

# Thermal Evolution of Isolated Neutron Star and the Influence of Equation of State

Akira Dohi (土肥 明)

(Kyushu Univ.)

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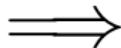
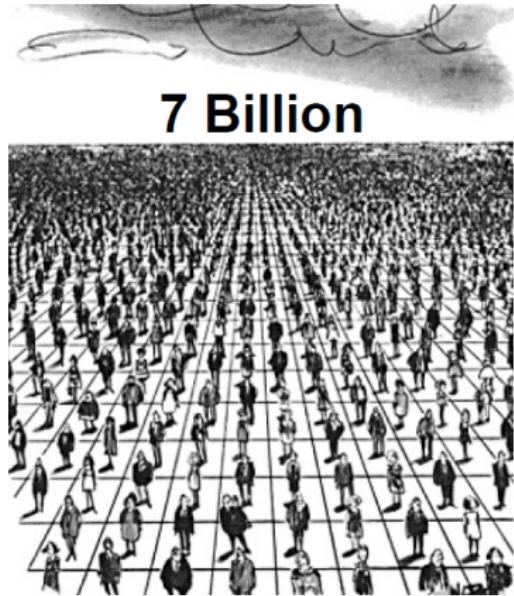
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- ① Introduction of Neutron Star Structure and Neutron Star Cooling
- ② Nuclear Equation of State
- ③ Cooling processes
- ④ Setup of Cooling Calculation in this study
- ⑤ Results and Discussion
- ⑥ Application to Neutron Star Cooling with Modified Gravity
- ⑦ Conclusion

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# The Density of Neutron Star (NS)



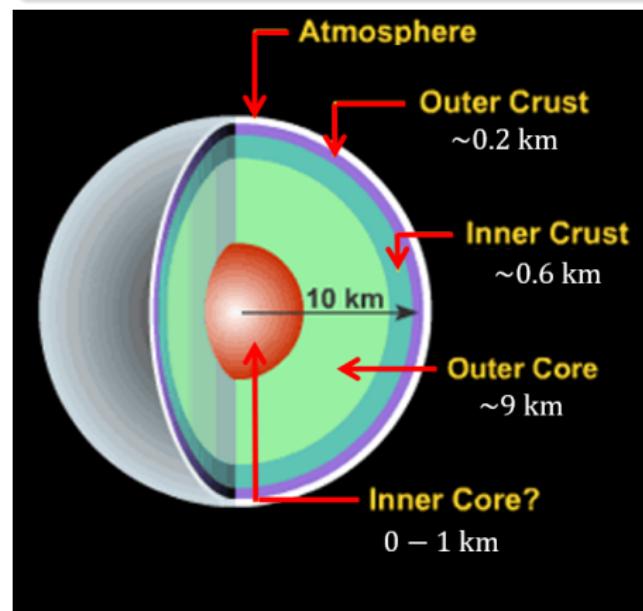
NS density is equivalent to the mass of the entire human population compressed to the size of a sugar cube.

# Basic NS Size and Composition

- $\sim$  The size of Manhattan city  
 $\Rightarrow$  Compact Object !!
- Supported by mainly nuclear pressure
- low temperature  $T \lesssim 1 \text{ MeV} \ll T_{\text{Fermi}}$
- At least neutrons, protons, electrons are degenerate.
- Possible exotic particles in NS core.

Typical values

$$M \sim 1.4 M_{\odot} \text{ & } R \sim 12 \text{ km}$$



# Neutrino Transparency (Shapiro's text)

$\lambda_i$ : mean free path of  $i$  particle

- $\nu_e - e$  Inelastic Scattering Off  $e$  (and  $\mu$ )

$$\lambda_e = (\sigma_e n_e)^{-1} \sim (9 \times 10^7 \text{ km}) (\rho_0/\rho_B)^{4/3} (100 \text{ keV}/E_\nu)^3$$

- $n - \nu$  Elastic Scattering Off  $n$

$$\lambda_n = (\sigma_n n_n)^{-1} \sim (300 \text{ km}) (\rho_0/\rho_B) (100 \text{ keV}/E_\nu)^2$$

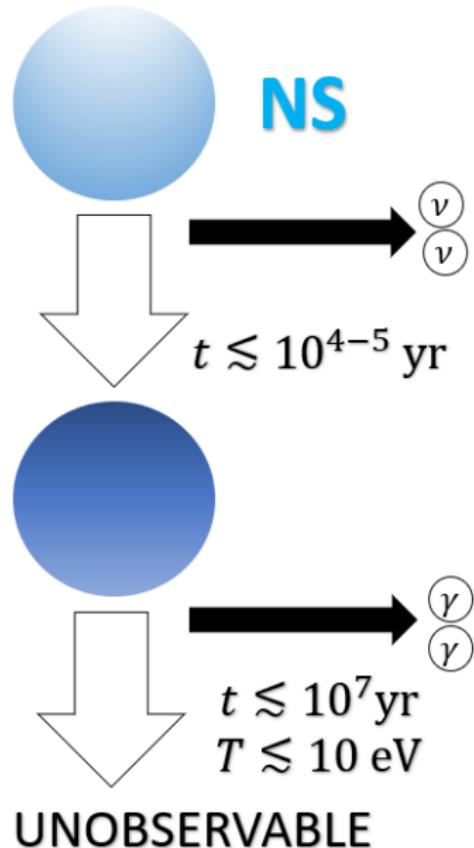
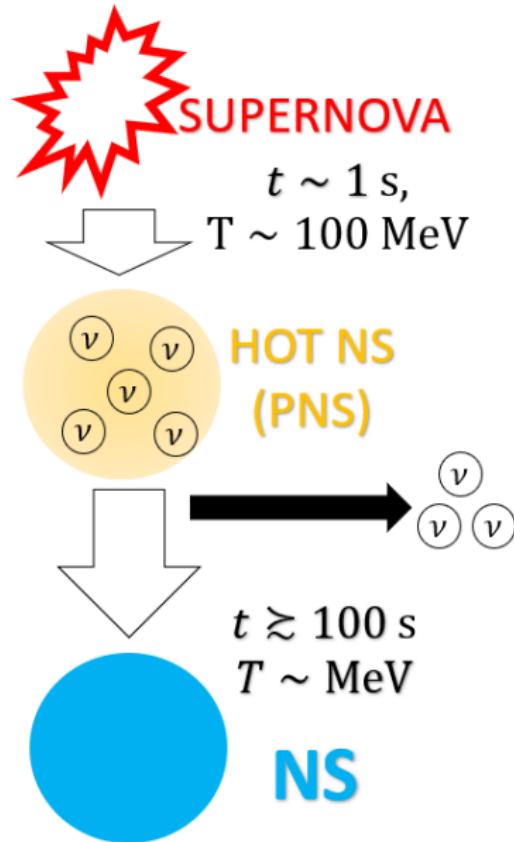
- The *effective* mean free path is

$$\lambda_{\text{eff}} = \sqrt{\lambda_n \lambda_e} \sim (2 \times 10^5 \text{ km}) (\rho_0/\rho_B)^{7/6} (100 \text{ keV}/E_\nu)^{5/2}$$

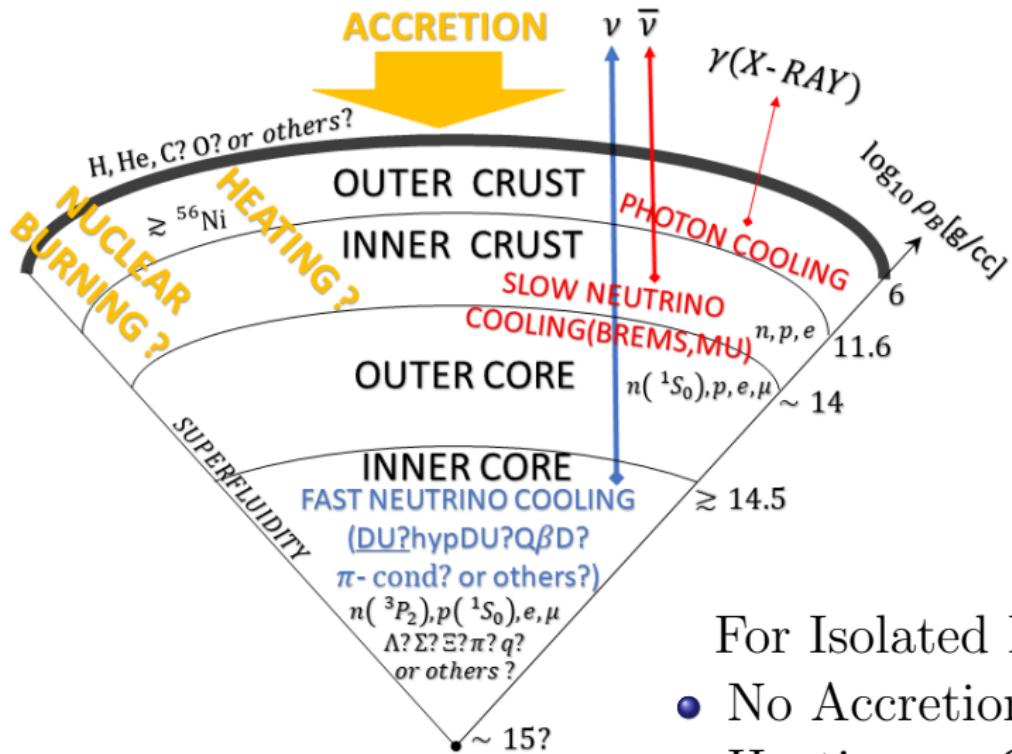
$\Rightarrow$  Since  $E_\nu \lesssim 100 \text{ keV}$  ( $T \lesssim 10^9 \text{ K}$ ),

$\lambda_{\text{eff}} \gg R \sim 10 \text{ km}$ , and that is why  
NS is transparent to neutrinos.

