QCD和則の基礎とその有限密度中のハドロンに対する応用

P. Gubler and M. Oka, Prog. Theor. Phys. **124**, 995 (2010).
P. Gubler, K. Morita and M. Oka, Phys. Rev. Lett. **107**, 092003 (2011).
K. Ohtani, P. Gubler and M. Oka, Eur. Phys. J. A **47**, 114 (2011).
K. Ohtani, P. Gubler and M. Oka, Phys. Rev. D **87**, 034027 (2013).
K. Suzuki, P. Gubler、K. Morita and M. Oka, Nucl. Phys. **A897**, 28 (2013). K-J. Araki, K. Ohtani、P. Gubler and M. Oka, arXiv:1403.6299 [hep-ph], to be published in PTP. P. Gubler and K. Ohtani, arXiv:1404.7701 [hep-ph].

Seminar
@ Chiba Institute of Technology
21.6.2014
Philipp Gubler (RIKEN, Nishina Center)

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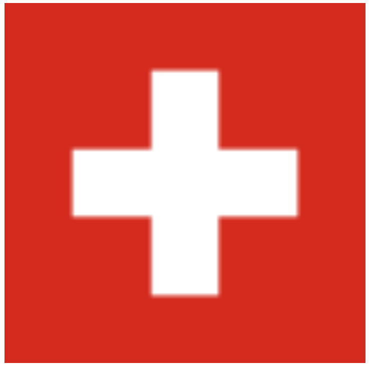
ワールドカップ速報：

今朝は5時前に起床し、スイス対フランス戦を見ました…



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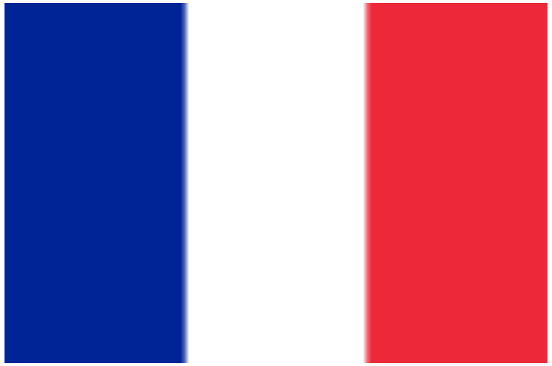

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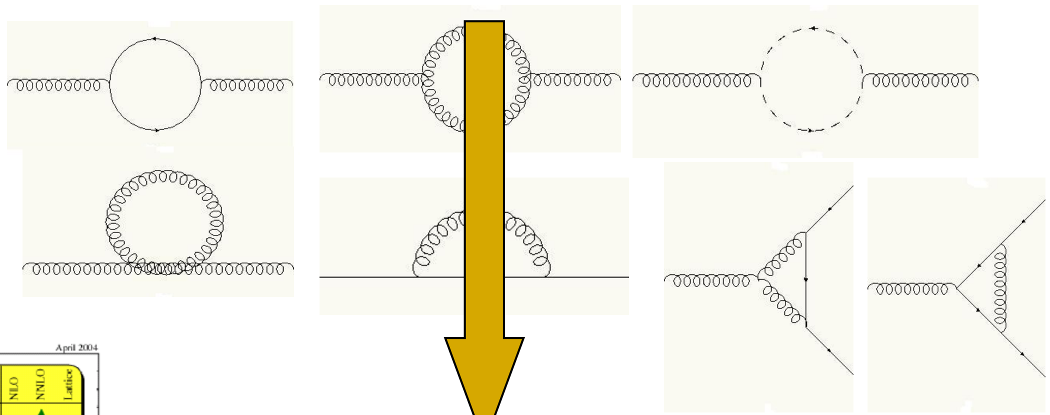
起きる意味がありませんでした…

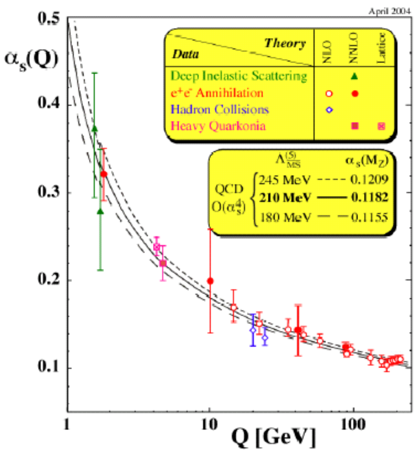
Contents

- Introduction, Motivation
  - QCD at low energies and the need for non-perturbative approaches
  - QCD sum rules, its strengths and weaknesses
- The tool for our novel analysis method: the maximum entropy method
  - Bayes' theorem
  - Shannon-Jaynes entropy
- Physical results
  - ρ meson
  - φ meson, nucleon, D meson at finite density
  - (Charmonium and bottomonium at zero and finite temperature)
- Conclusions and outlook

Introduction

$$\mathcal{L}_{QCD} = \sum_{q,j} \overline{q}^j(x)(i\partial^\mu \gamma_\mu - m_q)q^j(x) + g\sum_q \sum_{ika} \overline{q}^i(x)\gamma_\mu t^a_{ik} q^k(x) B^\mu_a(x)$$
$$-\frac{1}{4}\sum_a G^{\mu\nu}_a(x) G_{\mu\nu a}(x)$$





Asymptotic freedom

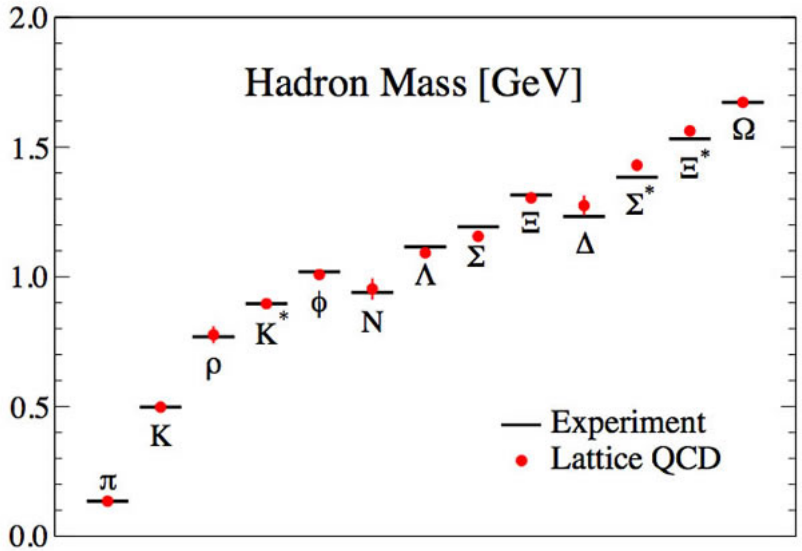
$$\alpha_S(Q^2) = \frac{12\pi}{(33-2N_f)\log(\frac{Q^2}{\Lambda^2_{QCD}})}$$

D.J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973).

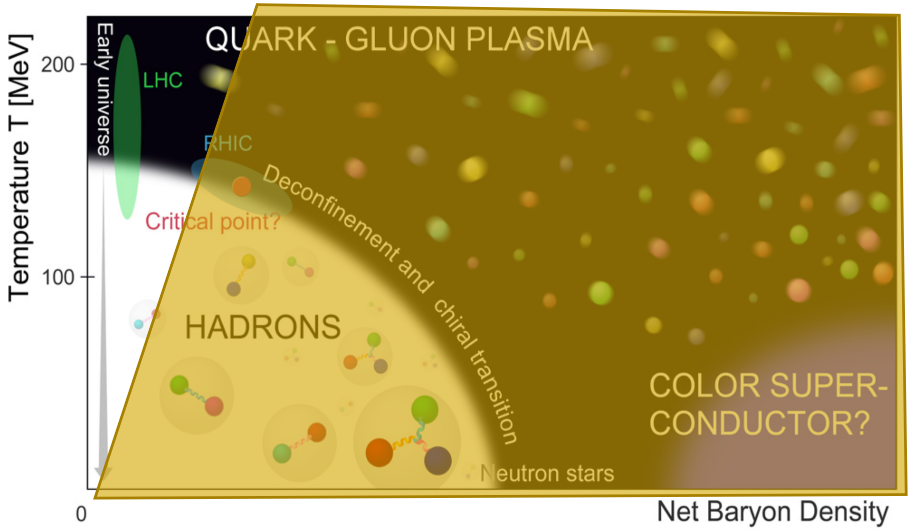
H.D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973).

Non-perturbative methods are needed!

adapted from:
S. Aoki et al., Phys. Rev. D **79**, 034503 (2009).



However…



<u>Lattice QCD</u>

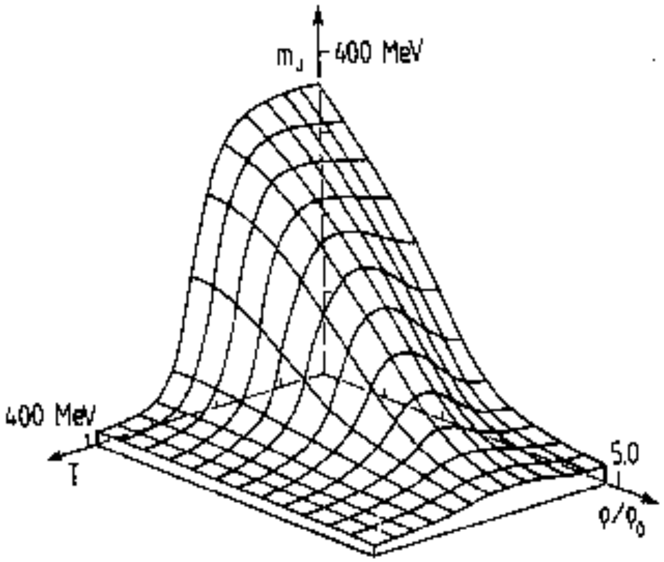
Best choice for most cases.

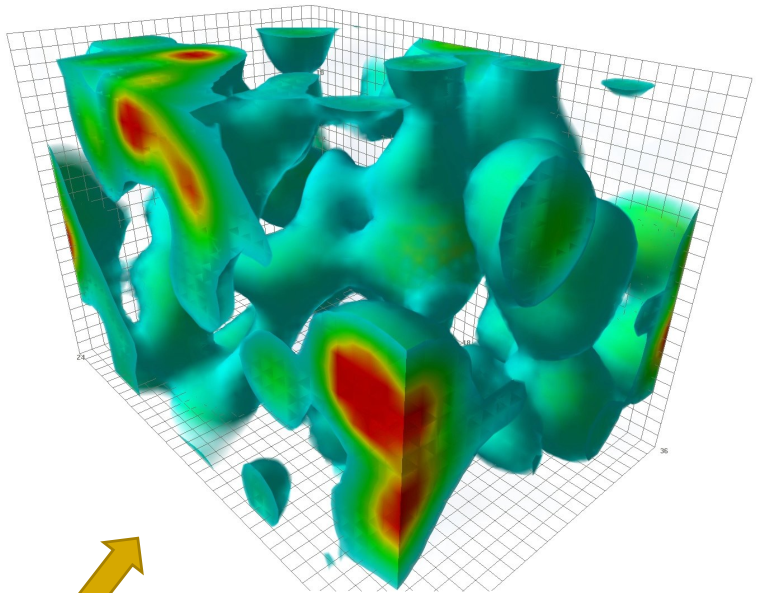
Sign problem at finite density! Lattice QCD studies for system with large $\frac{\mu}{T}$ are still not possible

Another peculiarity of QCD: Condensates

Most famous examples:

$$\langle 0|\overline{q}q|0\rangle \qquad \langle 0|G^a_{\mu\nu}G^{a\mu\nu}|0\rangle$$





Change as a function of temperature T and density ρ

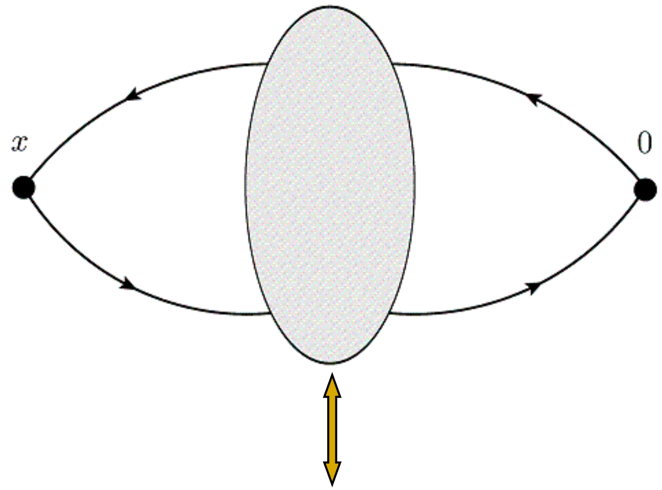
Basics of QCD sum rules

In QCD sum rules one considers the following correlator:

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 | T\{\chi(x)\overline{\chi}(0)\} | 0 \rangle$$

$$(\chi = \bar{q}q,\ qqq)$$

For example, mesons:



Complicated interaction governed by QCD.

