On String theoretic interpretation of Yang-Mills theory

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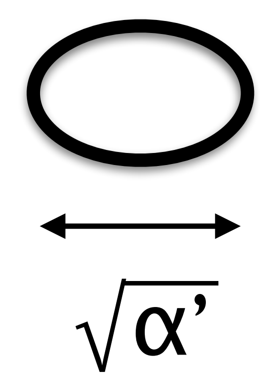
M.H.-Hyakutake-Nishimura-Takeuchi, PRL (2009)
M.H.-Hyakutake-Ishiki-Nishimura, Science (2014)
M.H.-Maltz-Susskind, PRD (2014)

Maldacena's conjecture: deconfining phase = black hole

SYM		STRING
$1/\lambda$	$\longleftrightarrow$	α'/R_{BH}^2
$g_{YM}^2 \sim 1/N$		g_s



$\lambda=\infty$, $N=\infty$ corresponds to supergravity.

assumed to be correct without proof, and applied to QGP

Is it correct?

Is it correct only at large-N, strong coupling?

(supergravity, or Einstein gravity)

Or correct including1/λ and 1/N corrections?

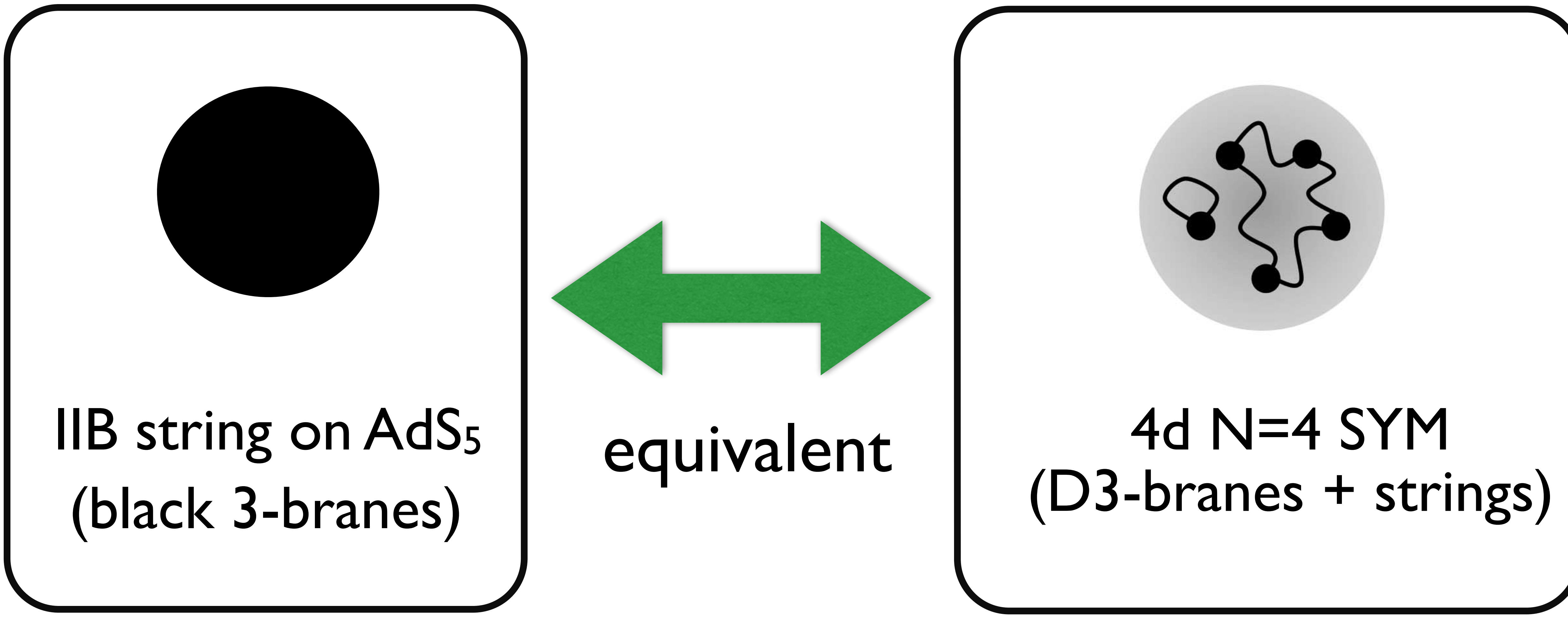
(superstring theory)

If correct, why? Can we understand it intuitively?