# How is NS cooled ?



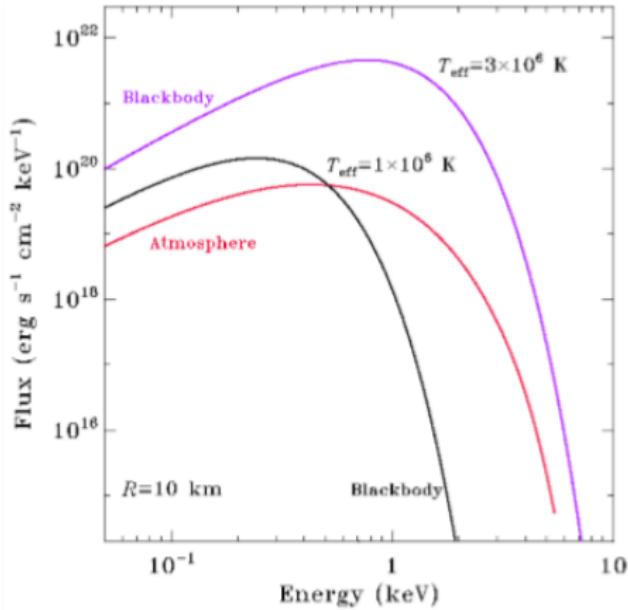
# NS Structure and Cooling



- For Isolated NS (INS)
- No Accretion
  - Heating  $\ll$  Cooling

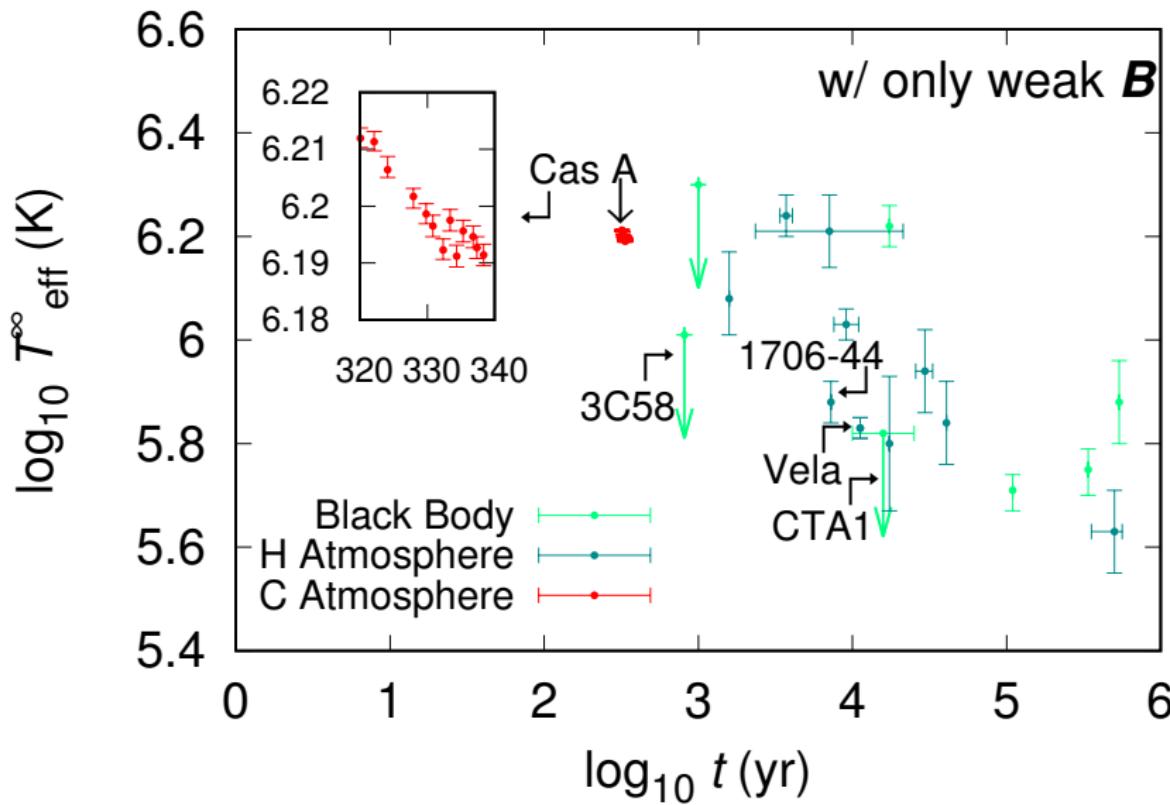
# Surface Models of INS observations

- Cooling history can be compared with the observed Flux of INS.
  - Blackbody:  
unrealistic
  - Atmosphere:  
at higher  $T$ , lower  $\mathcal{B}$
  - Solid/Liquid surface:  
at lower  $T$ , higher  $\mathcal{B}$
- We consider INS with low  $\mathcal{B}$ , that is, blackbody/atmosphere surface models.



(Pavlova *et al.* 2001,  
His eConf. slide in 2004)

# Age & Surface Temperature of INS



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# TOV Equations

① Mass Eq.

$$\frac{\partial M_{\text{tr}}}{\partial r} = 4\pi r^2 \rho$$

② Hydrostatic equilibrium Eq.

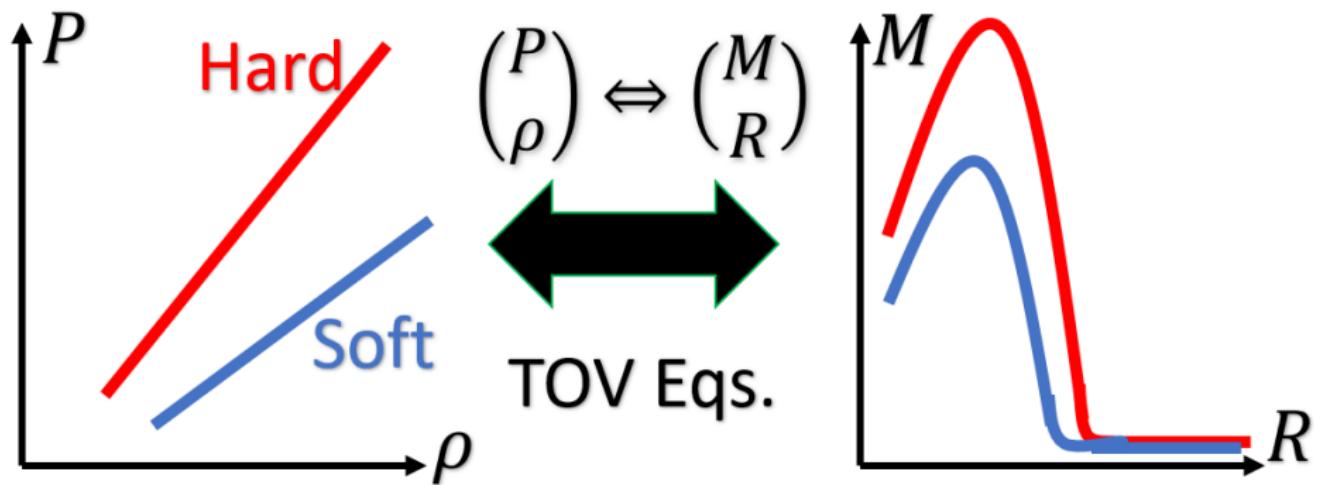
$$\frac{\partial P}{\partial r} = -\frac{G\rho}{r^2} \left(1 + \frac{P}{\rho c^2}\right) \left(M_{\text{tr}} + \frac{4\pi r^3 P}{c^2}\right) \left(1 - \frac{2GM_{\text{tr}}}{c^2 r}\right)^{-1}$$

under ideal fluid  $T^{\mu\nu} = \text{diag}(-\rho, P, P, P)$

$M_{\text{tr}}$ : total mass inside radius  $r$ ,  $\rho$ : total mass energy density

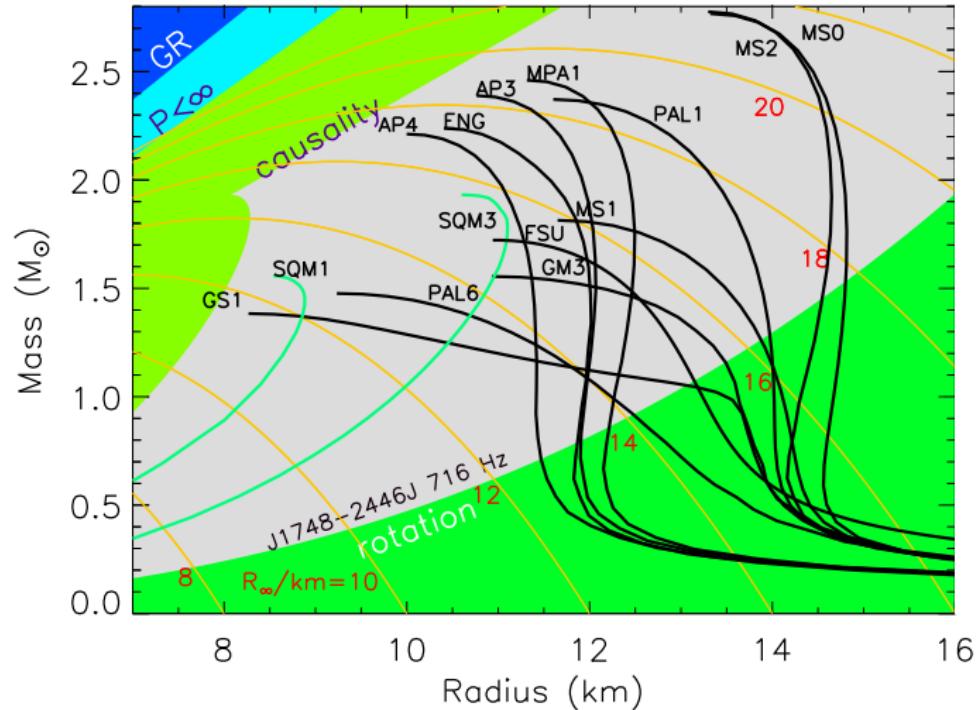
$\Rightarrow$  Since there are 3 variables  $P(r), \rho(r), M_{\text{tr}}(r)$  against inside a radius  $r$ , TOV Eqs. can be solved for given an EoS ( $P = P(\rho)$ ).

# Relation between $(P, \rho)$ and $(M, R)$



- Soft EoS  $\Leftrightarrow M \searrow$       Hard EoS  $\Leftrightarrow M \nearrow$

# Uncertainty of EoS: $P = P(\rho)$



(Lattimer 2013)

# Symmetry Energy of Nuclear Matter

The energy of nuclear matter can be expressed as

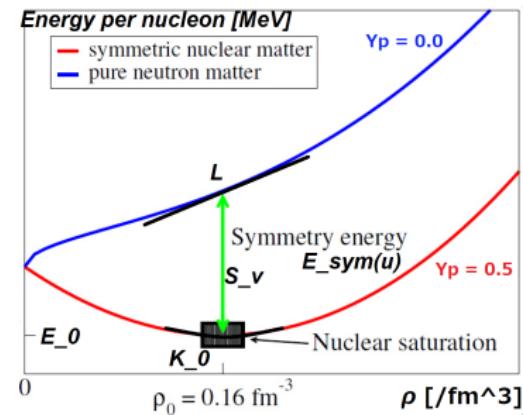
$$E_{\text{nuc}}(u, Y_p) = E_0 + \frac{K_0}{18} (u - 1)^2 + E_{\text{sym}}(u) (1 - 2Y_p)^2$$

$\Downarrow$  symmetry energy    ( $u \equiv \rho/\rho_0 = n_B/n_0$ )

$$E_{\text{sym}}(u) = \left[ S_v + \frac{L}{3} (u - 1) \right] + \dots$$

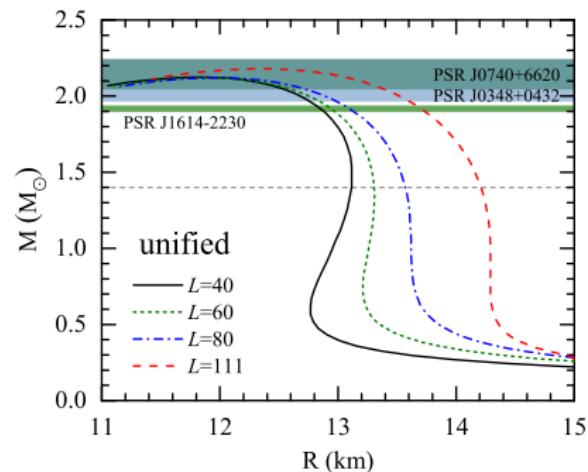
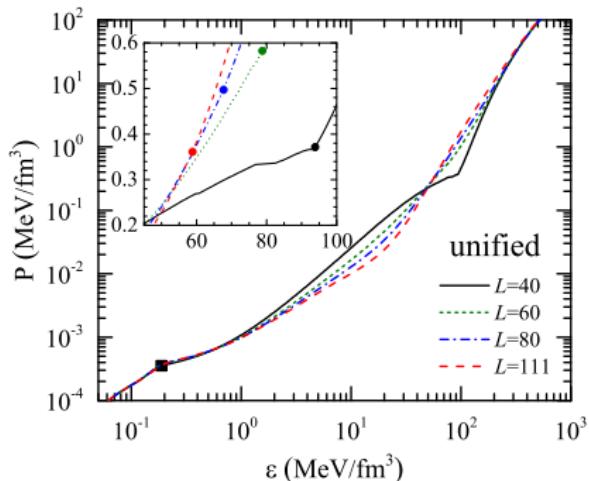
with saturation density  $\rho_0$ .

- $S_v \equiv E_{\text{sym}}(u = 1)$
- Slope parameter  
 $L \equiv 3 \left( u \frac{\partial E_{\text{sym}}(u)}{\partial u} \right)_{u=1}$



(M. Thiel Bormio 2015)

# $L$ dependence on EoS and $MR$



(Ji,  $\dots$ , Shen, 2019)

- indicates the transition between core and crust.
- The smaller  $L$  is, the smaller  $R$  is.