$$\Pi(q^2) = i\int d^4x e^{iqx}\langle 0|T\{\chi(x)\overline{\chi}(0)\}|0\rangle$$

In the region of Π(q) dominated by large energy scales such as

$$\boxed{-q^2 \gg \Lambda^2_{QCD}}$$

it can be calculated by the operator product expansion (OPE):

perturbative Wilson coefficients

$$i\int d^4x e^{iqx}\langle 0|T\{\chi(x)\overline{\chi}(0)\}|0\rangle = C_I(q^2)I + \sum_n C_n(q^2)\langle 0|O_n|0\rangle$$

$$\begin{aligned}\langle 0|O_n|0\rangle = \; & \langle 0|\overline{q}q|0\rangle, \\ & \langle 0|G^a_{\mu\nu}G^{a\mu\nu}|0\rangle, \\ & \langle 0|\overline{q}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^{a\mu\nu}q|0\rangle, \\ & \langle 0|\overline{q}q\overline{q}q|0\rangle, \ldots\end{aligned}$$

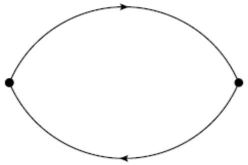
non-perturbative condensates

The OPE from Feynman diagrams:

leading term

α_s corrections

$C_I(q^2)I$:



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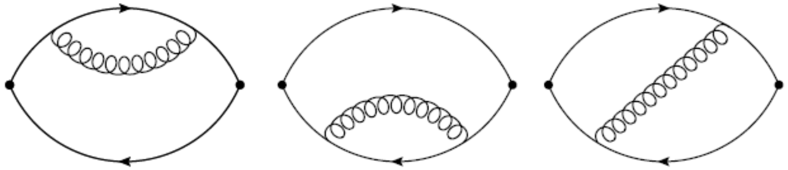

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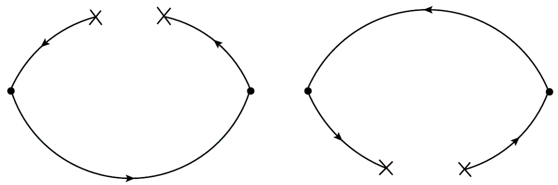
$C_{\langle \bar{q}q \rangle}(q^2)\langle \bar{q}q \rangle$:



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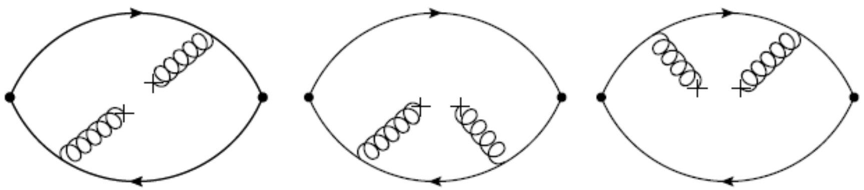
$C_{\langle G^2 \rangle}(q^2)\langle G^2 \rangle$:



+...

+...

$$\Pi(q^2) = i\int d^4x e^{iqx}\langle 0|T\{\chi(x)\overline{\chi}(0)\}|0\rangle$$

On the other hand, we consider the above correlator in the region of

$$\boxed{q^2 > 0}$$

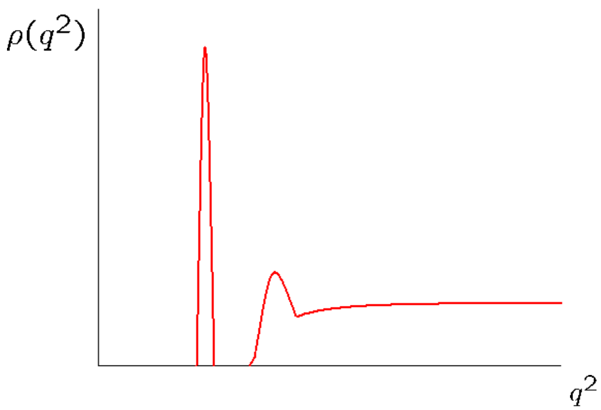
where the optical theorem (unitarity) gives

$$\frac{1}{\pi}\mathrm{Im}\Pi(q^2) = (2\pi)^3\sum_n \langle 0|\chi|n\rangle\langle n|\overline{\chi}|0\rangle d\tau_n \delta(q-p_n)$$

physical states

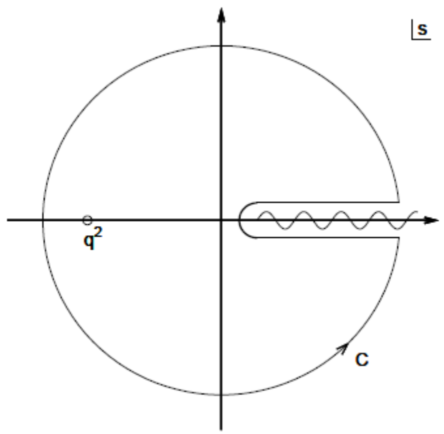
$$\equiv \rho(q^2)$$

the spectral function



Relating the two regions: the dispersion relation

$-q^2 \gg \Lambda^2_{QCD}$ $\Longleftrightarrow$ $q^2 > 0$



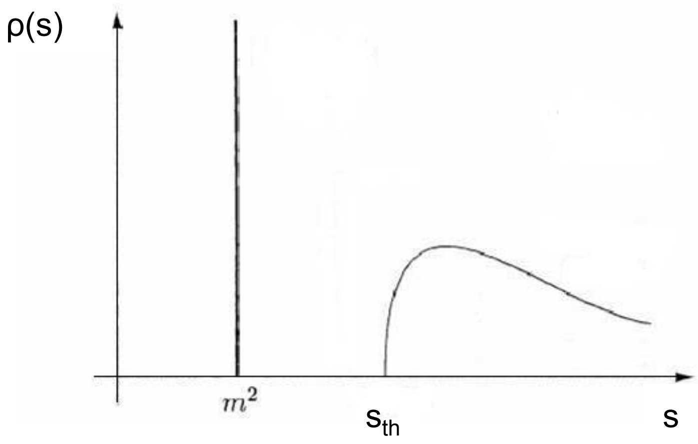
$$\to \Pi(q^2) = \frac{1}{\pi}\int_0^{\infty} ds \frac{\mathrm{Im}\Pi(s)}{s - q^2 - i\epsilon}$$

After the Borel transformation: $G(M) \equiv \lim_{\substack{-q^2, n\to\infty \\ (\frac{-q^2}{n} = M^2)}} \frac{(-q^2)^{n+1}}{n!}\left(\frac{d}{dq^2}\right)^n \Pi(q^2)$

$$G(M) = \int_0^{\infty} ds e^{-\frac{s}{M^2}} \rho(s)$$

The traditional analysis method:

This spectral function is approximated by a “pole + continuum” ansatz:



$$\rho(s) = \lambda^2 \delta(s - m^2) + \theta(s - s_{th}) \frac{1}{\pi} \mathrm{Im}\Pi^{OPE}(s)$$

Even though this ansatz is very crude, it works quite well in cases for which it is phenomenologically known to be close to reality.

e.g. - charmonium (J/ψ)

-ρ-meson

Inserting the "pole + continuum" ansatz into the sum rules, we get

$$\Pi^{OPE}(M^2) = \lambda^2 e^{-\frac{m^2}{M^2}} + \frac{1}{\pi}\int_{s_{th}}^{\infty} ds e^{-\frac{s}{M^2}} \mathrm{Im}\Pi^{OPE}(s)$$

From this equation, the mass m of the ground state can be obtained as:

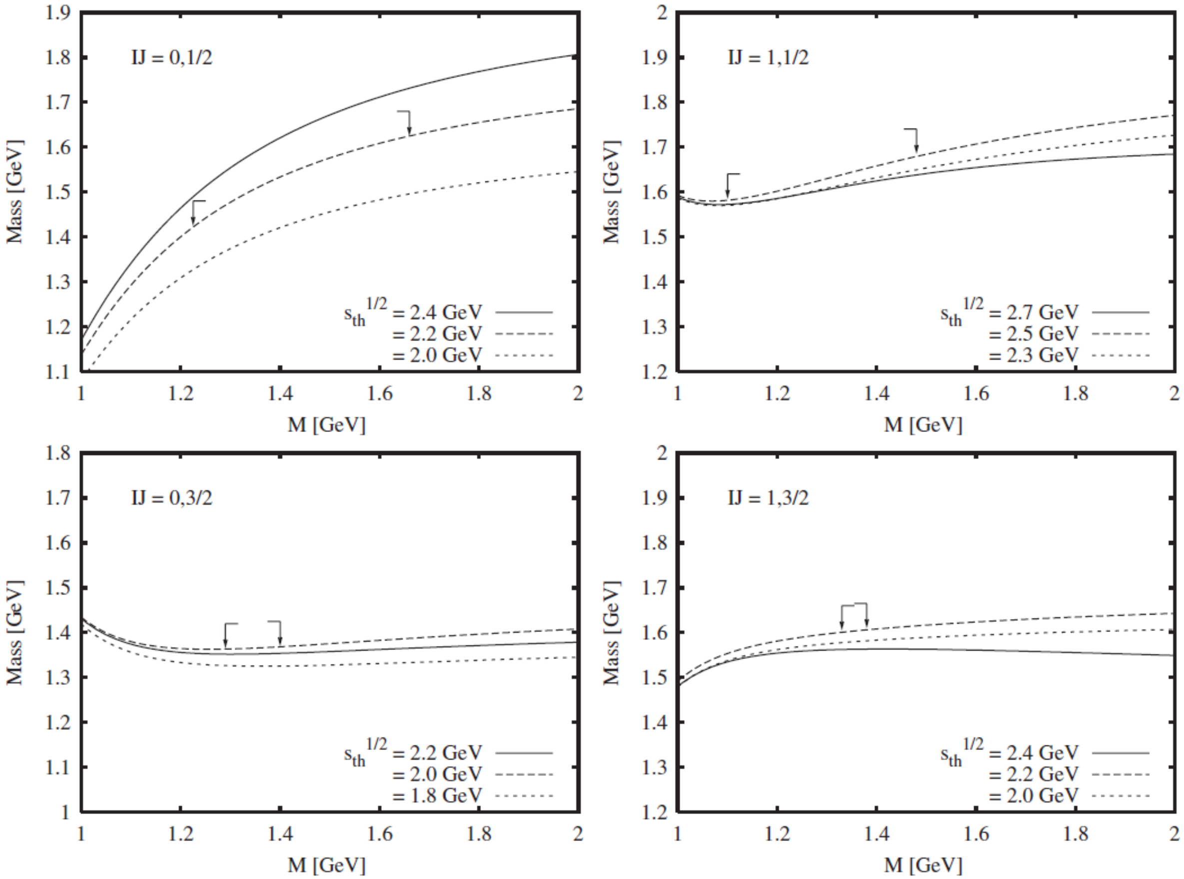
$$\lambda^2 e^{-\frac{m^2}{M^2}} = \frac{1}{\pi}\int_0^{s_{th}} ds e^{-\frac{s}{M^2}} \mathrm{Im}\Pi^{OPE}(s)$$

$$\rightarrow \quad m^2 = \frac{\frac{\partial}{\partial(-1/M^2)} \int_0^{s_{th}} ds e^{-\frac{s}{M^2}} \mathrm{Im}\Pi^{OPE}(s)}{\int_0^{s_{th}} ds e^{-\frac{s}{M^2}} \mathrm{Im}\Pi^{OPE}(s)}$$

Should not depend on M and s_{th}

Some examples:

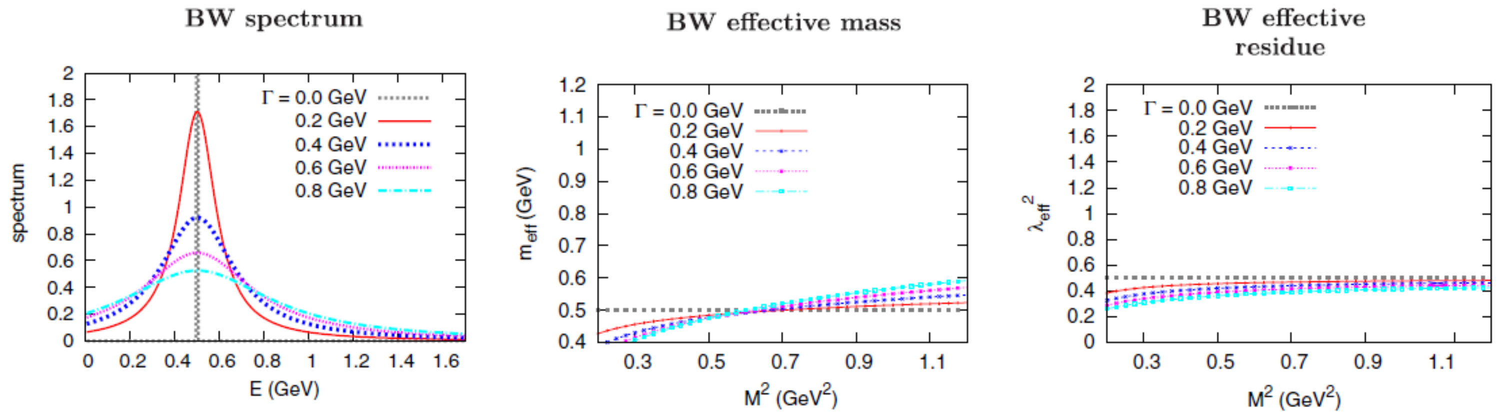
Study of various possible quantum numbers of the pentaquark $\Theta^+(1540)$:



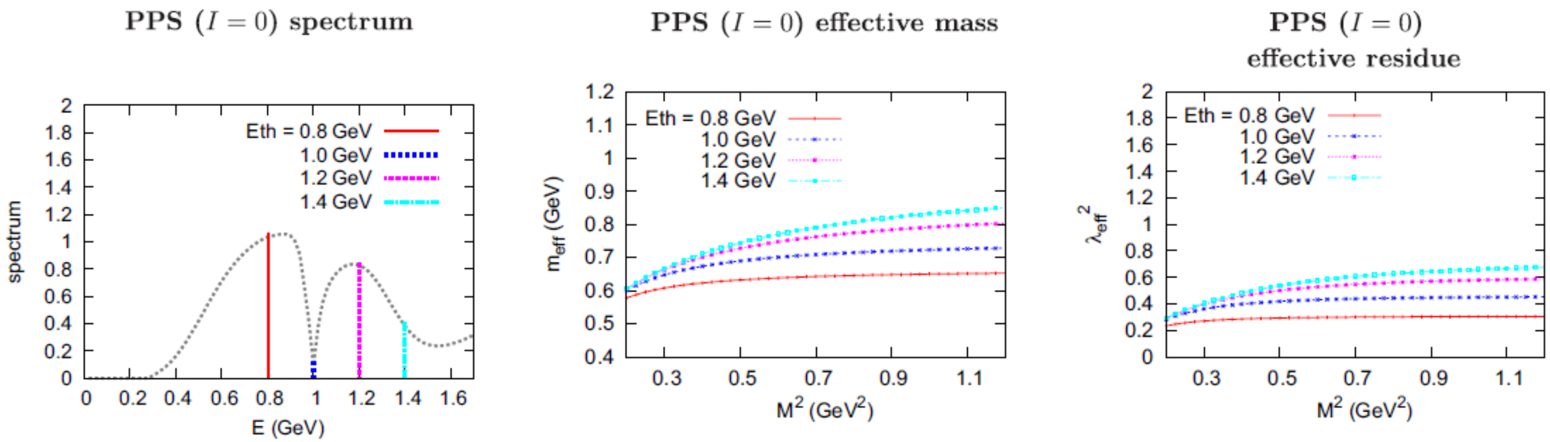
PG, D.Jido, T.Kojo, T.Nishikawa, M.Oka, Phys. Rev. D **80**, 114030 (2009).

A study of the σ-meson channel:

Spectrum with Breit-Wigner peak:



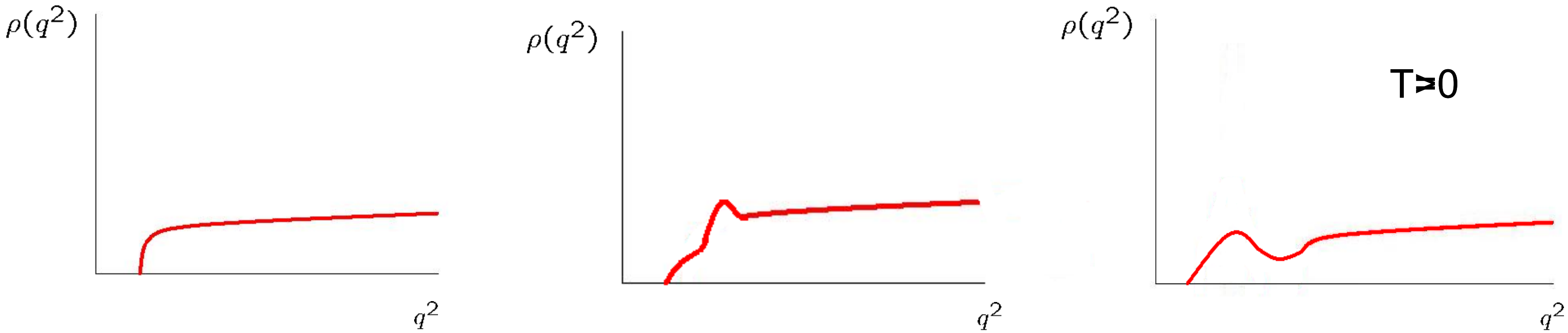
Spectrum with ππ scattering:



T.Kojo and D. Jido, Phys. Rev. D **78**, 114005 (2008).

This ansatz can, however, not always work!

For instance, for:



A more general analysis method is desirable

MEM analysis of QCD sum rules could be useful!

Basics of the Maximum Entropy Method

A mathematical problem:

$$G(M) = \int_0^\infty d\omega K(M,\omega)\rho(\omega)$$

given

(but only incomplete and
with error)

"Kernel"

?

This is an ill-posed problem!

But, one may have additional information on $\rho(\omega)$, such as:

$$\rho(\omega) \geq 0$$

How can one include this additional information and find the most probable image of $\rho(\omega)$?

$\rightarrow$ Bayes' Theorem

$$P[\rho|G,I] = \frac{P[G|\rho,I]\,P[\rho|I]}{P[G|I]}$$

likelihood function

prior probability

$$\rightarrow \quad \frac{\delta P[\rho|G,I]}{\delta\rho} = 0$$

Likelihood function

Gaussian distribution is assumed:

$$P[G|\rho, I] = \frac{1}{Z_L} e^{-L[\rho]}$$

$$L[\rho] = \frac{1}{2(M_{\max} - M_{\min})} \int_{M_{\min}}^{M_{\max}} dM \frac{\left[G_{OPE}(M) - G_{\rho}(M)\right]^2}{\sigma^2(M)}$$

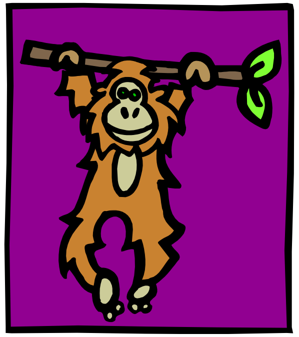
Minimalizing this likelihood function corresponds to the standard χ^2-fitting.

But, this procedure would not be stable in the present problem. The prior probability is therefore necessary.

Prior probability (1)

$$P[\rho|I]?$$

Monkey argument:

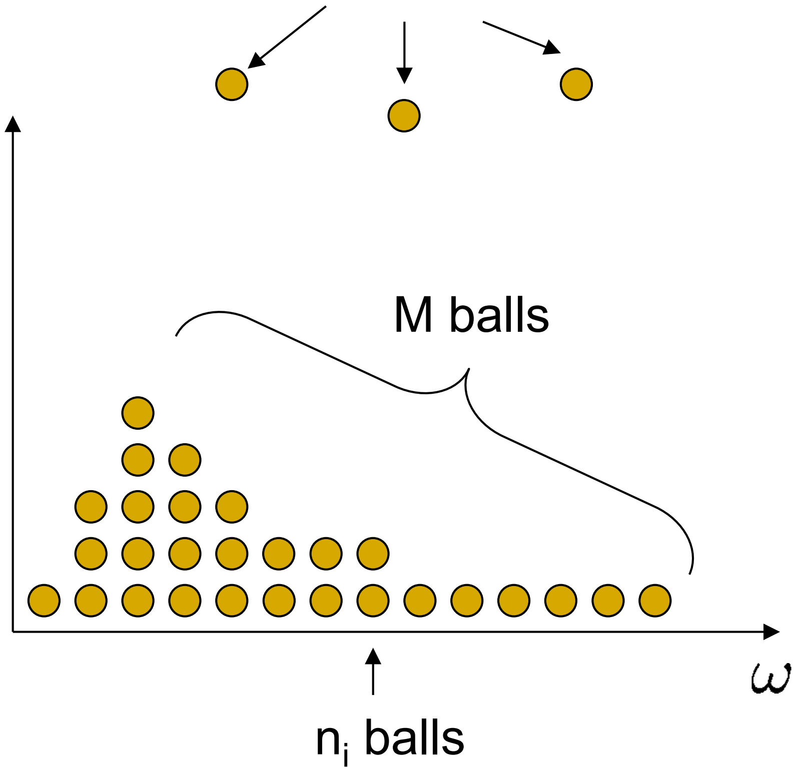


Probability of n_i balls falling into position i:

$$\frac{M!}{n_i!(M-n_i)!}p_i^{n_i}(1-p_i)^{M-n_i} \underset{(M\to\infty)}{\rightarrow} \frac{\lambda_i^{n_i}e^{-\lambda_i}}{n_i!}$$

Poisson distribution

$\rho(\omega)$



Probability of a certain image $(n_1, n_2, \ldots, n_N)$:

$$P_{\vec{\lambda}}(\vec{n}) = \prod_{i=1}^{N} \frac{\lambda_i^{n_i}e^{-\lambda_i}}{n_i!}$$

Prior probability (2)

To change the discrete image (n_1, n_2, …,n_N) into a continuous function, one takes a small number q and defines:

$$\rho_i = q n_i,\ m_i = q\lambda_i$$

Then, the probability for the image ρ(ω) to be in Π_i dρ_i becomes:

$$dP(\rho) = \frac{\prod_{i=1}^{N} d\rho_i}{q^N} \prod_{i=1}^{N} \frac{\lambda_i^{n_i} e^{-\lambda_i}}{(n_i)!} \simeq [d\rho] \frac{e^{\alpha S(\rho)}}{Z_S(\alpha)}$$

$$\alpha = q^{-1},\ [d\rho] = \prod_{i=1}^{N} \frac{d\rho_i}{\sqrt{\rho_i}},\ Z_S(\alpha) = \left(\tfrac{2\pi}{\alpha}\right)^{N/2}$$

$$S(\rho) = \sum_{i=1}^{N} \left[\rho_i - m_i - \rho_i \log\left(\frac{\rho_i}{m_i}\right)\right]$$

(Shannon-Jaynes entropy)

“default model”

Summary

$$P[\rho|G,I] = \frac{P[G|\rho,I]P[A|I]}{P[G|I]} \sim \frac{1}{Z_L Z_S} e^{\alpha S(\rho) - L(\rho)}$$

Finding the most probable image of ρ(ω) corresponds to finding the maximum of αS(ρ) – L(ρ), which can be proofed to be unique if it exists.

→ Bryan's method: R.K. Bryan, Eur. Biophys. J. **18**, 165 (1990).
M. Asakawa, T. Hatsuda and Y. Nakahara, Prog. Part. Nucl. Phys. **46**, 459 (2001).

- How is α determined?

→ The average is taken:

$$\rho_{\mathrm{final}}(\omega) = \int [d\rho] \int d\alpha \rho(\omega) P[\rho|G,I,\alpha] P[\alpha|G,I]$$
$$\simeq \int d\alpha \rho_\alpha(\omega) P[\alpha|G,I]$$

determined using Bayes' theorem

- What about the default model m(ω)?

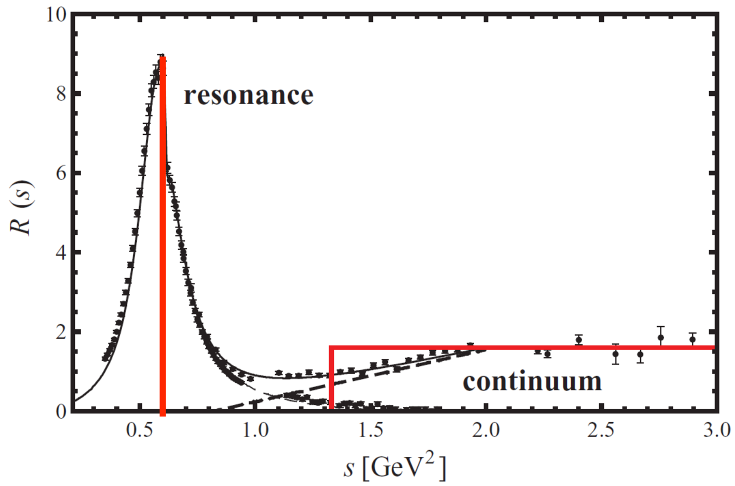
→ The dependence of the final result on the default model must be checked.

Application to the ρ meson channel

One of the first and most successful application of QCD sum rules was the analysis of the ρ meson channel.

The “pole + continuum” assumption works well in this case.

$e^+e^- \rightarrow n\pi$ (n: even)



Y. Kwon, M. Procura, and W. Weise, Phys. Rev. C **78**, 055203 (2008).

The experimental knowledge of the spectral function allows us generate realistic mock data.