I want to answer to these questions, because

(1) I want to understand quantum gravity.

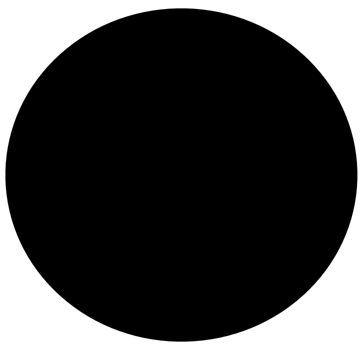
(2) I want to understand thermalization of QGP.



(Maldacena1997)

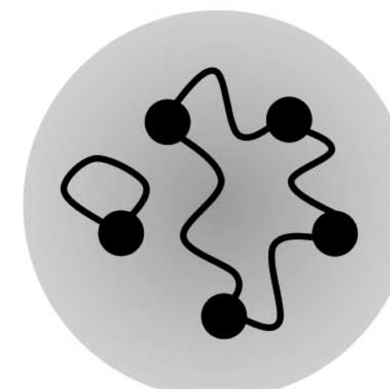
Black hole = bunch of D0-branes

(+ strings between them)



IIA string around
black 0-brane
(near horizon)

equivalent

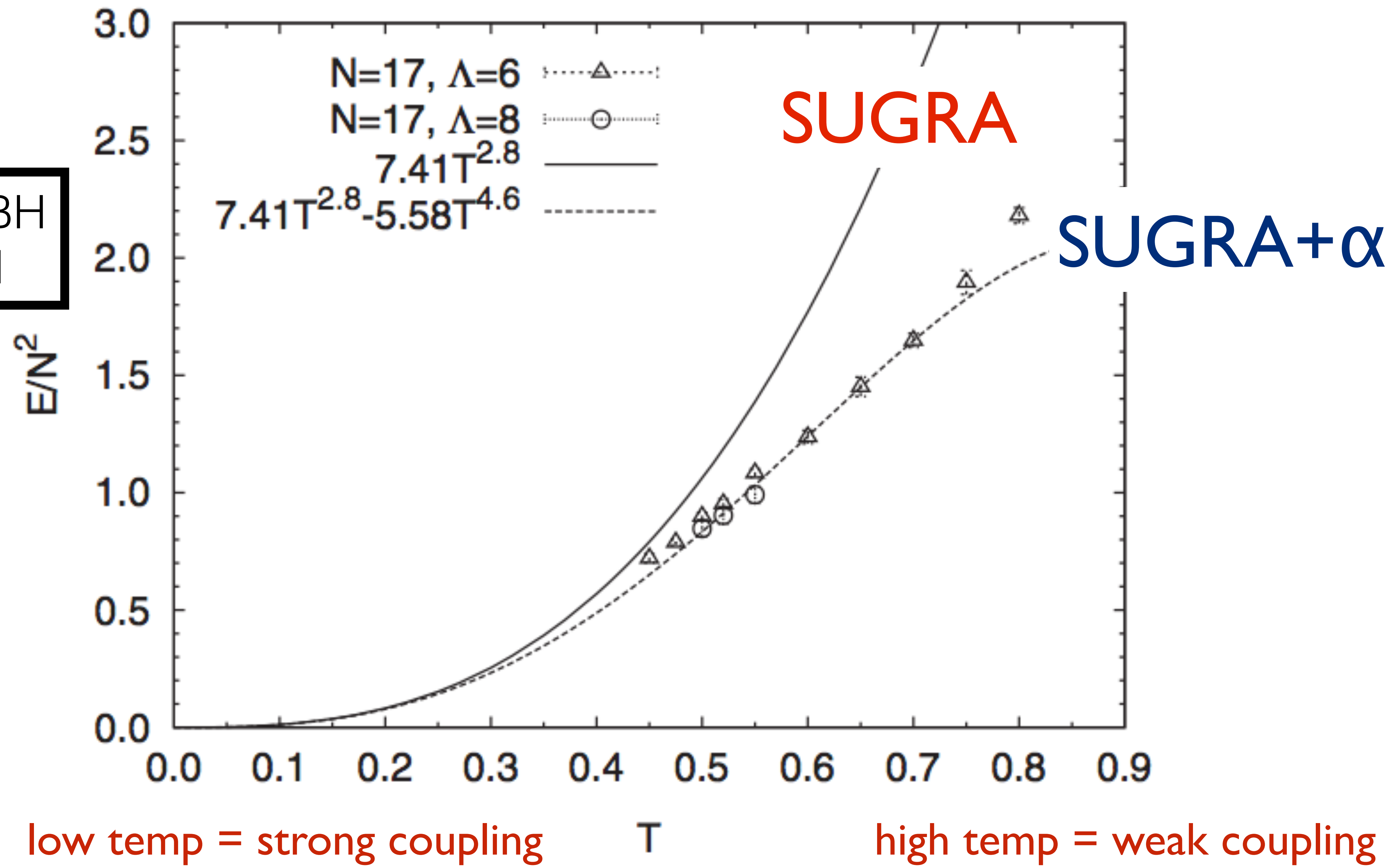


(0+1)-d maximal SYM

(Itzhaki-Sonnenschein-Maldacena-Yankielowicz 1998)

Quantitative test is possible by studying SYM numerically.