# Nuclear matter properties of the EoSs under finite temperature

Nuclear Interaction	$n_0$ (fm $^{-3}$ )	$E_0$ (MeV)	$K_0$ (MeV)	$S_v$ (MeV)	$L$ (MeV)	Type of int.
SKa	0.155	-16.0	263	32.9	74.6	Skyrme
LS180	0.155	-16.0	180	28.6	73.8	Skyrme
LS220	0.155	-16.0	220	28.6	73.8	Skyrme
LS375	0.155	-16.0	375	28.6	73.8	Skyrme
TM1(Shen)	0.145	-16.3	281	36.9	110.8	RMF
TMA	0.147	-16.0	318	30.7	90.1	RMF
NL3	0.148	-16.2	272	37.3	118.2	RMF
FSUgold	0.148	-16.3	230	32.6	60.5	RMF
FSUgold2.1	0.148	-16.3	230	32.6	60.5	RMF
IUFSU	0.155	-16.4	231	31.3	47.2	RMF
DD2	0.149	-16.0	243	31.7	55.0	RMF
SFH <sub>o</sub>	0.158	-16.2	245	31.6	47.1	RMF
SFH <sub>x</sub>	0.160	-16.2	239	28.7	23.2	RMF

(Oertel *et al.* 2017)

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SFHx	0.160	-16.2	239	28.7	23.2	RMF
Togashi	0.160	-16.1	241	30.0	35.0	Variational

Focus on Togashi EoS !!

(Oertel *et al.* 2017)

# Togashi EoS (Togashi *et al.* 2017)

- 1- & 2- body Hamiltonian as

$$H_{1,2} = - \sum_i \left[ m_i c^2 + \frac{\hbar^2}{2m_i} \nabla_i^2 + \sum_{j < i} V_{ij} \right]$$

$V_{ij}$ : Argonne V18 (AV18) (Wiringa *et al.* 1995)

- 3- body Hamiltonian as

$$H_3 = - \sum_{i < j < k} V_{ijk}$$

$V_{ijk}$ : Ubrana IX (UIX)

(Carlson *et al.* 1983, Pudliner *et al.* 1995)

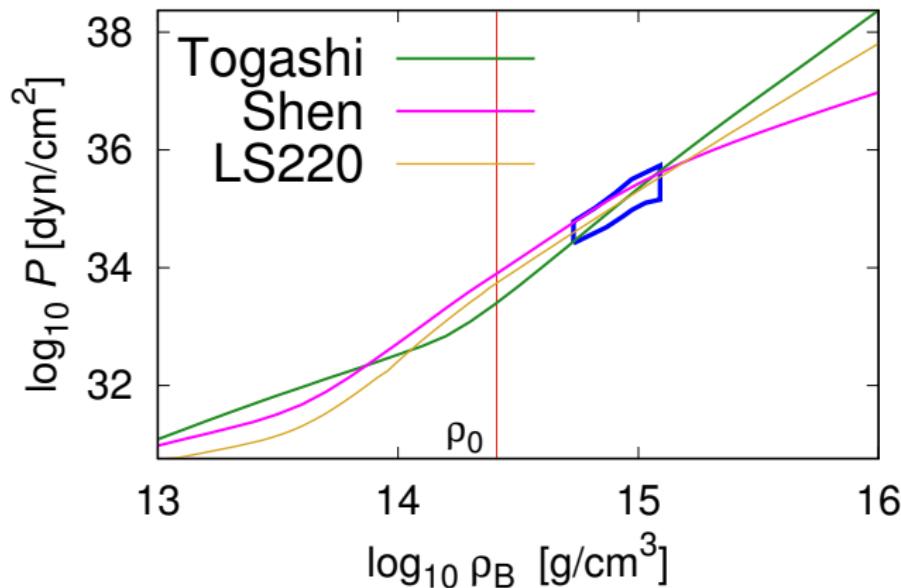
- Minimizing the free energy by using variation method of  $H = H_{1,2} + H_3$ , thermodynamic amounts  $p, \epsilon, S, C_V, \dots$  can be obtained.

# EoS Models

Nuclear EoS	$S_v$ (MeV)	$L$ (MeV)	Type of Int.	References
LS220	28.6	73.8	Skyrme	Lattimer & Swesty 1991
Shen	36.9	110.8	RMF	Shen <i>et al.</i> 1998, 2011
Togashi	30.0	35.0	Variational	Togashi <i>et al.</i> 2017

- Togashi EoS is NEW EoS constructed on realistic nuclear potentials under finite temperature.
- Other EoSs are not based on realistic methods.
- We use above three EoSs in order to examine the consistency of INS cooling for Togashi EoS.

# Pressure – Baryon Density Relation

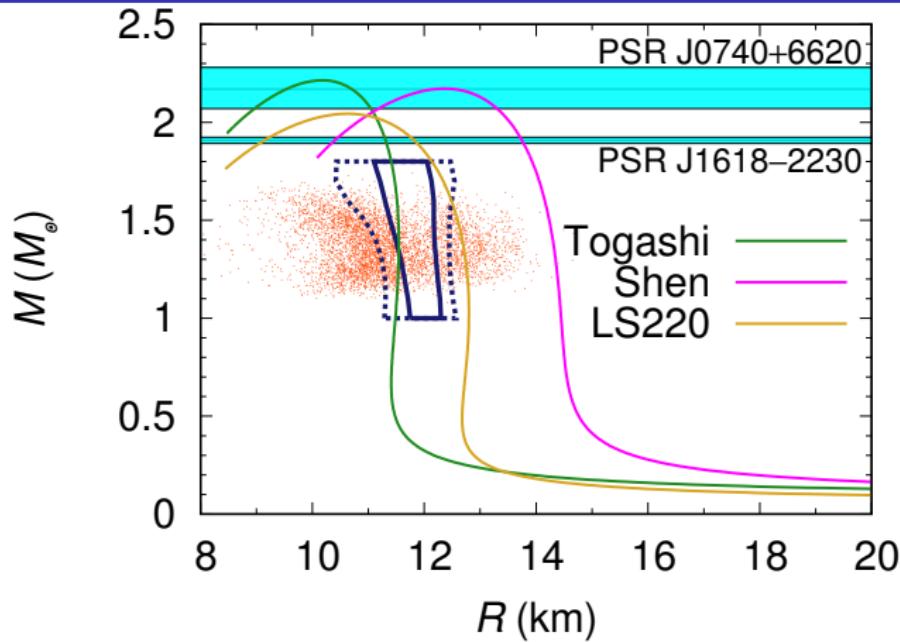


Togashi EoS is soft around  $\rho_B \sim \rho_0$  while stiff  $\rho_B \gg \rho_0$

The constraints on EoSs adopted in this work:

- ① Experimental results by Heavy Ion Collision between Au–Au (Danielewicz *et al.* 2002)

# Mass – Radius Relation

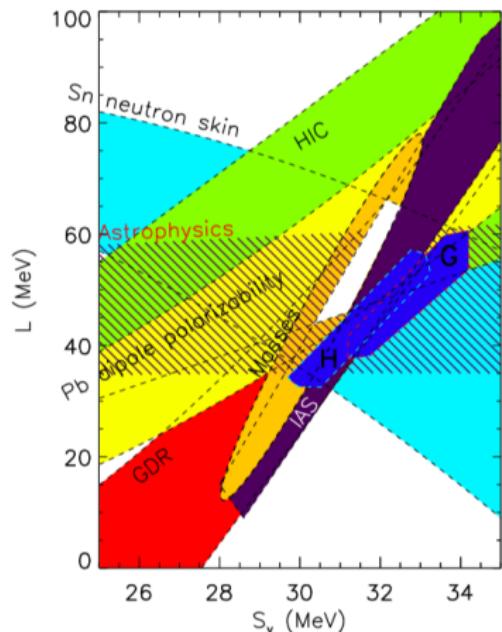


- ②  $M_{\max} \geq 2.17^{+0.11}_{-0.10} M_\odot$  (Cromartie *et al.* 2019)
- ③  $R \lesssim 13 \text{ km}$  by observations of GW and LMXB  
(Abbott *et al.* 2018, Steiner *et al.* 2010)

# EoS stiffness and constraints

- $S_v L$  constraints  
(Lattimer & Steiner 2014)
  - ④ Neutron skin thickness
  - ⑤  $^{208}\text{Pb}$  dipole polarization
  - ⑥ Centroids of giant dipole resonances (GDR)
  - ⑦ Neutron matter studies (G and H)
  - ⑧ Nuclear masses
  - ⑨ Isobaric Analog State
- $S_v \lesssim 36 \text{ MeV}$
- $L \lesssim 80 \text{ MeV}$   
(ex. Beloin *et al.* 2019)

EoS	$S_v$ [MeV]	Togashi	Shen	LS220
	$L$ [MeV]			
	30.0	36.9	28.6	
	35.0	111	73.8	



# Unitary Gas (UG) Criterion

## ⑩ UG bound

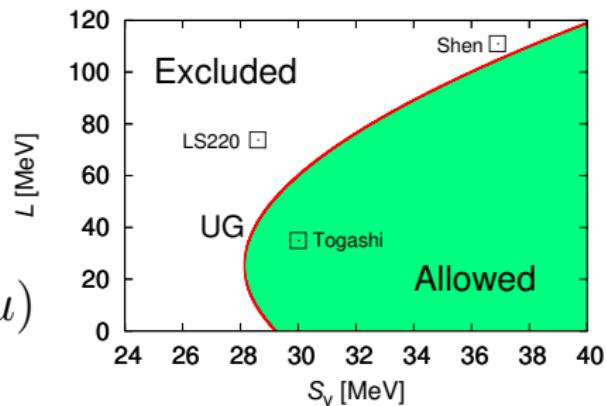
(Tews *et al.* 2017)

- Large Int. in NS  
⇒ UG approximation
- Energy of  $n$ -rich nuclei is higher than UG energy  $E_{\text{UG}}$ :

$$E_{\text{nuc}}(u, 0) \geq E_{\text{UG}} \approx 0.22 E_{\text{F}}(u)$$

here,  $E_{\text{F}}(u)$  is *Fermi* energy

EoS	Togashi	Shen	LS220
$S_v$ [MeV]	30.0	36.9	28.6
$L$ [MeV]	35.0	111	73.8



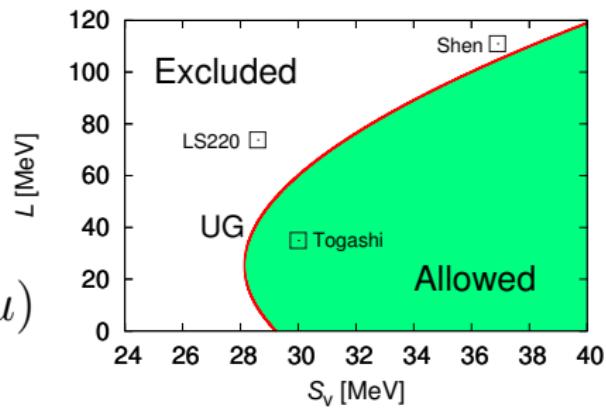
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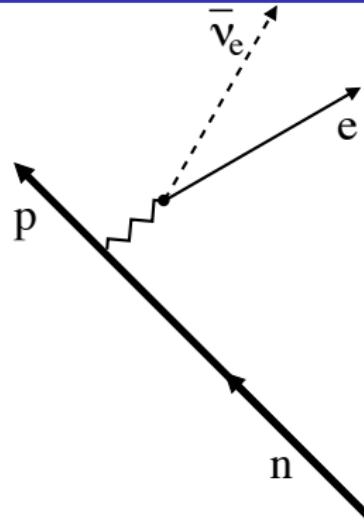


Togashi EoS is good in above conditions of ① ~ ⑩ !!