Analysis of the OPE data:

$$j_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$$

$$\frac{1}{4\pi^2}(1+\frac{\alpha_s}{\pi})+\Big(2m\langle\overline{q}q\rangle+\frac{1}{12}\langle\frac{\alpha_s}{\pi}G^2\rangle\Big)\frac{1}{M^4}-\frac{112\pi}{81}\alpha_s\kappa\langle\overline{q}q\rangle^2\frac{1}{M^6}+\dots$$

$$=\frac{2}{M^2}\int_0^\infty d\omega e^{-(\omega/M)^2}\omega\rho(\omega)$$

We use three parameter sets in our analysis:

	Colangelo *et al.* [13]	Narison [14]	Ioffe [15]
$\langle\bar{q}q\rangle$	$-(0.24 \pm 0.01)^3$ GeV^3	$-(0.238 \pm 0.014)^3$ GeV^3	$-(0.248 \pm 0.013)^3$ GeV^3
$\langle\frac{\alpha_s}{\pi}G^2\rangle$	0.012 ± 0.036 GeV^4	0.0226 ± 0.0029 GeV^4	0.009 ± 0.007 GeV^4
κ	1	2.5 ± 0.5	1.0 ± 0.1
α_s	0.5	0.50 ± 0.07	0.57 ± 0.08

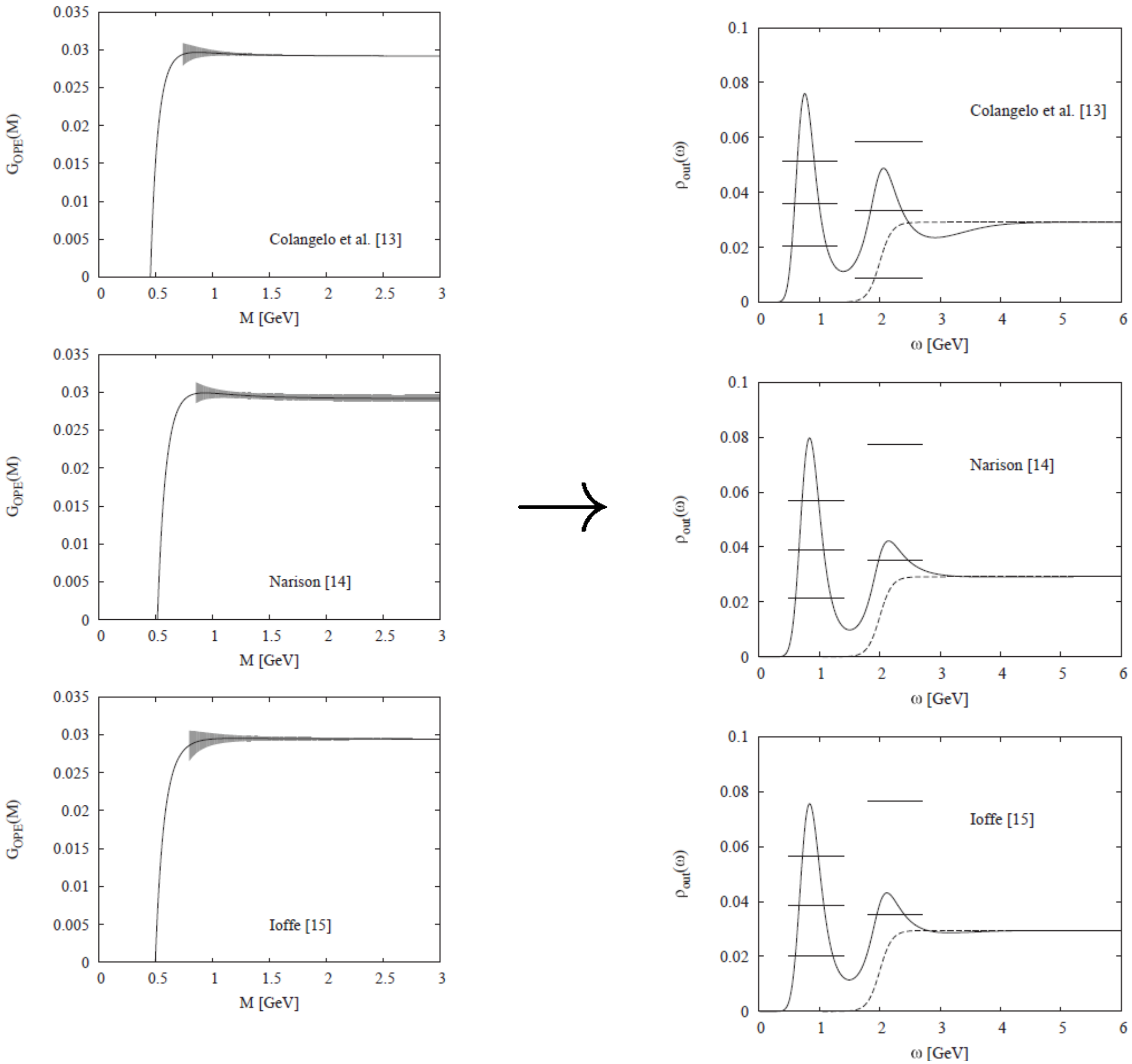
[13] P. Colangelo and A. Khodjamirian, *"At the Frontier of Particle Physics/Handbook of QCD"* (World Scientific, Singapore, 2001), Volume 3, 1495.
[14] S. Narison, *"QCD as a Theory of Hadrons"* (Cambridge Univ. Press, Cambridge, 2004).
[15] B.L. Ioffe, Prog. Part. Nucl. Phys. **56**, 232 (2006).

$$m\langle\bar{q}q\rangle = -8.5 \times 10^{-5}\,\mathrm{GeV}^4$$

(from the Gell-Mann-Oakes-Renner relation)

PG, M. Oka, Prog. Theor. Phys. **124**, 995 (2010).

Results (1)



$$m_\rho = 0.76 \pm 0.07\ \text{GeV}$$
$$F_\rho = 0.174 \pm 0.039\ \text{GeV}$$

$$m_\rho = 0.84 \pm 0.12\ \text{GeV}$$
$$F_\rho = 0.190 \pm 0.053\ \text{GeV}$$

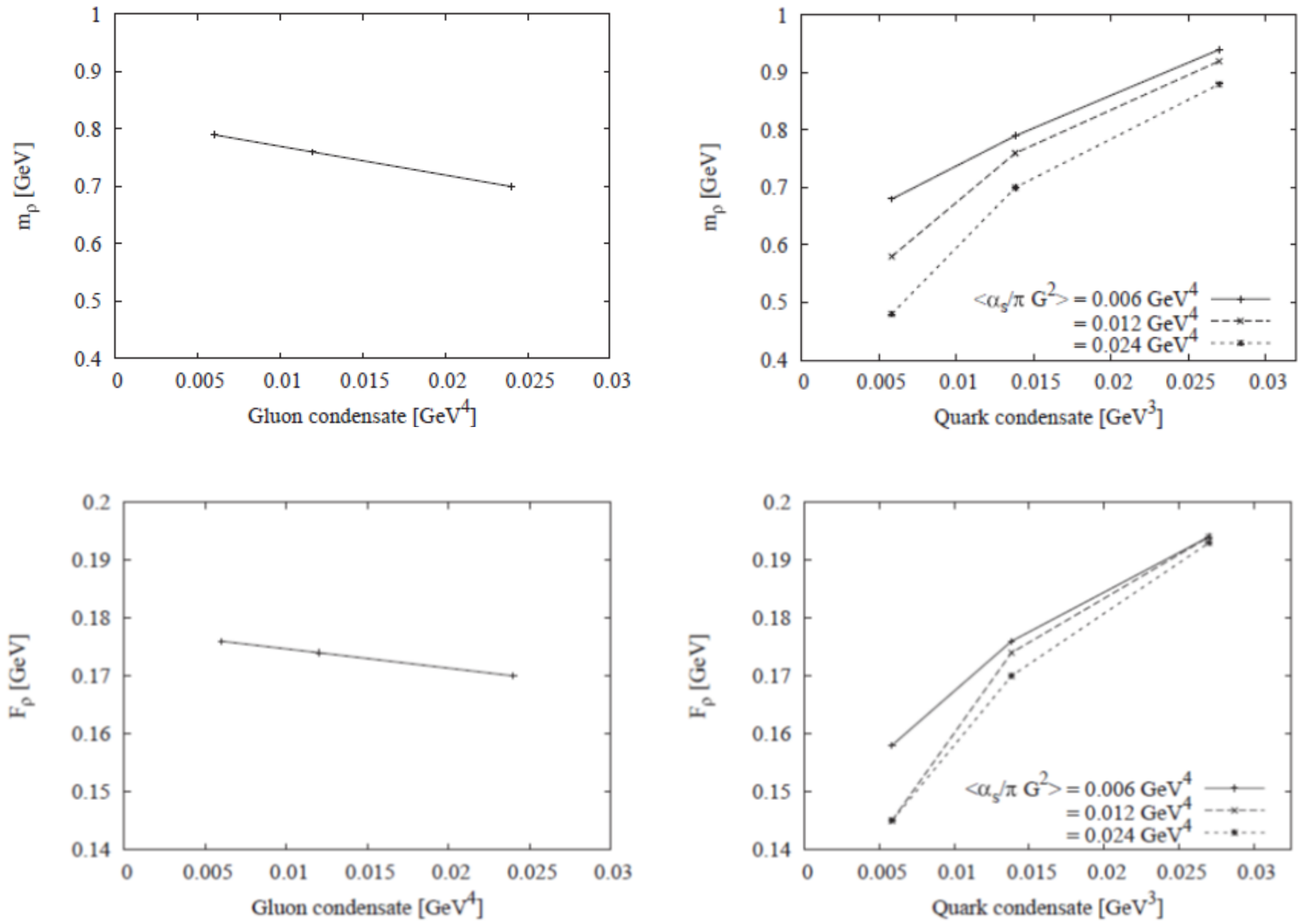
$$m_\rho = 0.84 \pm 0.10\ \text{GeV}$$
$$F_\rho = 0.187 \pm 0.050\ \text{GeV}$$

Experiment:
m_ρ= 0.77 GeV
F_ρ= 0.141 GeV

PG, M. Oka, Prog. Theor. Phys. **124**, 995 (2010).

Results (2)

The dependence of the ρ-meson properties on the values of the condensates:

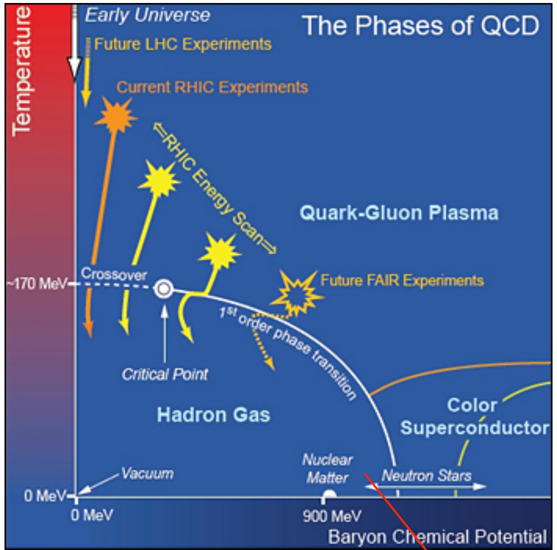


PG, M. Oka, Prog. Theor. Phys. **124**, 995 (2010).

Introduction: Vector mesons at finite density

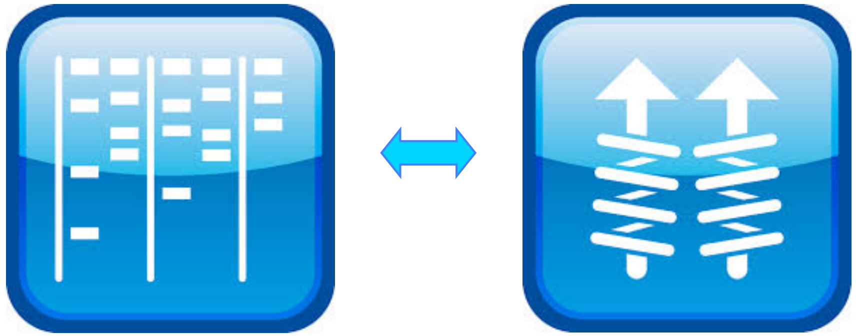
Basic Motivation:

Understanding the behavior of matter under extreme conditions



$$\boxed{\rho,\ \phi,\ \omega,\ D\ \ldots}$$

Understanding the origin of mass and its relation to chiral symmetry of QCD



- Vector mesons: clean probe for experiment
- To be investigated at J-PARC
- Firm theoretical understanding is necessary for interpreting the experimental results!