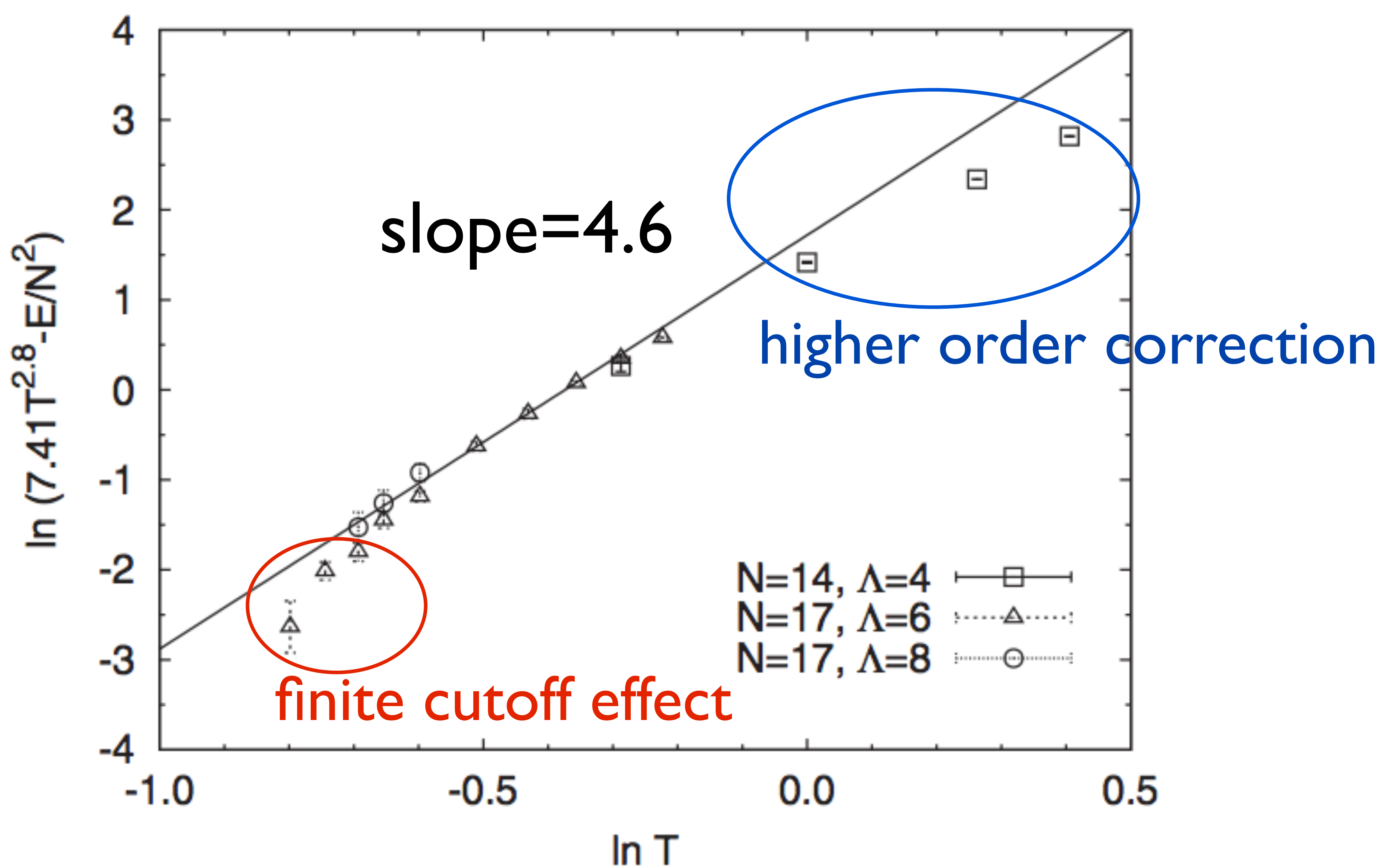
energy of BH and SYM



M.H.-Hyakutake-Nishimura-Takeuchi, PRL 2009

($\lambda^{-1/3}T$: dimensionless effective temperature)

Maldacena conjecture is correct at finite coupling & temperature!