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# Direct Urca process (DU)



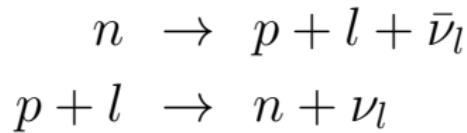
Direct Urca

- (Inverse)  $\beta$  decay as

$$n \rightarrow p + l + \bar{\nu}_l$$

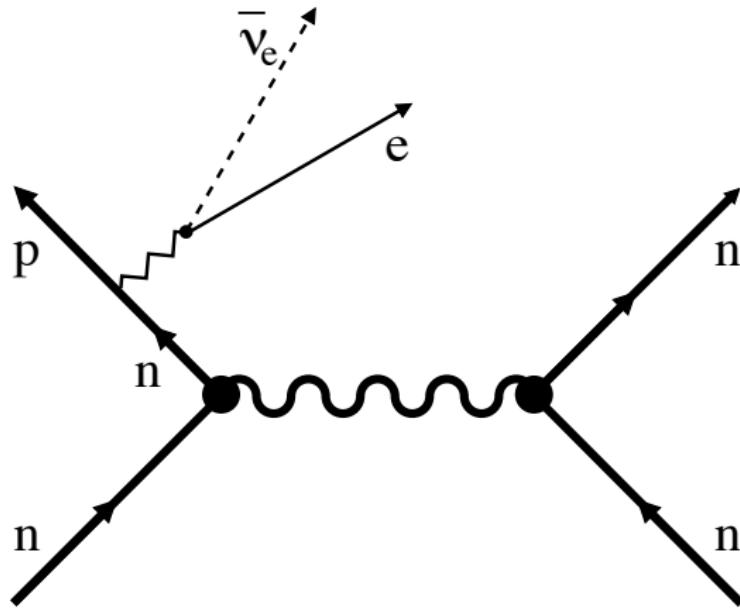
$$p + l \rightarrow n + \nu_l$$

# Direct Urca process (DU)



- $\epsilon_{\text{DU}} \sim 10^{26-27} (n_{\text{B}}/n_0)^{1/3} T_9^6 \text{ erg cm}^{-3} \text{ s}^{-1}$
- When the momentum conversation  $\mathbf{P}_{Fn} < \mathbf{P}_{Fp} + \mathbf{P}_{Fe} \Leftrightarrow Y_p > 1/9$  without muons is satisfied, DU occurs.
- With muons, using  $x_e = Y_e / (Y_e + Y_\mu)$ ,  
$$Y_p > \left( 1 + \left( 1 + x_e^{1/3} \right)^3 \right)^{-1} \gtrsim 0.1477$$

# Modified Urca (MU) Process

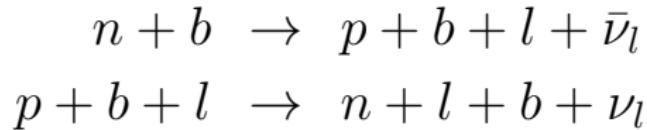


Modified Urca

$$n + b \rightarrow p + b + l + \bar{\nu}_l$$

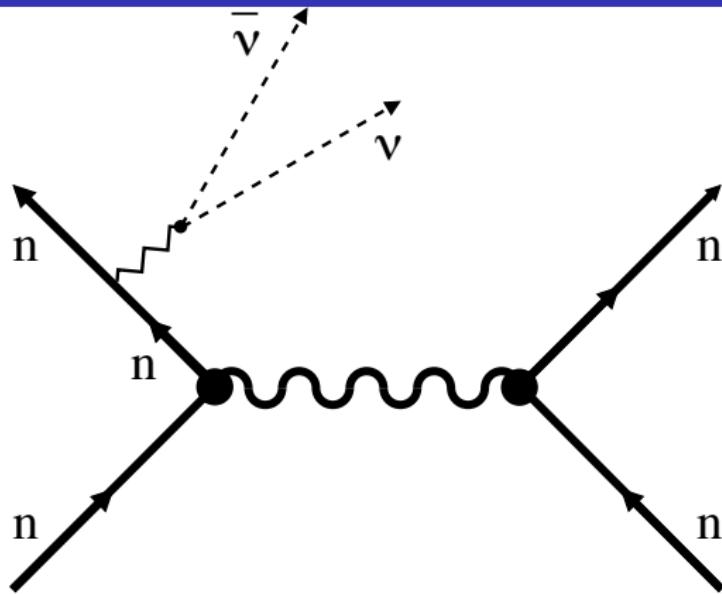
$$p + b + l \rightarrow n + l + b + \nu_l$$

# Modified Urca process (MU)



- $\epsilon_{\text{MU}} \sim 10^{20-21} (n_B/n_0)^{2/3} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}$
- In case of  $b = n$ , since the momentum conservation  $\mathbf{P}_{Fp} < 3\mathbf{P}_{Fn} + \mathbf{P}_{Fe}$  is satisfied, MU with  $b = n$  always occurs
- In case of  $b = p$ , when the momentum conservation  $\mathbf{P}_{Fn} < 3\mathbf{P}_{Fp} + \mathbf{P}_{Fe} \Leftrightarrow Y_p > 1/65$  is satisfied, MU with  $b = p$  occurs.

# Bremsstrahlung



Bremsstrahlung

- $b + b' \rightarrow b + b' + \nu + \bar{\nu}$
- $\epsilon_{\text{Brems}} \sim 10^{18-20} (n_B/n_0)^{2/3} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}$
- Always occur

# Neutrino Emissions in $npe\mu$

Classified with **Fast**, **Slow**, **Medium** Cooling Process

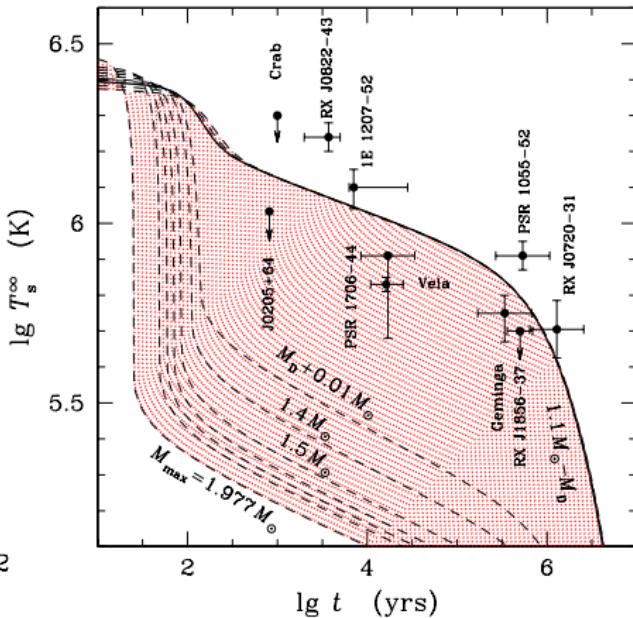
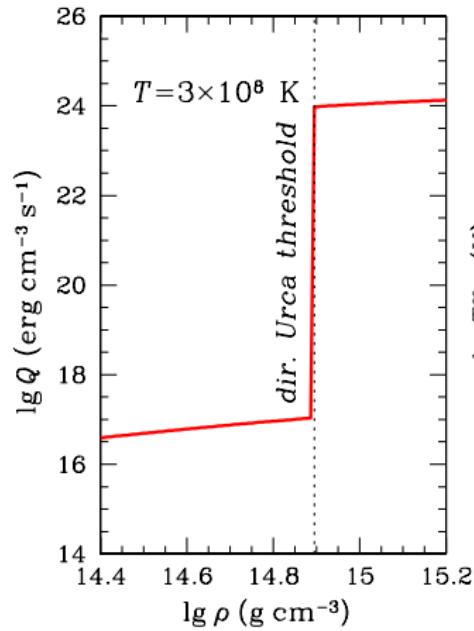
Direct Urca	$\epsilon_\nu \sim 10^{25-27} T_9^6 \text{ erg cm}^{-3} \text{ s}^{-1}$
$n \rightarrow p + l + \bar{\nu}_l$	$p + l \rightarrow n + \nu_l$ (Not always)
Modified Urca	$\epsilon_\nu \sim 10^{18-21} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}$
$n + b \rightarrow p + b + l + \bar{\nu}_l$	$p + b + l \rightarrow n + l + b + \nu_l$ (Always)
Brmsstrahlung	$\epsilon_\nu \sim 10^{16-20} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}$
$b + b' \rightarrow b + b' + \nu + \bar{\nu}$	(Always)
Pair Breaking Formation	$\epsilon_\nu \sim 10^{21-22} T_9^7 \text{ erg cm}^{-3} \text{ s}^{-1}$
$b + b \rightarrow [bb] + \nu + \bar{\nu}$	(Only $T \leq T_{\text{cr}}$ )

$\epsilon_\nu$ : Neutrino emissivity of each process

$T_9$ : temperature  $T$  in units of  $10^9$  K ( $\sim 0.1$  MeV)

$T_{\text{cr}}$ : Superfluid transition temperature

# Cooling Effect of Neutrino Processes



$M_D$ : Critical mass of Direct URCA (Yakovlev & Pethick. 2004)

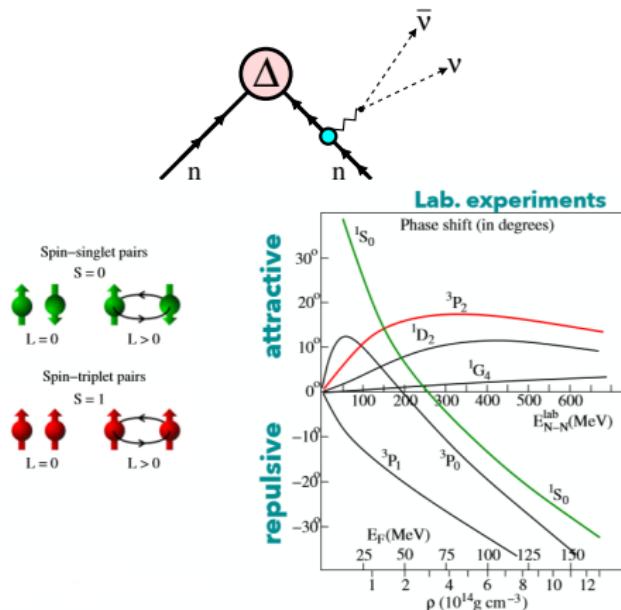
# Nucleon Superfluidity in NS

- Cooper paring occurs due to the attractive nuclear force:

$$b + b \rightarrow [bb] + \nu + \bar{\nu}$$

at  $T < T_{\text{cr}}^b \sim 10^{8-9} \text{ K}$

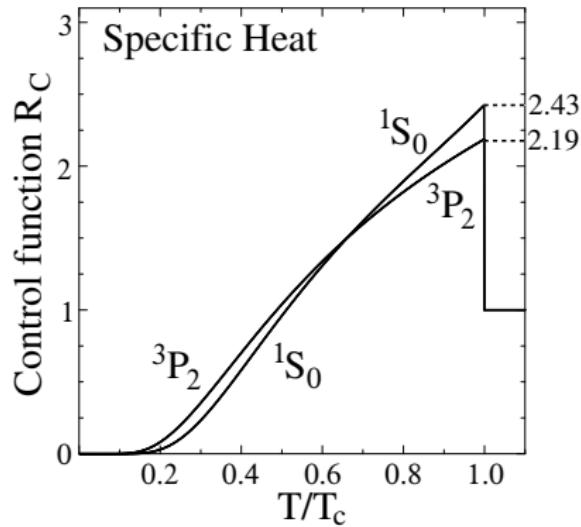
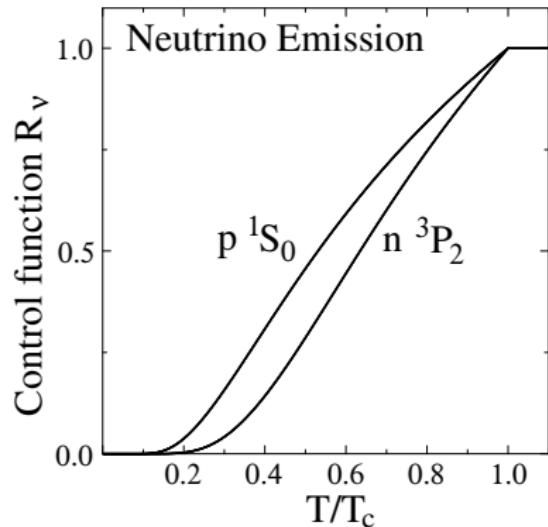
- SF in NS core
  - $p$  singlet pairing ( ${}^1S_0$ )
  - $n$  triplet pairing ( ${}^3P_2$ )
- SF in NS crust
  - $n$  singlet pairing ( ${}^1S_0$ )