The OPE for vector mesons

$$\frac{1}{M^2}\int_0^\infty ds e^{-\frac{s}{M^2}}\rho(s) = c_0 + \frac{c_1}{M^2} + \frac{c_2}{M^4} + \frac{c_3}{M^6} + \dots$$

<u>Vacuum</u>

$\boxed{\rho,\ \omega}$

$$c_0 = 1 + \frac{\alpha_s}{\pi}$$

$$c_1 = -6m_q^2 \sim 0$$

$$c_2 = \frac{\pi^2}{3}\langle\frac{\alpha_s}{\pi}G^2\rangle + 8\pi^2 m_q\langle\overline{q}q\rangle$$

$$c_3 = -\frac{896}{81}\kappa\pi^3\alpha_s\langle\overline{q}q\rangle^2$$

$\boxed{\phi}$

$$c_0 = 1 + \frac{\alpha_s}{\pi}$$

$$c_1 = -6m_s^2$$

$$c_2 = \frac{\pi^2}{3}\langle\frac{\alpha_s}{\pi}G^2\rangle + 8\pi^2 m_s\langle\overline{s}s\rangle$$

$$c_3 = -\frac{896}{81}\kappa\pi^3\alpha_s\langle\overline{s}s\rangle^2$$

The large strange quark mass leads to different behavior of the OPE results

Density effects on the condensates

$$\langle \bar{q}q \rangle_\rho = \langle \bar{q}q \rangle_0 + \langle N|\bar{q}q|N\rangle \rho = \langle \bar{q}q \rangle_0 + \frac{\sigma_{\pi N}}{2m_q}\rho$$

$$\langle \bar{s}s \rangle_\rho = \langle \bar{s}s \rangle_0 + \langle N|\bar{s}s|N\rangle \rho = \langle \bar{s}s \rangle_0 + \frac{\sigma_{sN}}{m_s}\rho$$

$$= \langle \bar{s}s \rangle_0 + y\frac{\sigma_{\pi N}}{2m_q}$$

$$\frac{\langle N|\bar{s}s|N\rangle}{\langle N|\bar{q}q|N\rangle}$$

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle_\rho = \langle \frac{\alpha_s}{\pi} G^2 \rangle_0 - \frac{8}{9} M_N^0 \rho$$

$$2\int_0^1 dx\, x\left[q(x) + \bar{q}(x)\right]$$

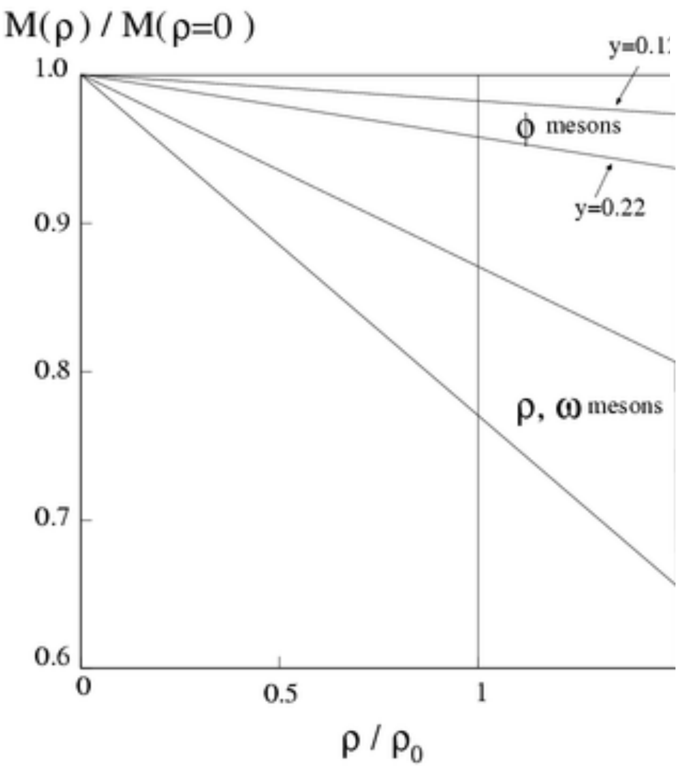
$$c_2(\rho) = c_2(0) + 4\pi^2 A_1^{q,s} M_N \rho$$

$$c_3(\rho) = c_3(0) - \frac{20}{3}\pi^2 A_3^{q,s} M_N^3 \rho$$

$$2\int_0^1 dx\, x^3\left[q(x) + \bar{q}(x)\right]$$

Important early study

T. Hatsuda and S.H. Lee, Phys. Rev. C **46**, R34 (1992).



Vector meson masses mainly drop due to changes of the quark condensates.

The most important condensates are:

$$\langle \overline{q}q\overline{q}q \rangle_\rho \quad \text{for} \quad \rho,\ \omega$$

$$m_s \langle \overline{s}s \rangle_\rho \quad \text{for} \quad \phi$$

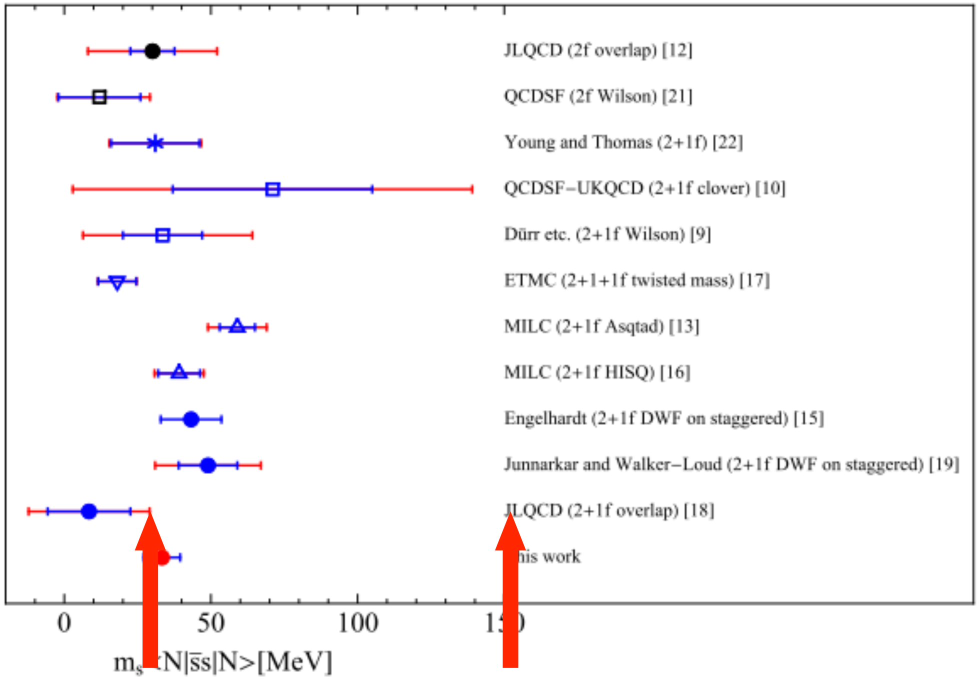
Important assumption:

$$\left.\langle \overline{q}q\overline{q}q \rangle_\rho = \langle \overline{q}q \rangle_\rho^2,\ \langle \overline{s}s\overline{s}s \rangle_\rho = \langle \overline{s}s \rangle_\rho^2 \right\} \quad \text{Might be wrong!}$$

The strangeness content of the nucleon: recent developments

$$y = \frac{\langle N|\bar{s}s|N\rangle}{\langle N|\bar{q}q|N\rangle}$$

Taken from M. Gong et al. (χQCD Collaboration), arXiv:1304.1194 [hep-ph].

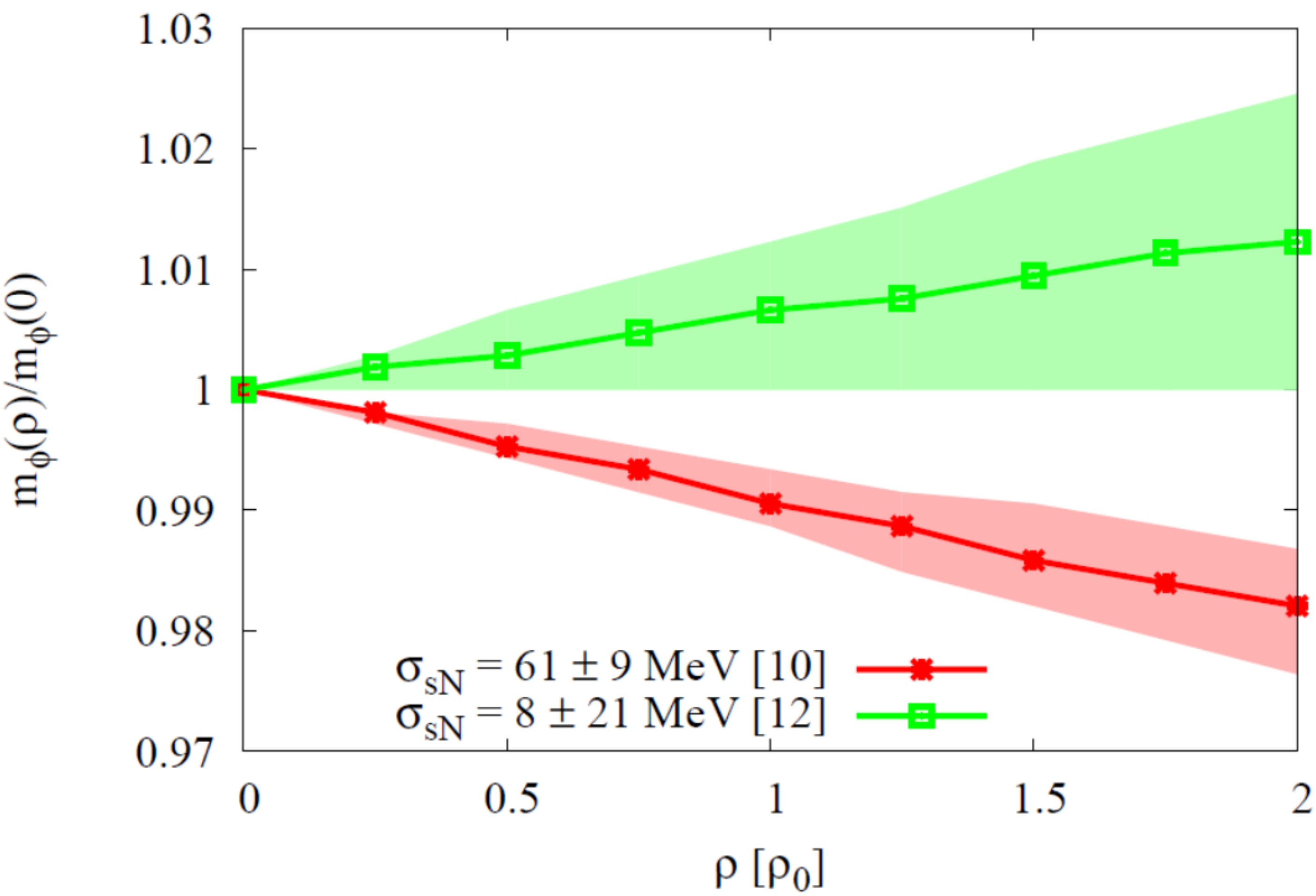


y ~ 0.04

Value used by Hatsuda and Lee: y=0.2
Too big!!

The value of y has shrinked by a factor of about 5: a new analysis is necessary!

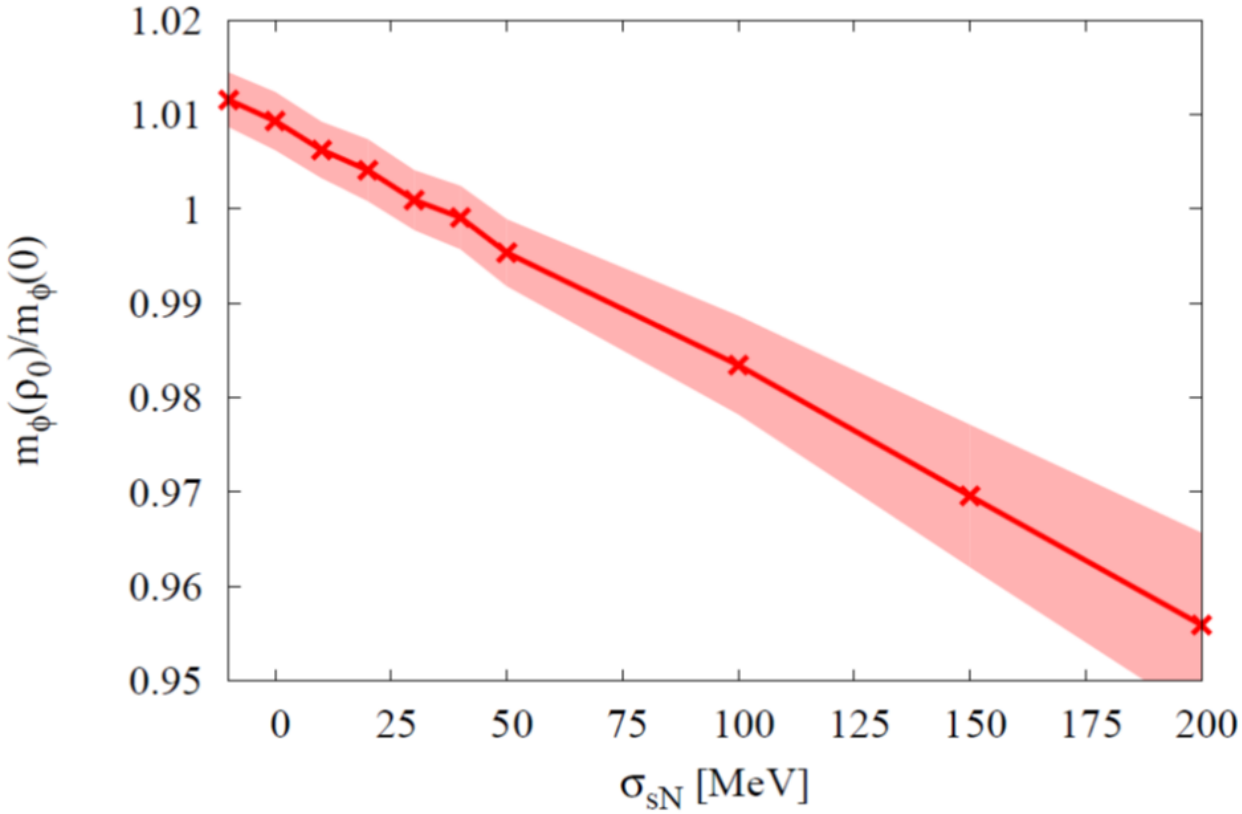
ϕ meson at finite density



Measuring the φ meson mass shift in nuclear matter provides a strong constraint to the strangeness content of the nucleon.