M.H.-Hyakutake-Nishimura-Takeuchi, PRL 2009

1/N correction

Dual gravity prediction (Y. Hyakutake, PTEP 2014)

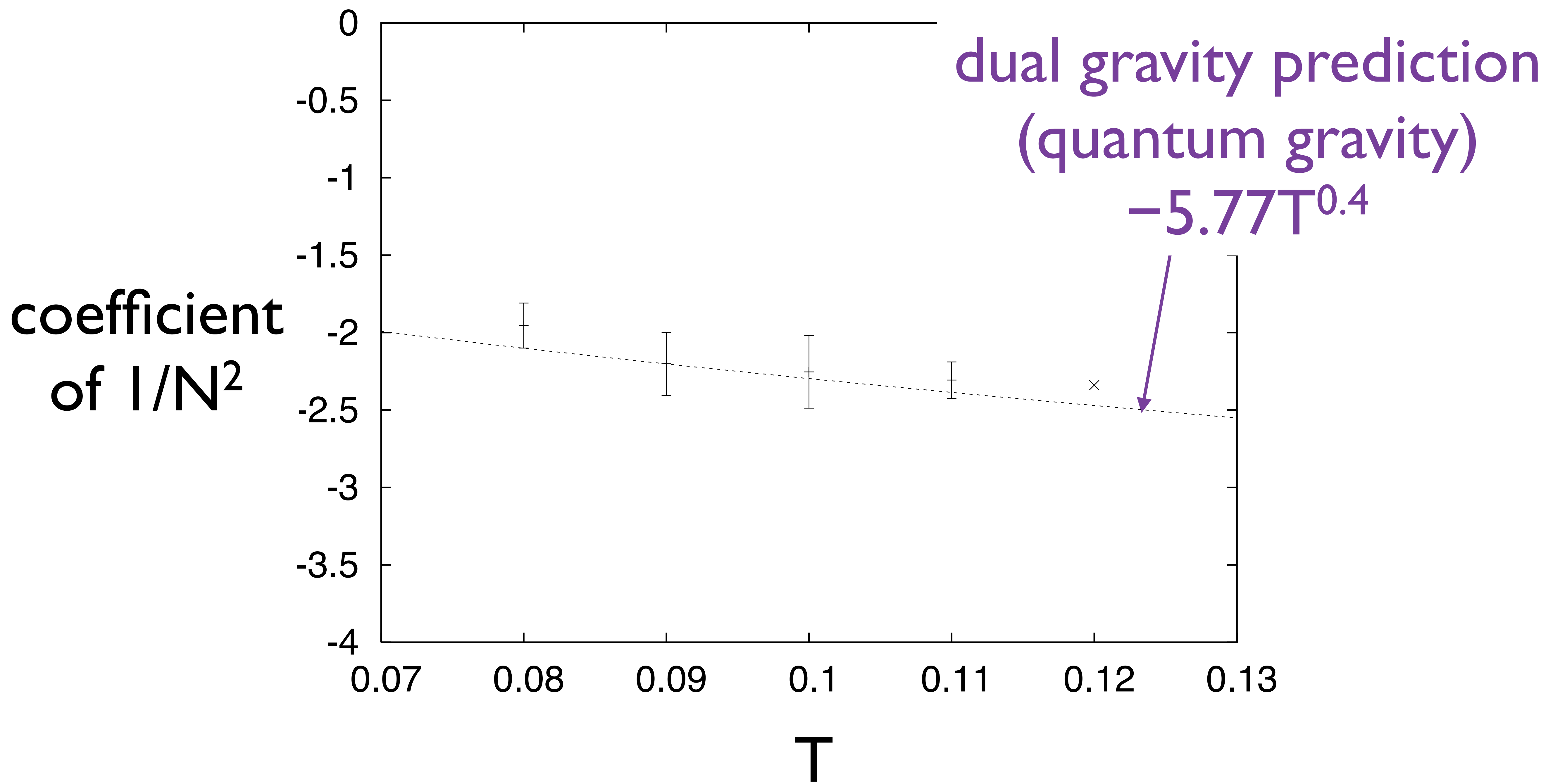
$$E/N^2 = 7.41T^{2.8} - 5.58T^{4.6} + \ldots.$$