(Page et al. 2013)

# Nucleon Superfluid (SF) Effect

## ① Cooling Suppression

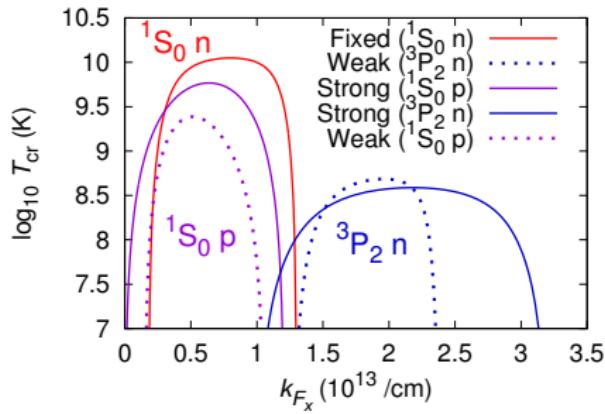
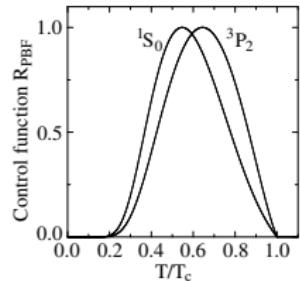


- $\epsilon_\nu^{\text{SF}} = R_\nu \epsilon_\nu^{\text{NONSF}}$ ,  $C_b^{\text{SF}} = R_C C_b^{\text{NONSF}}$
- Above all,  $[n^3P_2 n^3P_2]$  suppresses  $\nu$  emission

# Nucleon SF Effect

## ② Pair Breaking Formation (PBF)

- Release of Latent Heat
- $\epsilon_\nu \sim 10^{22} T_9^7 R_{\text{PBF}} \left( \frac{T}{T_{\text{cr}}} \right) \text{ erg cm}^{-3} \text{ s}^{-1}$
- Medium Cooling ( $>$  MU, Brems)
- $T_{\text{cr}}$  can be obtained from gap equations, but is unknown due to uncertain nuclear force.
- In this work, we adopt the right SF models  
(Ho *et al.* 2015)



# Photon Emission

- Dominant for  $t \gtrsim 10^{4-5}$  yr
- Blackbody  $\gamma$  emission, we have

$$\begin{aligned} L_\gamma &= 4\pi R^2 a T_{\text{eff}}^4 \\ &= 7 \times 10^{36} \text{ erg s}^{-1} \left( \frac{R}{10 \text{ km}} \right) a T_{\text{eff},7}^4 \end{aligned}$$

$a$ : Stefan-Boltzmann constant

$T_{\text{eff}}$ : Effective temperature in units of  $10^7$  K

- With atmosphere as INS envelop,  $L_\gamma$  becomes higher due to the opacity  $\kappa$ .  
⇐ Generally, the lighter the surface composition is, the higher  $L_\gamma$  is.

# Radiative Opacity $\kappa_{\text{rad}}$

- free-free absorption

(Schatz 1999)

$$\kappa_{ff} \propto \rho T^{-7/2}$$

- Scattering

(Paczynski 1983)

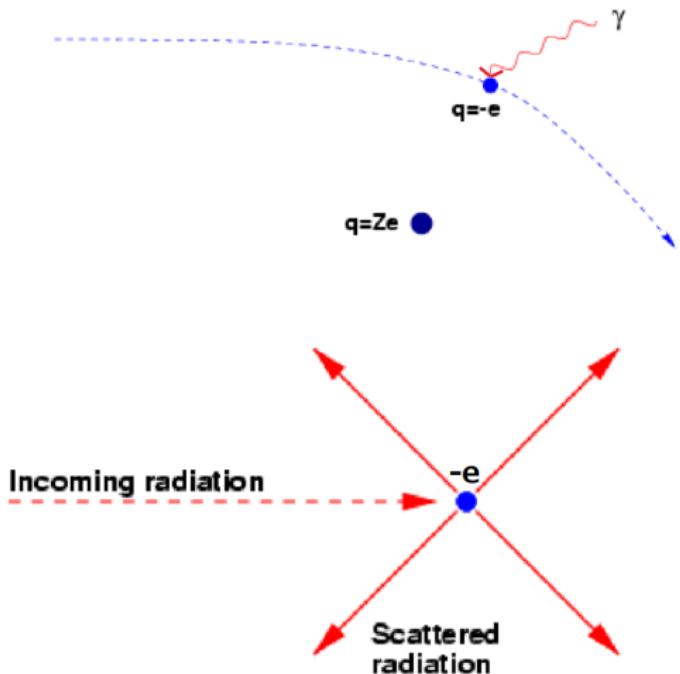
$$\kappa_{es} = \sigma_{\text{Th}} \frac{(1+X)}{2m_p}$$

- $\kappa_{\text{rad}} = \kappa_{ff} + \kappa_{es}$

$m_p$ : proton mass

$X$ : hydrogen mass fraction

$\sigma_{\text{Th}}$ : Thomoson scattering cross section



# Thermal Conductive Opacity $\kappa_{\text{cond}}$

$$\kappa_{\text{cond}} = \left( \sum_{i=n,p,e^-, \mu^-} \kappa_i^{-1} \right)^{-1} = \left( \sum_{i=n,p,e^-, \mu^-} \left( \frac{\pi^2 T n_B Y_i}{3 m_i^* \nu_c} \right)^{-1} \right)^{-1}$$

$\nu_c$ :  $i$ -particle collision frequency    $m_i^*$ :  $i$ -particle effective mass

Regions	Outer Crust	Inner Crust	Core
Dominant particles for $\kappa_{\text{cond}}$	$e^-$	$n$	$e^-, \mu^-$ and $n$

- We calculate the conductive opacities considering only electrons (Potekhin *et al.* 2015) and neutrons (Baiko *et al.* 2001).
- Total opacity  $\kappa$  is given by

$$\kappa = (\kappa_{\text{rad}}^{-1} + \kappa_{\text{cond}}^{-1})^{-1}$$

# $T_s - T_b$ relation

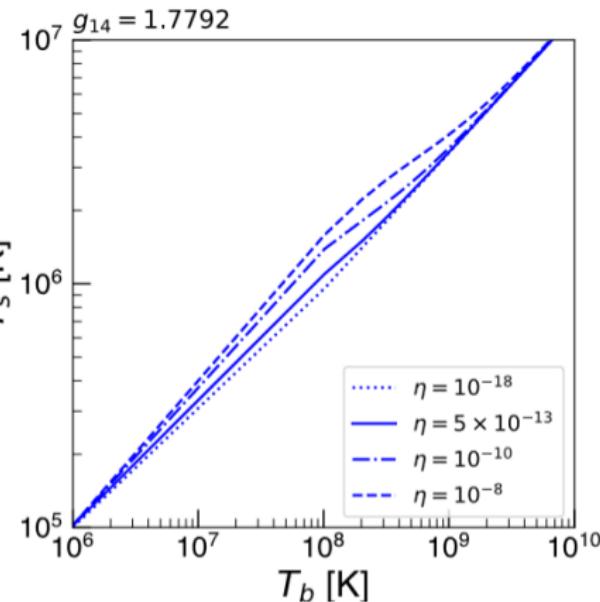
Since observed value is the surface temperature  $T_s$ , we have to connect  $T_s$  with interior temperature  $T_b$

- Parameter for the amount of light elements  $\Delta M$ :

$$\eta = \frac{g}{10^{14} \text{ cm s}^{-2}} \frac{\Delta M}{M}$$

with surface gravity  $g$ .

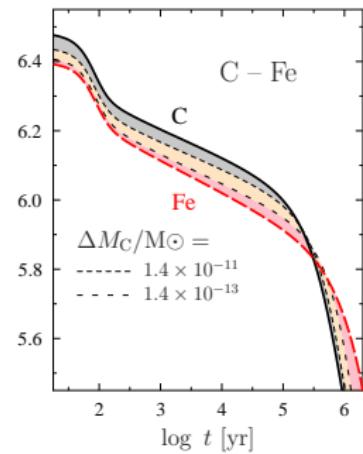
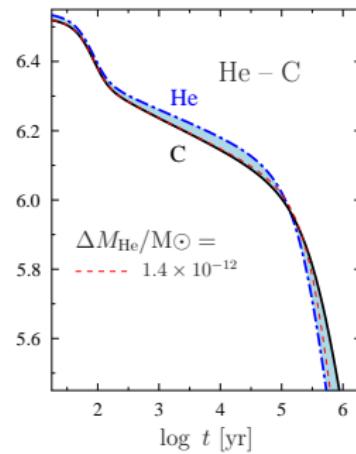
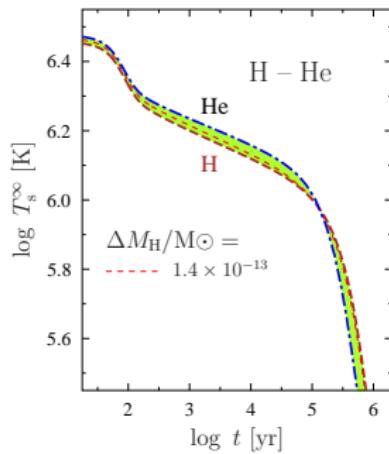
- The higher  $\eta$  is, the higher  $T_s$  is.



(Hamaguchi *et al.* 2018)

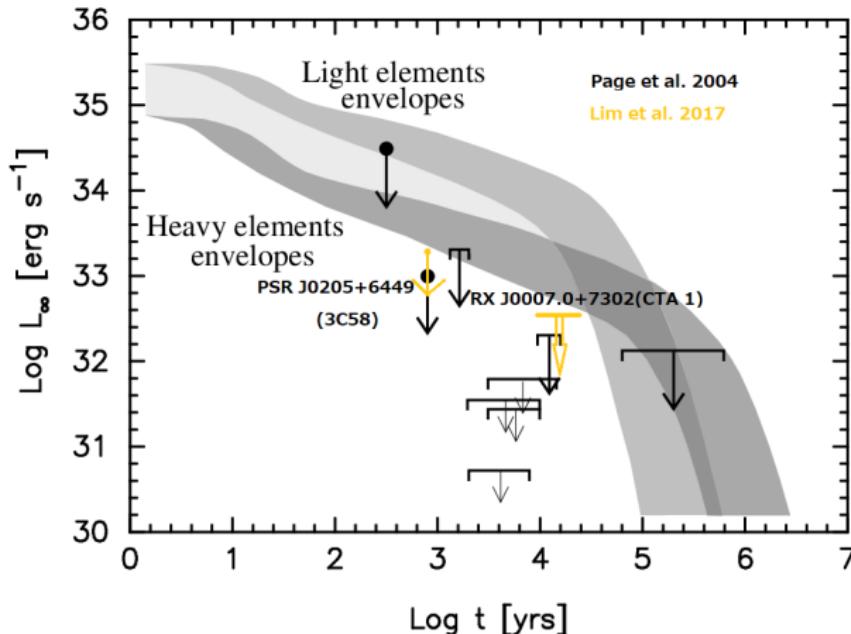
# Surface composition of INS envelop

- $T_{\text{eff}}$ : He>H>C>⋯>Fe(Ni) on INS envelop
- In this slides, we adopt Ni 100%, and H 73%, He 25%, and Ni 2% with  $\Delta M = 5 \times 10^{-13} M_{\odot}$ .