P. Gubler and K. Ohtani, arXiv:1404.7701 [hep-ph].

Relation between the ϕ meson mass shift and the strange sigma term



$$\frac{m_\phi(\rho)}{m_\phi(0)} = 1 - B\left[\left(\frac{\sigma_{sN}}{1\,\mathrm{MeV}}\right) - 34.6\right]\frac{\rho}{\rho_0}$$

$$B = (2.64 \pm 0.45)10^{-4}$$

$$\frac{m_\phi(\rho)}{m_\phi(0)} = 1 - C[y - 0.057]\frac{\rho}{\rho_0}$$

$$C = (1.61 \pm 0.38)10^{-1}$$

P. Gubler and K. Ohtani, arXiv:1404.7701 [hep-ph].

How can this result be understood?

Let us examine the OPE at finite density more closely:

$$c_2(\rho) =$$

$$c_2(0) + \rho\Big[-\frac{2}{27}M_N^0 + 2m_s\langle N|\bar{s}s|N\rangle + A_1^s M_N - \frac{23}{36}\frac{\alpha_s}{\pi}A_2^g M_N\Big]$$

$$-82 \text{ MeV} \qquad 2\sigma_{sN} \qquad 38 \text{ MeV} \qquad -34 \text{ MeV}$$

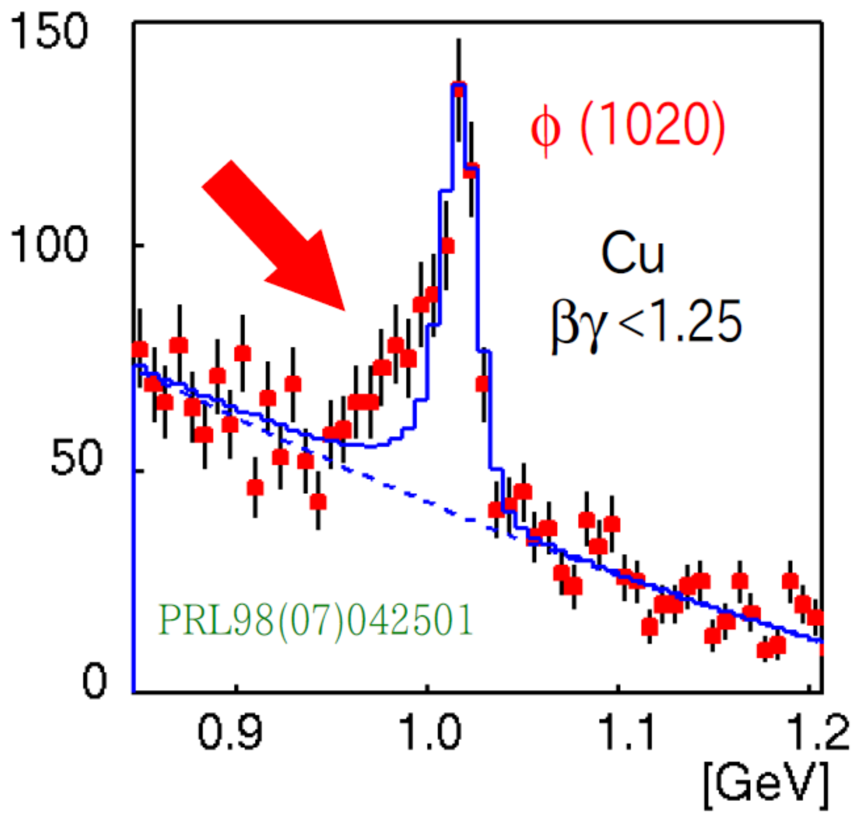
$$\sim 2\rho\Big[\Big(\frac{\sigma_{sN}}{1\text{MeV}}\Big) - 34\Big]\text{MeV}$$

Dimension 4 terms govern
the behavior of the φ meson.

However…

Experiments seem to suggest something else:

Result of the E325 experiment at KEK



R. Muto et al, Phys. Rev. Lett. 98, 042501 (2007).

35 MeV mass reduction of the φ meson at nuclear matter density!

Will be measured at the E16 experiment at J-PARC with 10 times increased statistics.

What could be wrong?

1. So far neglected condensates

Terms containing higher orders of m_s and other so far neglected terms could have a non-negligible effect.

$$m_s^3 \langle \bar{s}s \rangle, \; m_s^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle, \; m_s \langle \bar{s} g \sigma G s \rangle, \; \ldots$$

The effects of these terms are small

2. α_s corrections

These corrections are small

3. Underestimated density dependence of four-quark condensates

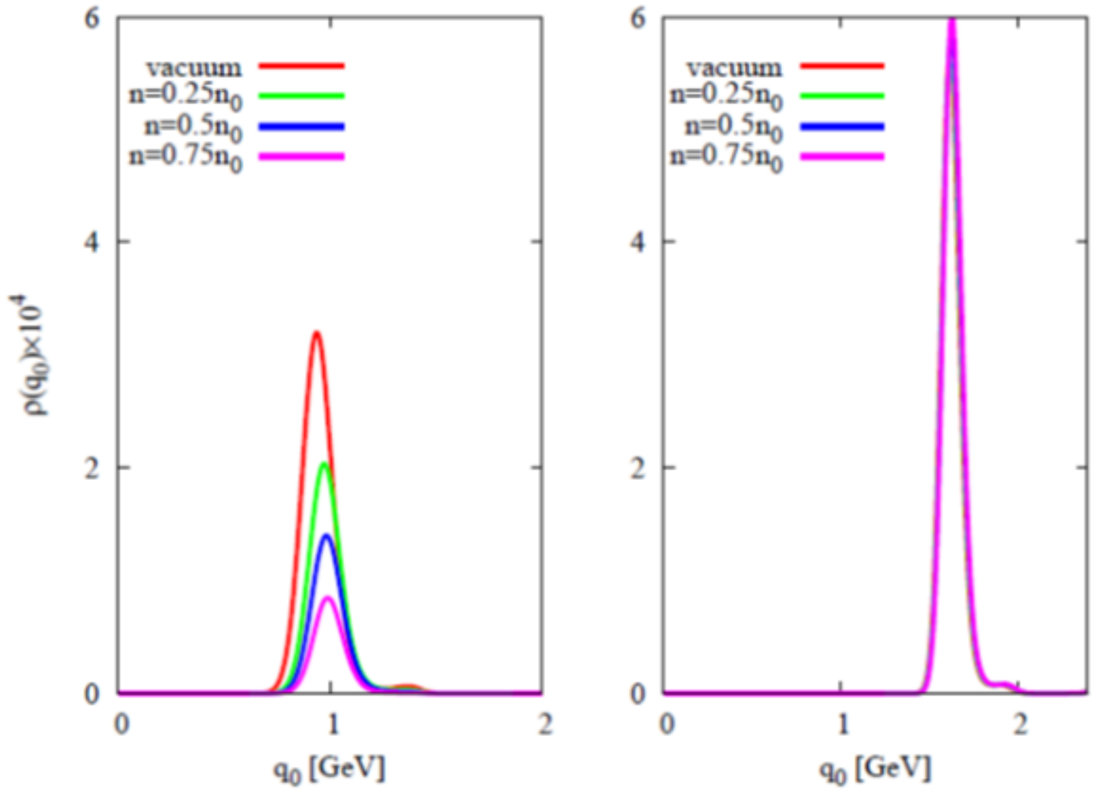
$$\langle 0 | \alpha_S (\bar{s} \gamma_\mu \gamma_5 \lambda^a s)^2 | 0 \rangle + \frac{4}{9} \langle 0 | \alpha_S (\bar{s} \gamma_\mu \lambda^a s) \sum_{q=u,d,s} \bar{q} \gamma^\mu \lambda^a q | 0 \rangle$$

At this moment, we do not know…

Other hadrons that we have studied at finite density

Preliminary

The nucleon and its excited states



ground state

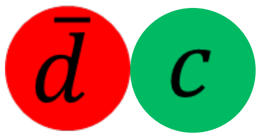
First excited negative parity state

K. Ohtani, P. Gubler and M. Oka, in preparation.

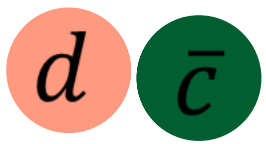
D^+ and D^- at finite density

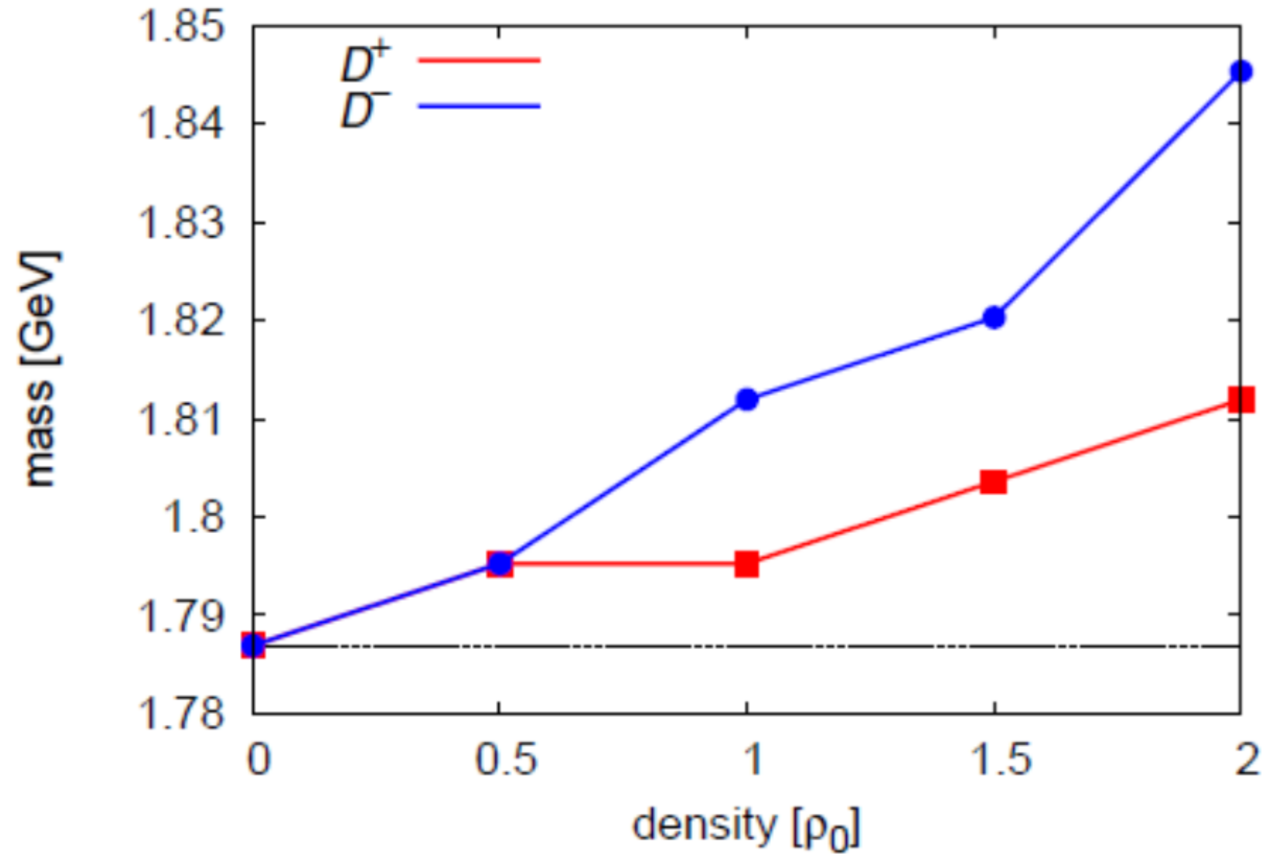
Preliminary

D^+



D^-





K. Suzuki, P. Gubler and M. Oka, in preparation.

Conclusions

- QCD sum rules are useful for studying the behavior of hadrons at finite density
- We have shown that MEM can be applied to QCD sum rules
- The "pole + continuum" is not a necessity
- The resolution of the method is limited, therefore it is generally difficult to obtain the peak-width

Outlook

- Application to the Unitary Fermi Gas
  - Work in collaboration with Y. Nishida, N. Yamamoto and T. Hatsuda
- A more detailed extraction of the spectral function can be obtained using the OPE on the complex Borel plane
  - K-J. Araki, K. Ohtani、P. Gubler and M. Oka, arXiv:1403.6299 [hep-ph], to be published in PTP.

