	0.388	0.333	0.255	1.11

TABLE I: Properties of quark matter at $P = 0$.

	BC			RB			
α	0.25	1	2	0.5	4	5	20
$B_{min}[Mevfm^{-3}]$	56.5	56.1	50.2	48.6	53.5	55.1	63.1
$B_{max}[Mevfm^{-3}]$	50.5	50.2	52.8	43.3	51.8	56.6	79

TABLE II: Constraints on B_{DS} for strange quark matter hypothesis.

Strange Quark Stars



Summary

- ➢ **Quark matter with DSE:**
  - **Harder EOS than bag model。**
- ➢ **Hadron-quark phase transition and EOS:**
  - **Excluding hyperon by hands**
  - **Global effects of gluon propagator and vertices.**
- ➢ **Hybrid Quark Stars:**
  - **Possibility of hybrid stars with 2-solar-mass；**
  - **Small effects of rotation on mass**
- ➢ **Strange quark matter and strange quark star**
  - **Possible stable strange quark matter**
  - **Strange quark stars slightly lighter than 2-solar mass**

Thank You！

Welcome to CUG, Wuhan

Internal structure of compact stars

- **TOV Eq.** (Spherical and static)

$$\frac{dp}{dr} = -\frac{Gm\varepsilon}{r^2}\frac{(1+p/\varepsilon)(1+4\pi r^3 p/m)}{1-2Gm/r},$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon,$$

$$M_G \equiv m(R) = 4\pi \int_0^R dr r^2 \varepsilon(r)$$

At low densities：
Negele and Vautherin in the medium-density regime,
and the ones by Feynman-Metropolis-Teller and Baym-Pethick-Sutherland for the outer crust of NS.
and Shen EoS at low density for PNS