(Beznogov *et al.* 2016)

# Minimal Cooling Scenario



- Slow Cooling + Superfluidity (Page *et al.* 2004)
  - Inconsistent with stars in SNR 3C58 and CTA 1
- Rapid cooling processes are necessary !!

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# Uncertain Physics of INS cooling

Table: The list of several unknown parameters or INS cooling

Uncertain Parameters for given $M$ or central density	Affected amounts of Basic Equation
<b>1. Equation of State (EoS)</b> ⇒ $P - \rho_B$ or $M - R$ relations ⇒ How many particles in NS core ?	$\nu$ Loss Energy
2. Nucleon Superfluidity (SF) ⇒ $\rho_B Y_{\text{baryon}} - T_{\text{cr}}$ relations	$\nu$ Loss Energy
3. Surface Composition ⇒ Is NS envelop light or heavy ?	$T_{\text{eff}} - T_b$ Relation $\gamma$ Loss Energy

$T_{\text{cr}}$ : SF Transition Temperature  $T_b$ : Temperature outside NS Crust as  $\rho \sim 10^{8-10} \text{ g/cc}$

# Relativistic Heat Diffusion (Maarten Kater's M thesis 2011)

- The action of heat to describe diffusion of a scalar  $\phi$  field with a sink term  $Q$  is

$$\begin{aligned} S_{\text{diffuse}} &= \int dx^4 \frac{\sqrt{-g}}{2} \left( \phi \left( C_V n^\mu \partial_\mu - \frac{\kappa}{\sqrt{-g}} \partial_\mu \gamma^{\mu\nu} \partial_\nu \right) \phi + Q\phi \right) \\ &\approx \int dx^4 \frac{\sqrt{-g}}{2C_V} \left| \left( C_V n^\mu \partial_\mu - \frac{\kappa}{\sqrt{-g}} \partial_\mu \gamma^{\mu\nu} \partial_\nu \right) \phi + Q\phi \right|^2 \end{aligned}$$

where  $g$  is determinant of  $g^{\mu\nu} = n^\mu n^\nu + \gamma^{\mu\nu}$

$n^\mu = (\exp(-\phi), 0, 0, 0)$ ,  $\gamma^{\mu\nu}$ : spacial part

- Equation of motion of scalar field  $\phi$

$$\left( C_V n^\mu \partial_\mu - \frac{\kappa}{\sqrt{-g}} \partial_\mu \gamma^{\mu\nu} \partial_\nu \right) \phi + Q = 0$$

# Cooling Transport Equation

Solving Einstein Eq. with ideal fluid, we obtain

$$\frac{\partial(L_r e^{2\phi/c^2})}{\partial M_r} = -e^{2\phi/c^2} \left( \varepsilon_\nu + \varepsilon_\gamma + e^{-\phi/c^2} C_V \frac{\partial T}{\partial t} \right),$$
$$\frac{\partial(T_r e^{\phi/c^2})}{\partial r} = \frac{L_r}{4\pi r^2 \kappa}$$

This is enhanced version of classical *Heat Equation* as

$$\nabla \cdot \mathbf{J} = -C_V \frac{dT}{dt} - q$$
$$\kappa \nabla T = -\mathbf{J}$$

$\mathbf{J}$ : Heat Current,  $q$ : nonrelativistic loss of particles

# Basic Equations (Thorne 1977)

- Henyey method (Henyey *et al.* 1964)
- ① TOV equations
- ② Energy Transport

$$\frac{\partial \ln T}{\partial \ln P} = \frac{3}{4} \frac{\kappa L_r P}{act^4} \left( \frac{\partial P}{\partial M_r} \right)^{-1} + \left( 1 - \left( 1 + \frac{P}{\rho c^2} \right)^{-1} \right) ,$$

- ③ Energy conversation

$$\frac{\partial (L_r e^{2\phi/c^2})}{\partial M_r} = -e^{2\phi/c^2} \left( \epsilon_\nu + \epsilon_\gamma + e^{-\phi/c^2} C_V \frac{\partial T}{\partial t} \right) .$$

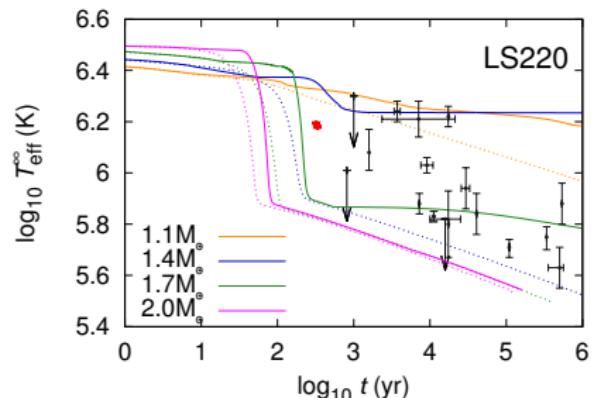
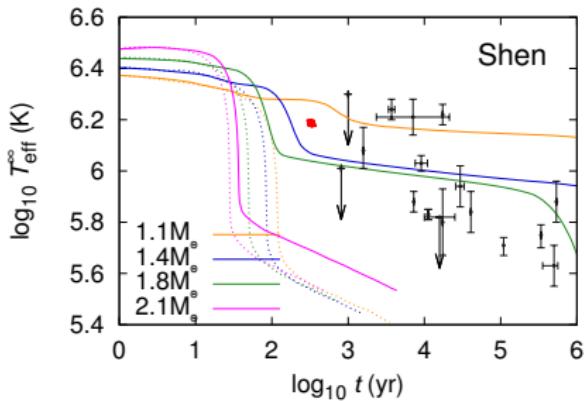
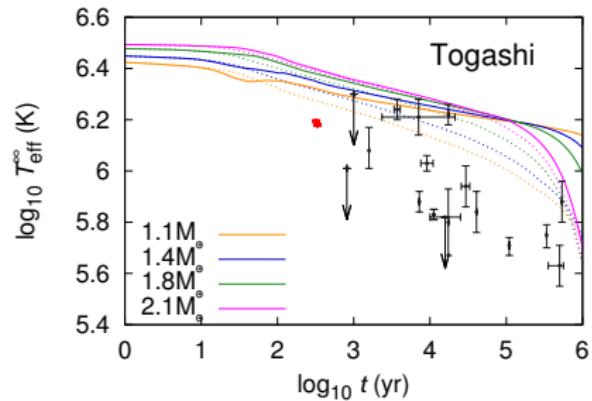
$\phi$ : gravitational potential,  $L_r$ : luminosity inside radius  $r$ ,  
 $\epsilon_\nu, \epsilon_\gamma$ : neutrino and photon emissivities, respectively,  
 $C_V$ : Capacity of NS,  $\kappa$ : opacity,  
 $\rho_0, \rho$ : rest and total mass density inside radius  $r$ , respectively,  
 $M_r, M_{tr}$ :rest and total mass inside radius  $r$ , respectively, which obeys a relation as

$$\frac{\partial M_{tr}}{\partial M_r} = \frac{\rho}{\rho_0} \left( 1 - \frac{2GM_{tr}}{c^2 r} \right)^{1/2} .$$

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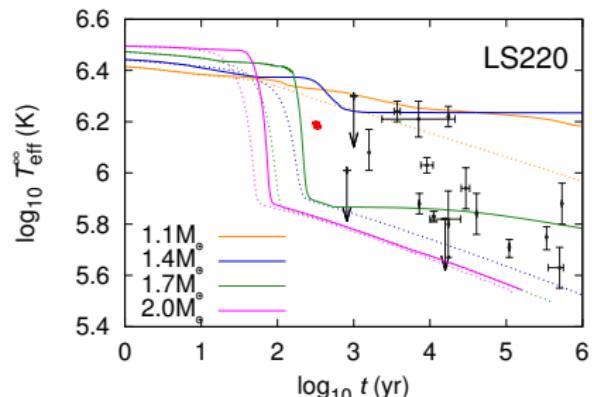
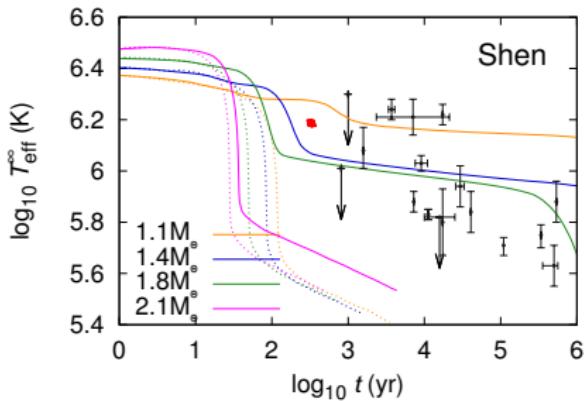
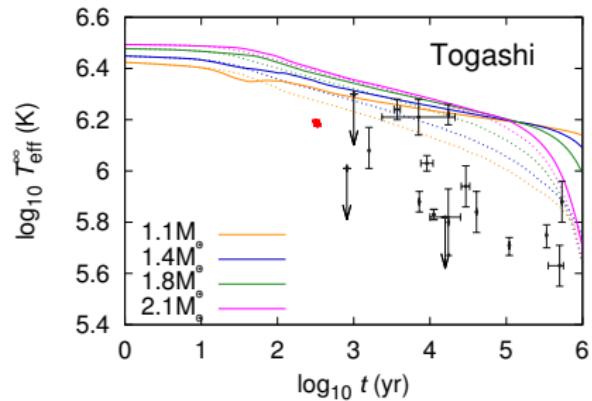
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# EoS dependence on cooling curves



..... : w/o SF model  
——— : w/ SF model

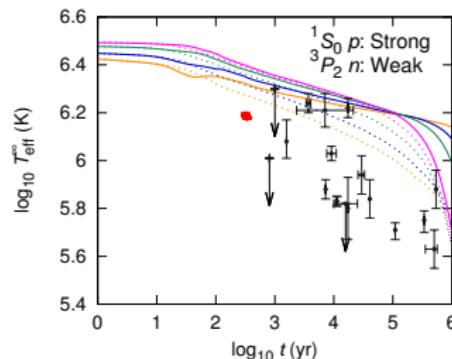
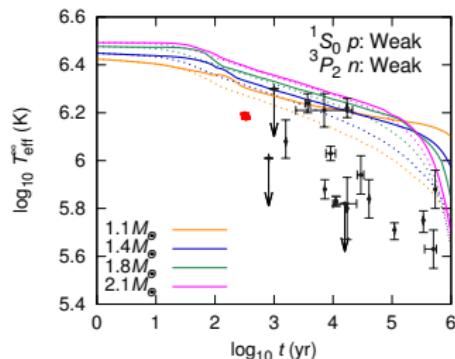
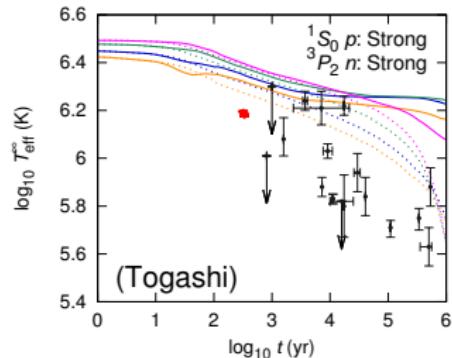
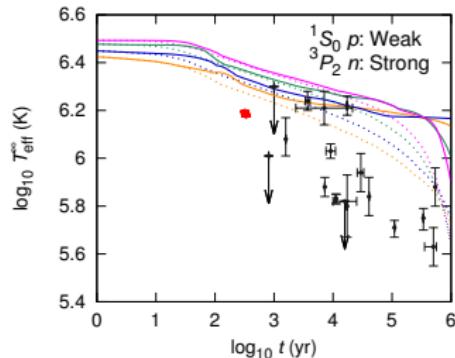
# EoS dependence on cooling curves