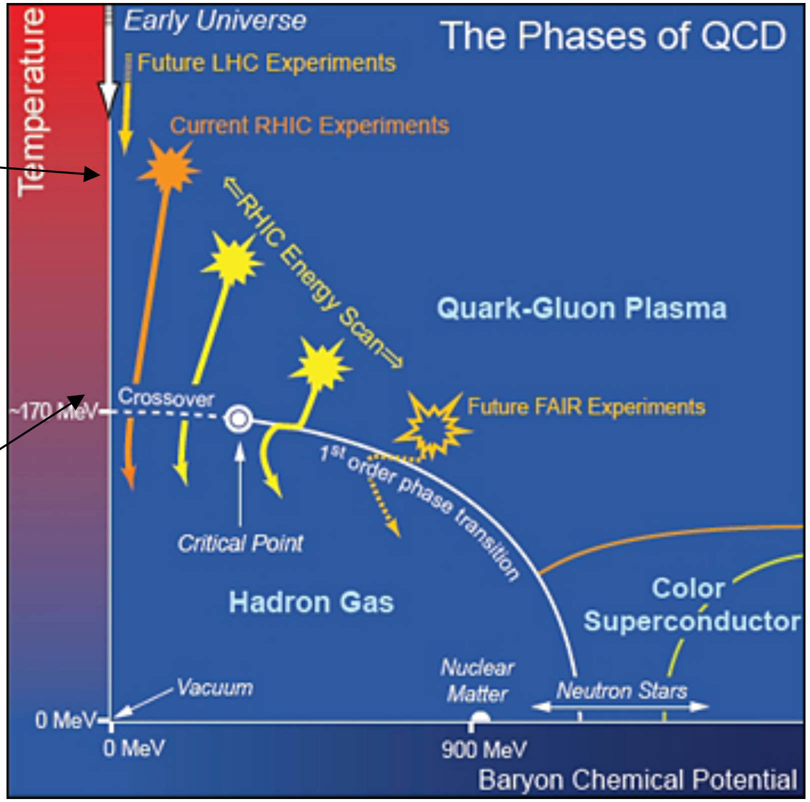
Backup slides

Quarkonia at finite T

General Motivation: Understanding the behavior of matter at high T.

$\Upsilon,\ \eta_b\ \ldots$

$J/\psi,\ \eta_c\ \ldots$



- Phase transition:

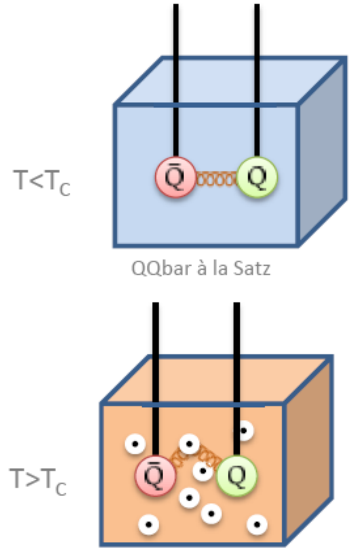
  QGP ($T>T_c$) $\leftrightarrow$ confining phase ($T<T_c$)

- Currently investigated at RHIC and LHC

- Heavy Quarkonium: clean probe for experiment

Why are quarkonia useful?

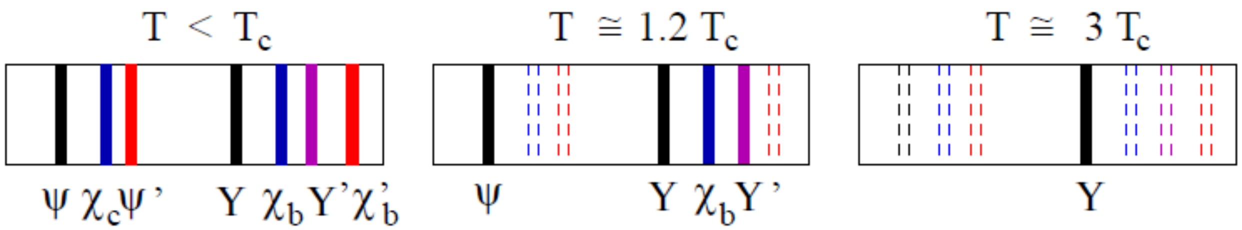
Prediction of J/ψ suppression above T_c
due to Debye screening:



T. Matsui and H. Satz, Phys. Lett. B **178**, 416 (1986).
T. Hashimoto et al., Phys. Rev. Lett. **57**, 2123 (1986).

Lighter quarkonia melt at low T, while heavier ones melt at higher T

→ Thermometer of the QGP



The charmonium sum rules at finite T

The application of QCD sum rules has been developed in:

A.I. Bochkarev and M.E. Shaposhnikov, Nucl. Phys. B 268, 220 (1986).

T.Hatsuda, Y.Koike and S.H. Lee, Nucl. Phys. B 394, 221 (1993).

$$M(\nu) = \int_0^\infty e^{-\nu t}\rho(4m_c^2 t)dt \qquad \left(\nu \equiv \frac{M^2}{4m_c^2}\right)$$

$$M(\nu) = A(\nu)\Big[1 + a(\nu)\alpha_s(\nu) + b(\nu)\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle_T}{m_c^4} + c(\nu)\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle_{T,2}}{m_c^4} + d(\nu)\frac{\langle g^3G^3\rangle_T}{m_c^6}\Big]$$

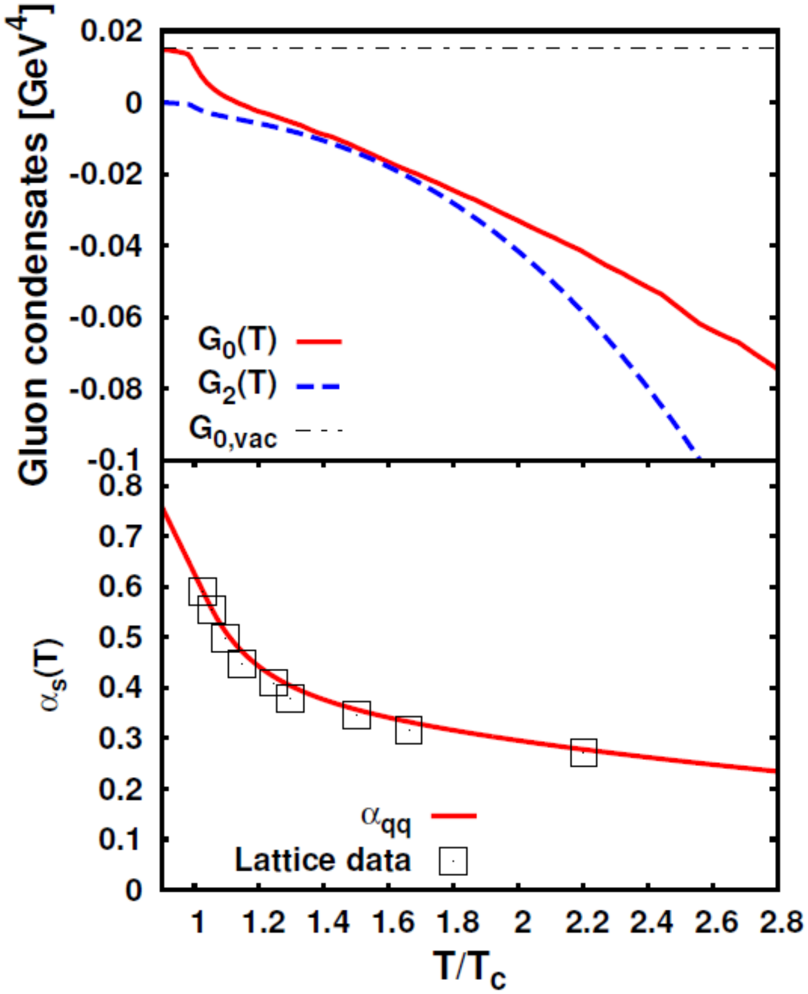
depend on T

Compared to lattice:

No reduction of data points that can be used for the analysis, allowing a direct comparison of T=0 and T≠0 spectral functions.

The T-dependence of the condensates

K. Morita and S.H. Lee, Phys. Rev. Lett. 100, 022301 (2008).



taken from:

K. Morita and S.H. Lee, Phys. Rev. D82, 054008 (2010).

Considering the trace and the traceless part of the energy momentum tensor, one can show that in tht quenched approximation, the T-dependent parts of the gluon condensates by thermodynamic quantities such as energy density ε(T) and pressure p(T).

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle_T = \langle \frac{\alpha_s}{\pi} G^2 \rangle_{\text{vac.}} - \frac{8}{11}(\epsilon - 3p)$$

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle_{T,2} = -\frac{\alpha_s(T)}{\pi}(\epsilon + p)$$

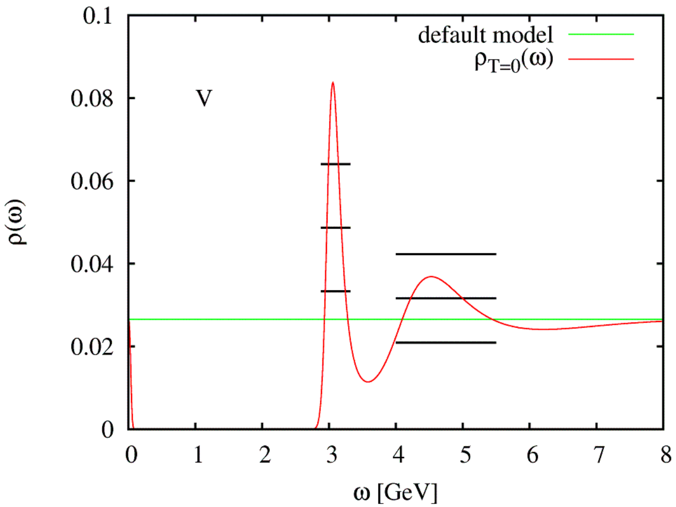
The values of ε(T) and p(T) are obtained from quenched lattice calculations:

G. Boyd *et al*, Nucl. Phys. B 469, 419 (1996).

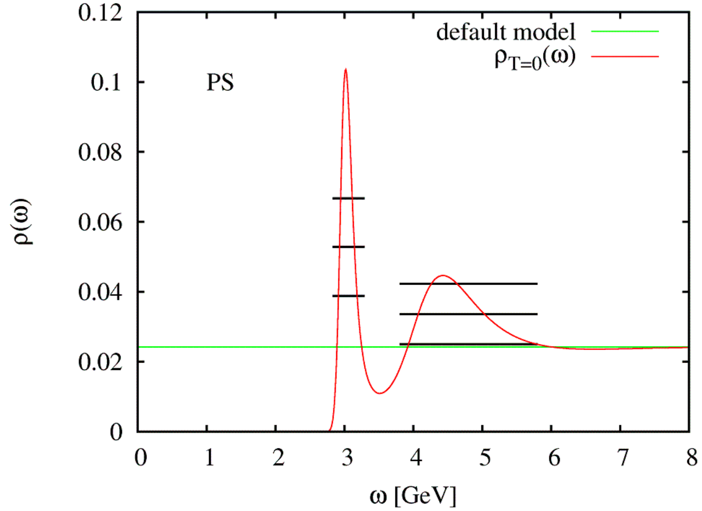
O. Kaczmarek *et al*, Phys. Rev. D 70, 074505 (2004).