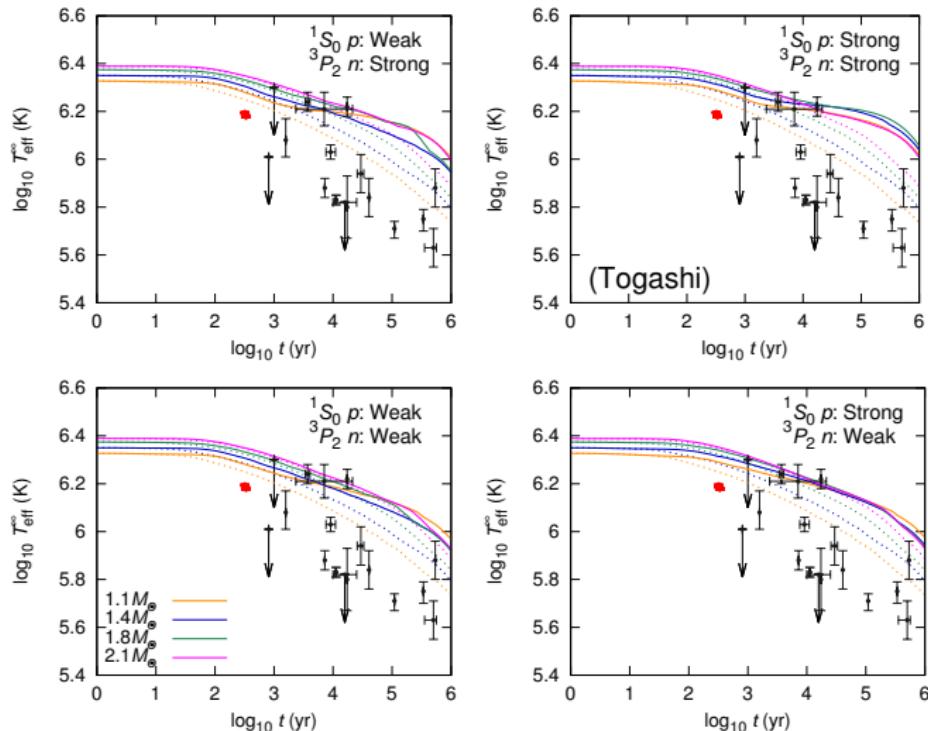
..... : w/o SF model  
——— : w/ SF model

- Even with  $2.1 M_{\odot}$ , Direct URCA is prohibited with the Togashi EoS !!

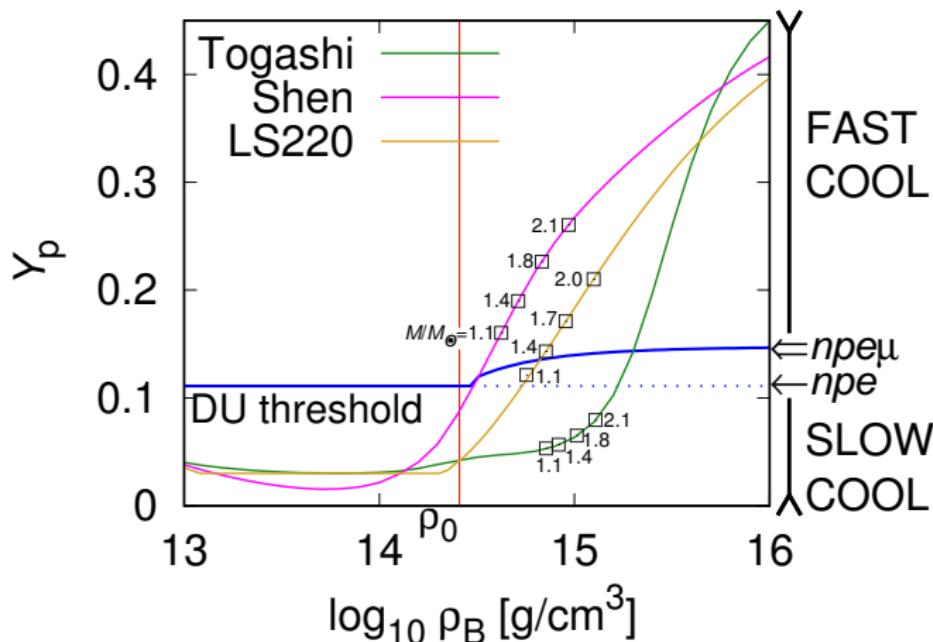
# SF-model dependence with Togashi EoS (with light compositions)



# SF-model dependence with Togashi EoS (with heavy compositions)

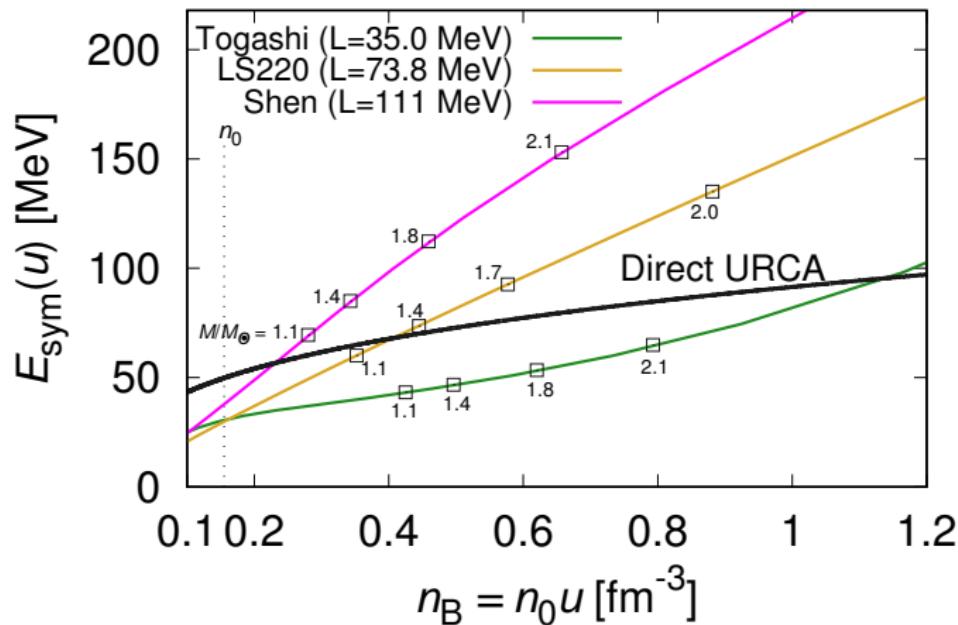


# Density dependence of $Y_p$



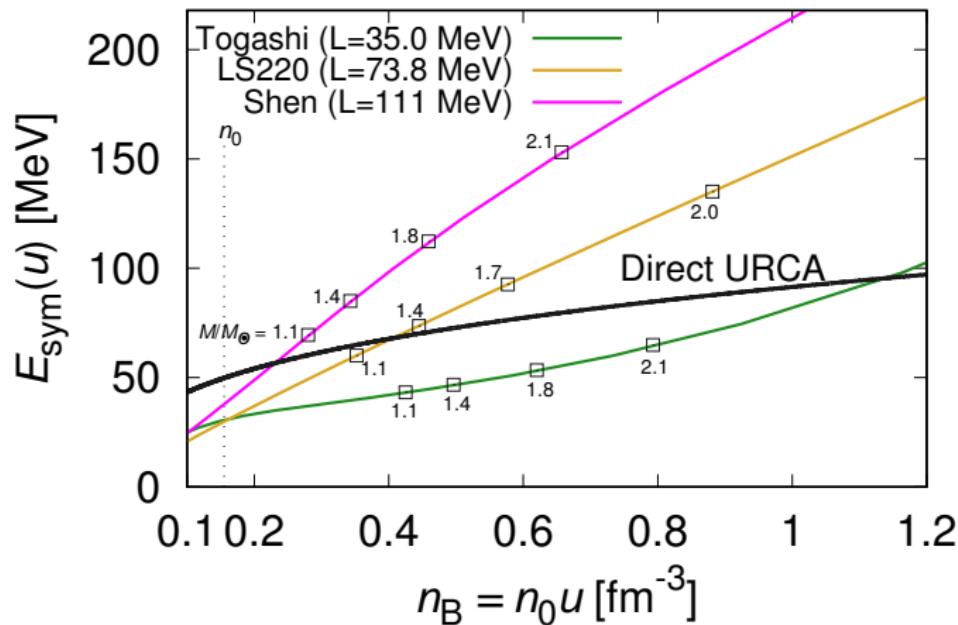
- The Togashi EoS has low value of  $Y_p$  and is inconsistent with INS observations.

# Symmetry Energy and Rapid Cooling



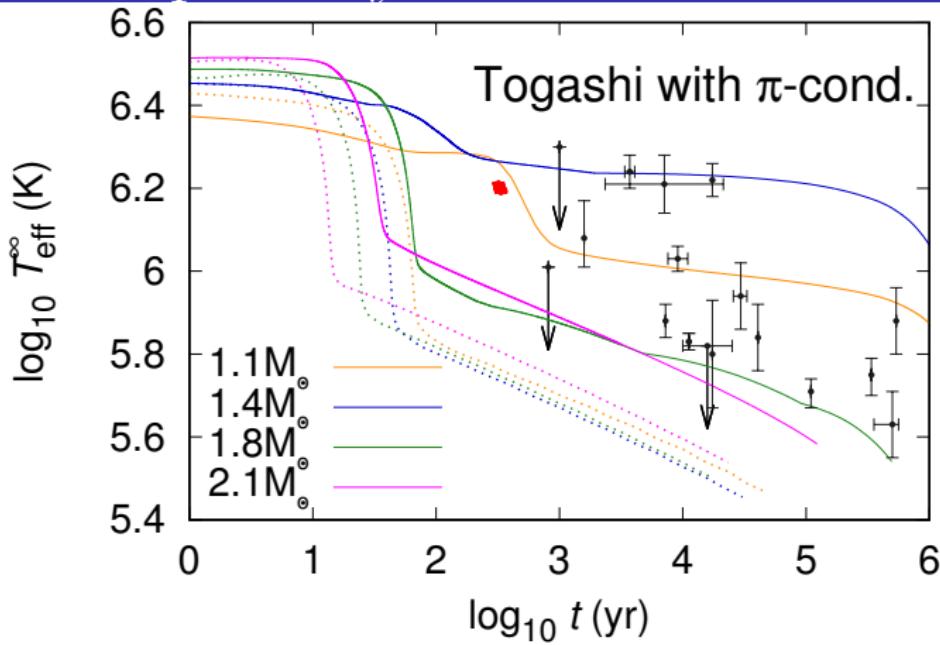
- Direct URCA is hard to occur for the model with small  $E_{\text{sym}}(u)$  or  $L$ , that is, small  $R$  values

# Symmetry Energy and Rapid Cooling

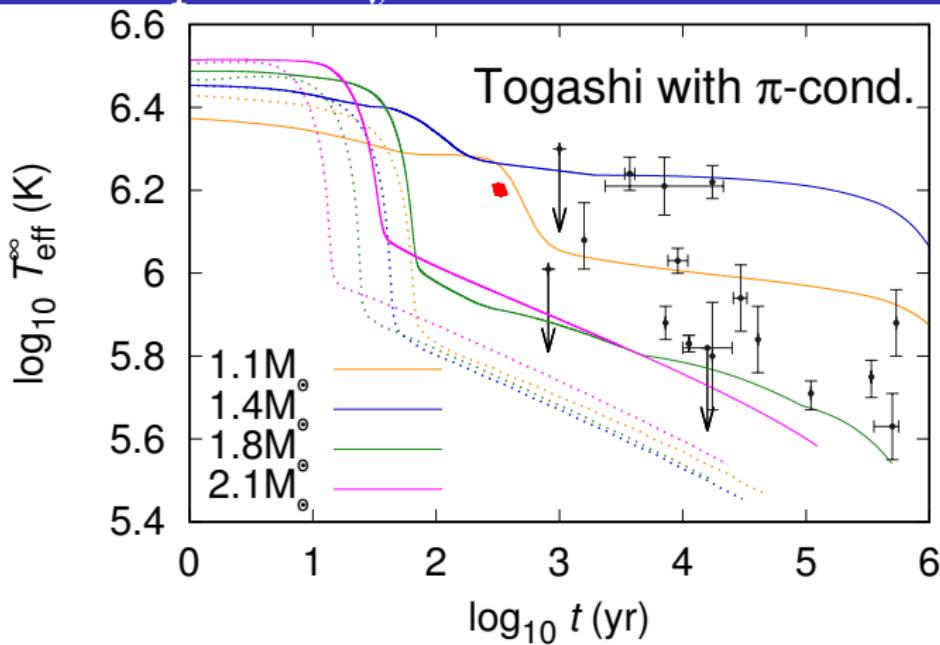


- Direct URCA is hard to occur for the model with small  $E_{\text{sym}}(u)$  or  $L$ , that is, small  $R$  values  
⇒ Another strong cooling process is needed !!