MEM Analysis at T=0

S-wave

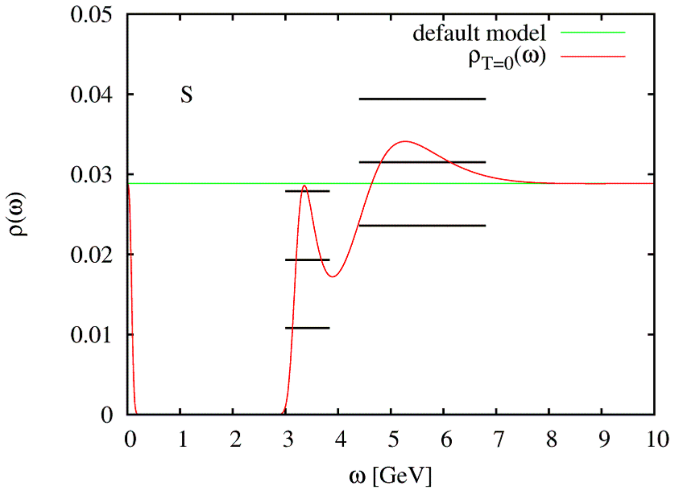


$m_{J/\psi}$=3.06 GeV (Exp: 3.10 GeV)

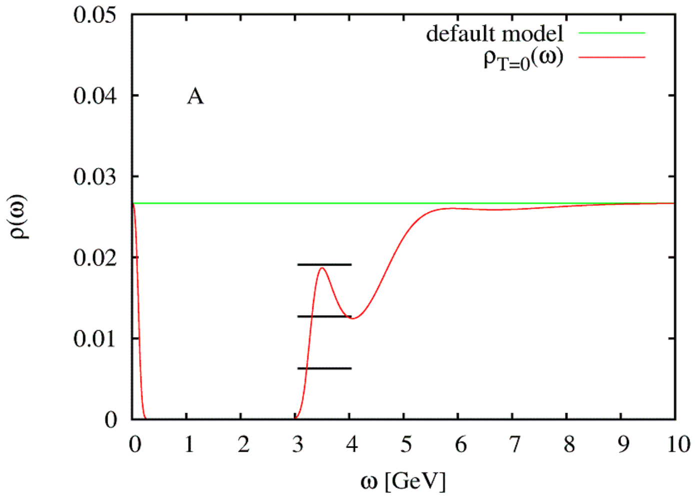


$m_{\eta c}$=3.02 GeV (Exp: 2.98 GeV)

P-wave



$m_{\chi 0}$=3.36 GeV (Exp: 3.41 GeV)

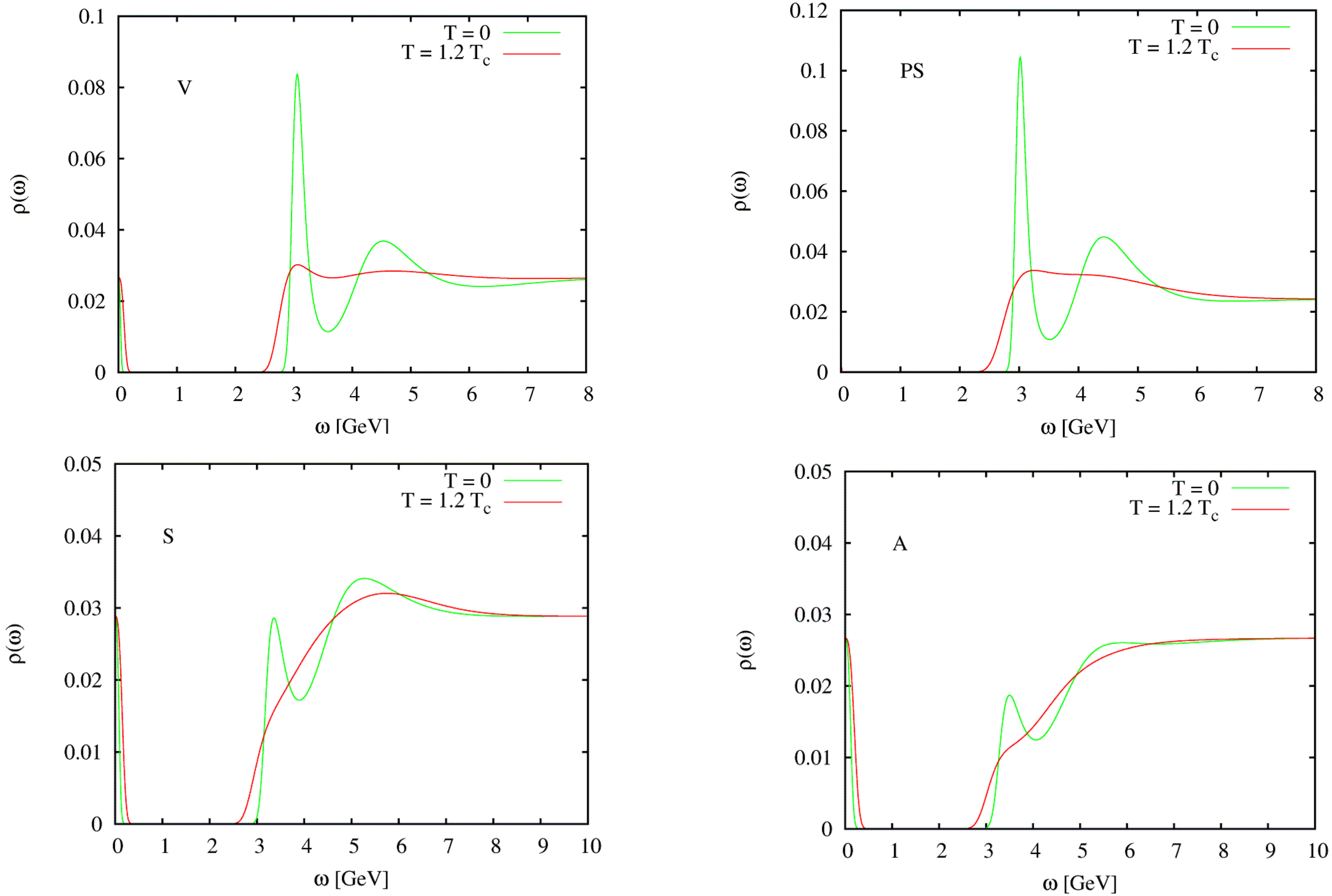


$m_{\chi 1}$=3.50 GeV (Exp: 3.51 GeV)

PG, K. Morita and M. Oka, Phys. Rev. Lett. **107**, 092003 (2011).

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \pm 0.0036 \text{ GeV}^4$$
$$m_c = 1.277 \pm 0.026 \text{ GeV}$$

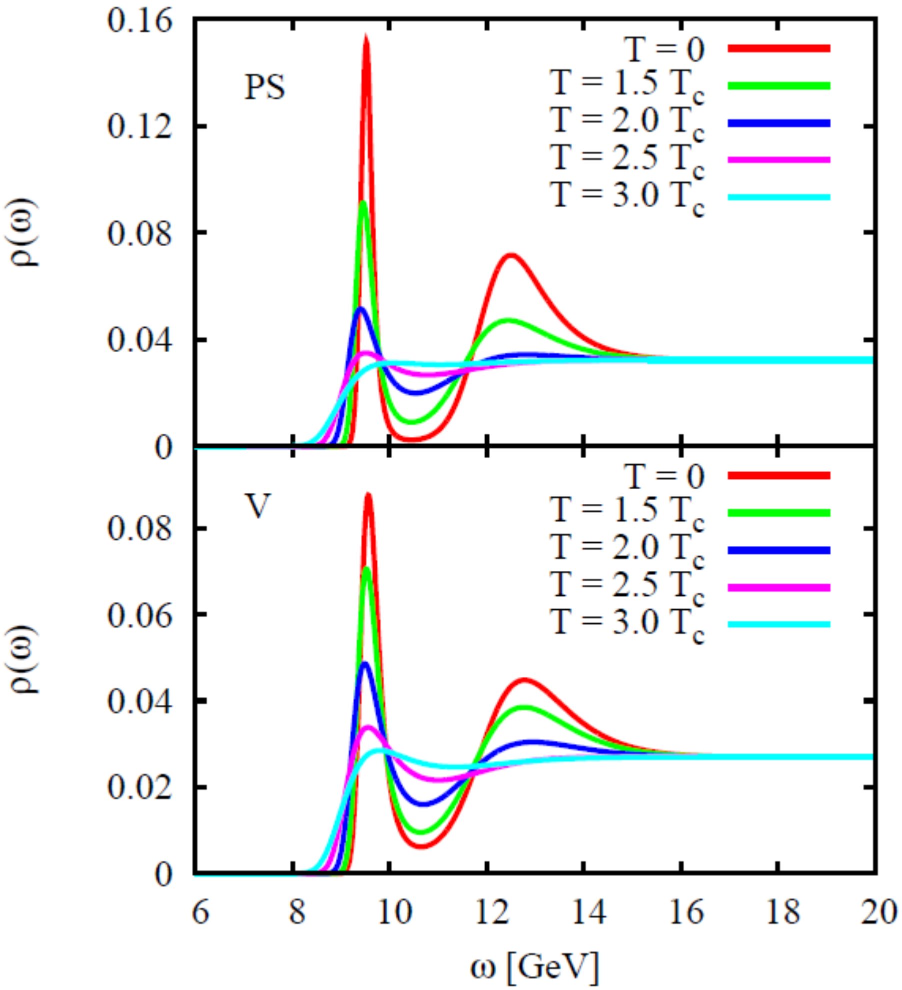
The charmonium spectral function at finite T



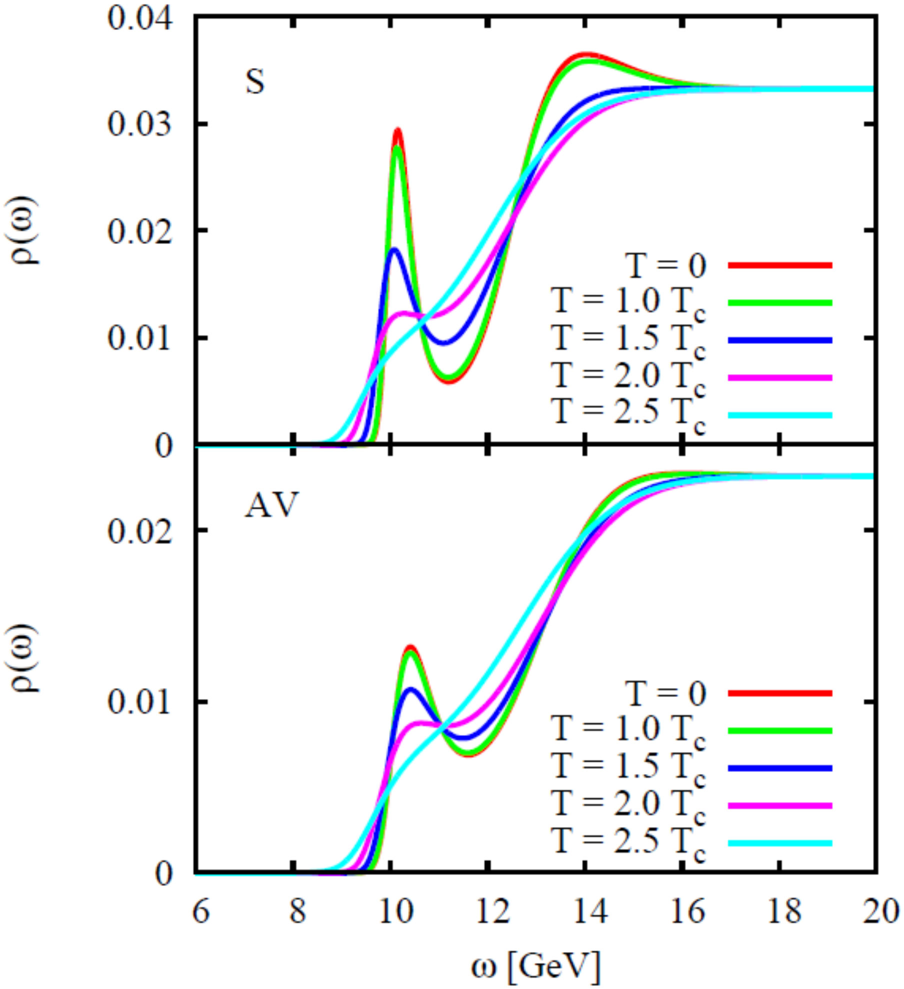
PG, K. Morita and M. Oka, Phys. Rev. Lett. **107**, 092003 (2011).

Results for bottomonium

S-wave



P-wave



K. Suzuki, PG, K. Morita and M. Oka, Nucl. Phys. **A897**, 28 (2013).

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \pm 0.0036 \ \text{GeV}^4$$
$$m_b = 4.167 \pm 0.013 \ \text{GeV}$$

Where might we have problems?

- Higher order gluon condensates?
  - Probably not a problem, but needs to be checked
- Higher orders is α_s?
  - Maybe. Can be tested for vector channel
- Division between high- and low-energy contributions in OPE?
  - Could be a problem at high T. Needs to be investigated carefully.

Conclusions for quarkonia at finite T

- We could observe the melting of the S-wave and P-wave charmonia using finite temperature QCD sum rules and MEM
- J/ψ, η_c, χ_{c0}, χ_{c1} melt between $T \sim 1.0\ T_c$ and $T \sim 1.2\ T_c$, which is below the values obtained in lattice QCD
- As for bottomonium, $\Upsilon(1S)$ survives until $3.0\ T_c$ or higher. Furthermore,η_b melts at around $3.0\ T_c$, while χ_{b0} and χ_{b1} melt at around $2.0 \sim 2.5\ T_c$

Outlook

- Check possible problems of our method
  - α_s, higher twist, division of scale
- Calculate higher order gluon condensates on the lattice