# Cooling curves with pion condensation process adopted by Muto *et al.* 1993



# Cooling curves with pion condensation process adopted by Muto *et al.* 1993



- ⇒ Togashi EoS with pion condensation is consistent !!
- ⇒ Some exotic particles may be in neutron stars !?

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# How to dissolve the gap between INS observations and EoS with small $L$ value?

An EoS with small  $L$  can account for radius constraint but cannot INS observations.

⇒ How to dissolve ?

## ① Considering exotic particles

- Some enhanced cooling processes by exotic particles
- However, EoS becomes softer and NS may not support  $2M_{\odot}$  (ex. *Hyperon puzzle*).

## ② Changing the NS structure

- Changing the TOV equations by introducing a modified gravity

We introduce the modified gravity as Second Scalar-Tensor (ST) theory.

# Action of Gravity

- Einstein-Hilbert Action (GR)

$$S_G = \frac{M_{pl}^2}{2} \int dx^4 \sqrt{-g} R$$

- Action of ST theory with massless Brans-Dicke model:

$$S_G = \int dx^4 \sqrt{-g} \left[ \frac{M_{pl}^2}{2} F(\phi) R - (1 - 6Q^2) F(\phi) X \right]$$

$R$ : Ricci scalar     $M_{pl} = (8\pi G)^{-1/2}$ : Reduced Planck Mass

$g$ : the determinant of metric tensor  $g_{\mu\nu}$

$Q$ : parameter to characterize the coupling between the field  $\phi$  and the gravity sector. (GR if  $Q \rightarrow 0$ )

$$X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi / 2, \quad F(\phi) = \exp(-2Q\phi/M_{pl})$$

(we adopt natural units  $c = \hbar = 1$ )

# TOV Equations

- ① Mass Eq.

$$\frac{\partial M}{\partial r} = 4\pi r^2 \rho$$

- ② Hydrostatic equilibrium Eq.

$$\frac{\partial P}{\partial r} = -\frac{G}{r^2} (\rho + P) (M + 4\pi r^3 P) \left(1 - \frac{2GM}{r}\right)^{-1}$$

- 3 Variables as  $P(r)$ ,  $\rho(r)$ ,  $M(r)$
- Boundary condition:  $\rho(r = 0) = \rho_c$

# Eq. of Motions (Kase & Tsujikawa. 2019)

① Mass Eq.

$$\begin{aligned} M' &= -4\pi F^{-1} r^2 \left[ (1 - 2Q^2) \rho + 6Q^2 P \right] \\ &+ \phi' \frac{2QM_{pl}M + 8QM_{pl}\pi r^3 F^{-1}P + r\phi' \left( 4\pi r M_{pl}^2 - M \right) (1 + 2Q^2)}{2M_{pl} (M_{pl} - Qr\phi')} \end{aligned}$$

② Hydrostatic equilibrium Eq.

$$-\frac{2P'}{\rho + P} = \frac{2M_{pl}^2 (h - 1) - 2F^{-1} r^2 P + hr\phi' [(6Q^2 - 1) r\phi' - 8QM_{pl}]}{2hrM_{pl} (M_{pl} - Qr\phi')}$$

③ Euler-Lagrange Eq. of  $\phi$

$$\begin{aligned} \phi'' &= -\frac{\phi'}{2M_{pl}^2 rh} \left[ 2(h + 1) M_{pl}^2 + r^2 F^{-1} (P - \rho + 2(\rho - 3P)Q^2) \right] \\ &+ \frac{Q(\rho - 3P)}{M_{pl} h F} \end{aligned}$$

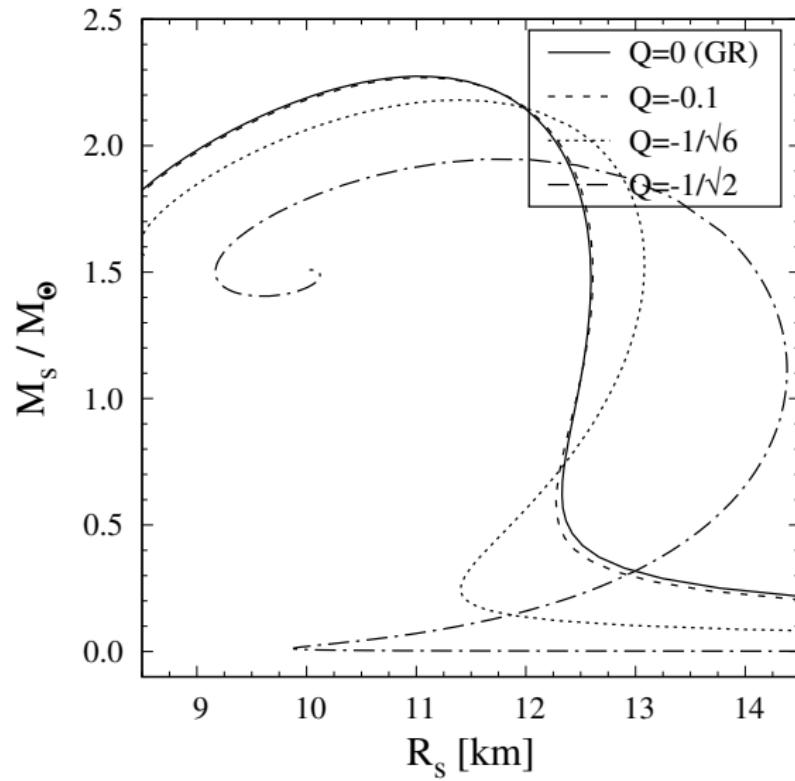
• 4 Variables:  $P(r), \rho(r), M(r), \phi(r)$ , due to  $h(r) = 1 - 2M/r$

• Boundary conditions:

$$\rho(0) = \rho_c, \quad \phi(0) = \phi'(0) = 0, \quad \phi(\infty) \rightarrow \phi_\infty, \quad \phi'(\infty) \rightarrow 0$$

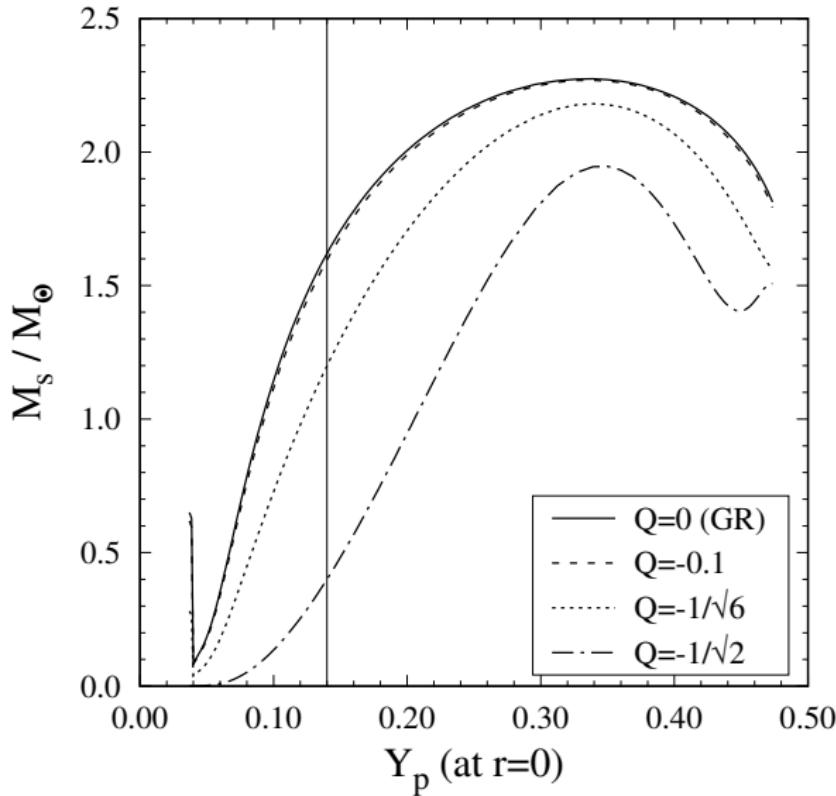
# $MR$ relation (Calculated by R. Kase)

BSk21

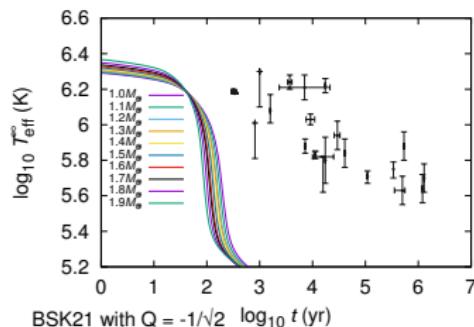
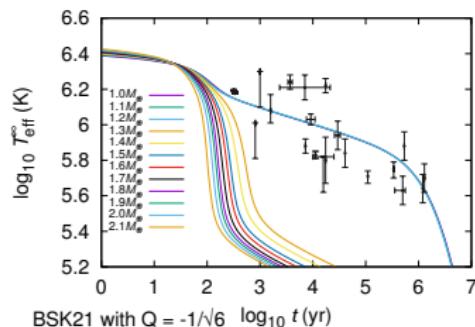
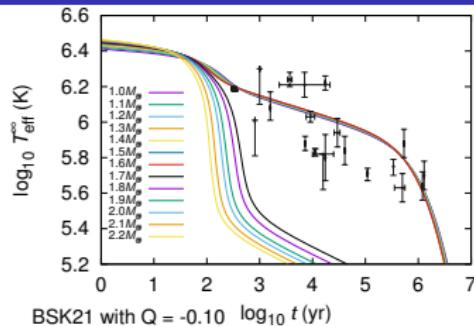
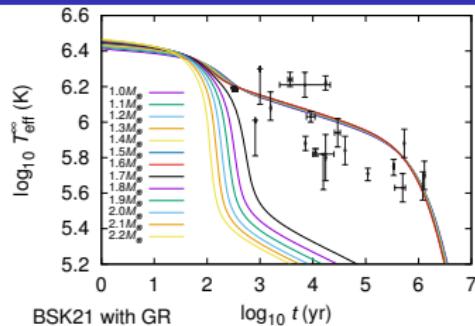


# $Y_p$ distribution

BSk21



# Cooling Curves with modified gravity



The smaller  $Q$  is, DU occurs with smaller mass.

⇒ Can modified gravity can dissolve the problem between EoS constraint and INS observation ?

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# Summary & Future Work

- For Togashi EoS, mass and radius constraints are satisfied, but the cooling curves cannot account for the thermal data.
- However with some strong coolings, such a EoS becomes "consistent" model !!
  - ⇒ Exotic particles may be in the neutron star !!
  - ⇒ Or possibility of modified gravity ?

## IN FUTURE WORK

- The cooling simulation for the Togashi EoS with other exotic particle (above all  $\Lambda$ , mesons)
- Cooling Calculations with massive Brans-Dicke (but Pessimistic due to *Chameleon Mechanism*)