$$\left.\begin{aligned} &+(1/N^2)(-5.77T^{0.4} + aT^{2.2} + \ldots) \\ &+(1/N^4)(bT^{-2.6} + cT^{-2.0} + \ldots) \\ &+\ldots.. \end{aligned}\right\}$$

QUANTUM string effect

Can it be reproduced from YM?

Maldacena conjecture is correct at finite-N !



M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

But why does it hold? We want to understand it intuitively, so that we can understand physics behind it.

It should give us new perspective for both QGP and BH.

microscopic descriptions of the black hole (black brane)

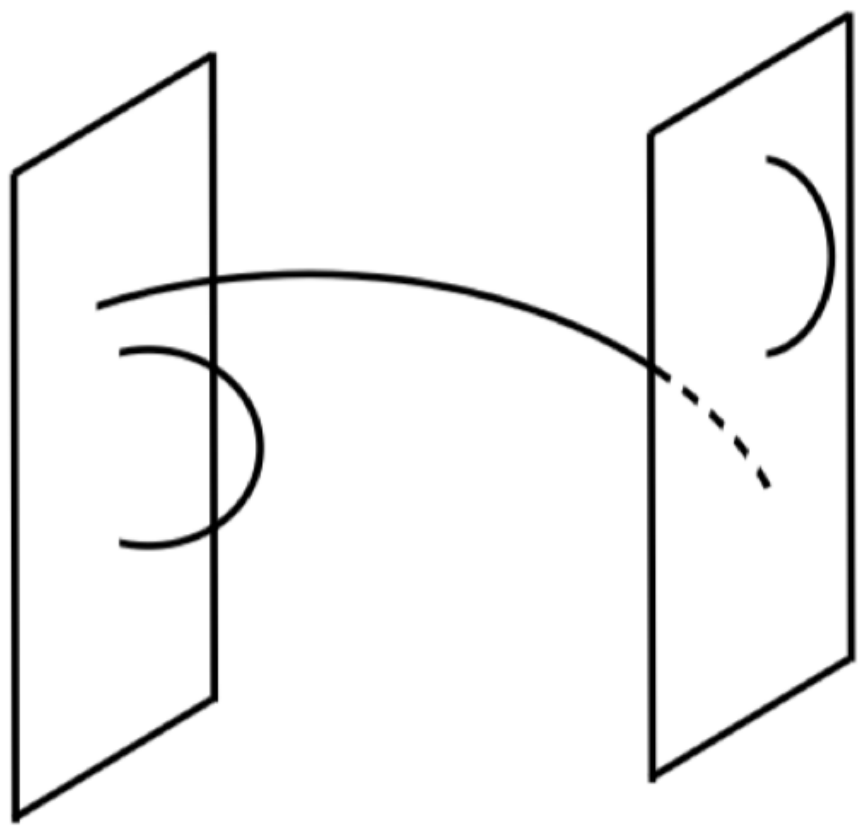
(1) D-branes + open strings

Polchinski, …

(2) condensation of closed strings

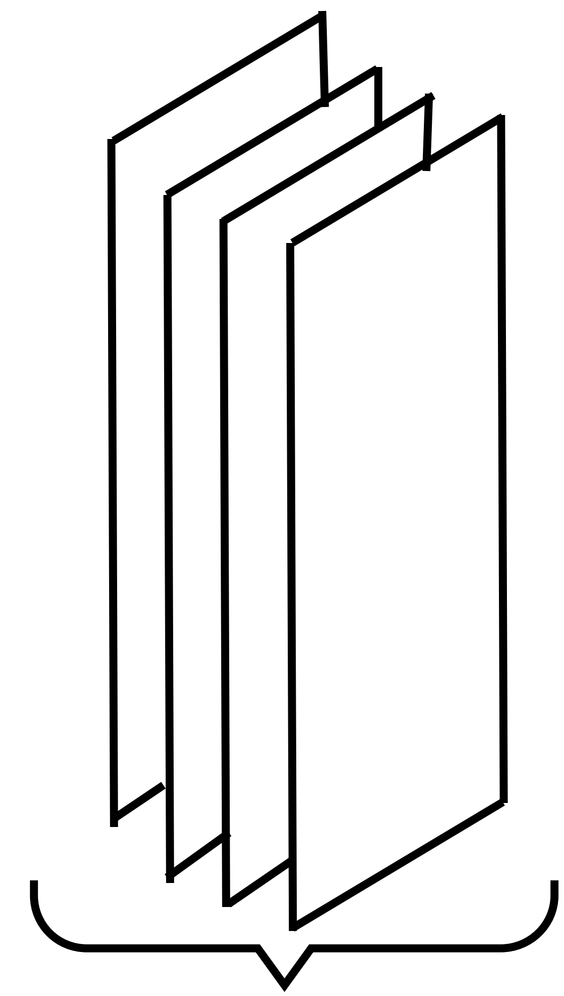
Susskind, Horowitz-Polchinski, …

$\boxed{\text{BH = D-branes + open strings}}$



U(2) YM

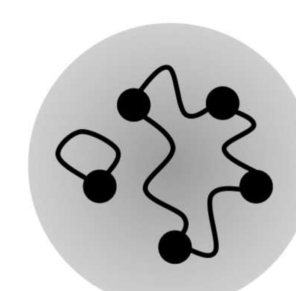
U(N) YM



N Dp-branes

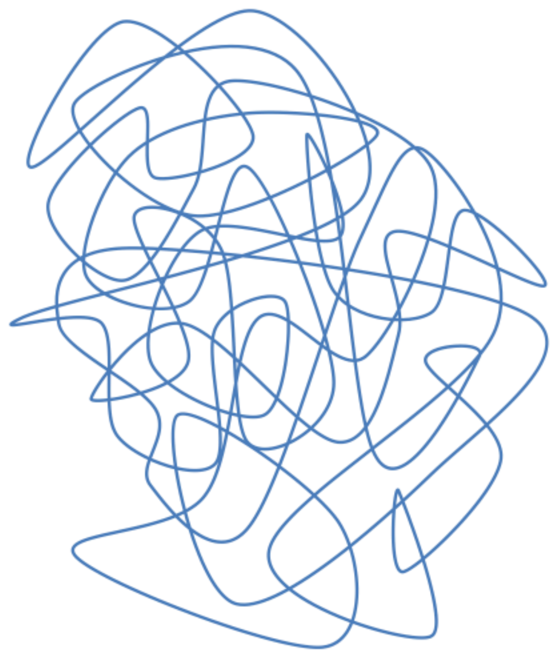
(i,j)-component of matrices
= string between i-th and j-th D-branes

$\boxed{\text{large N} \rightarrow \text{heavy} \rightarrow \text{BH}}$



Black hole from closed string

(e.g. Susskind 1993)

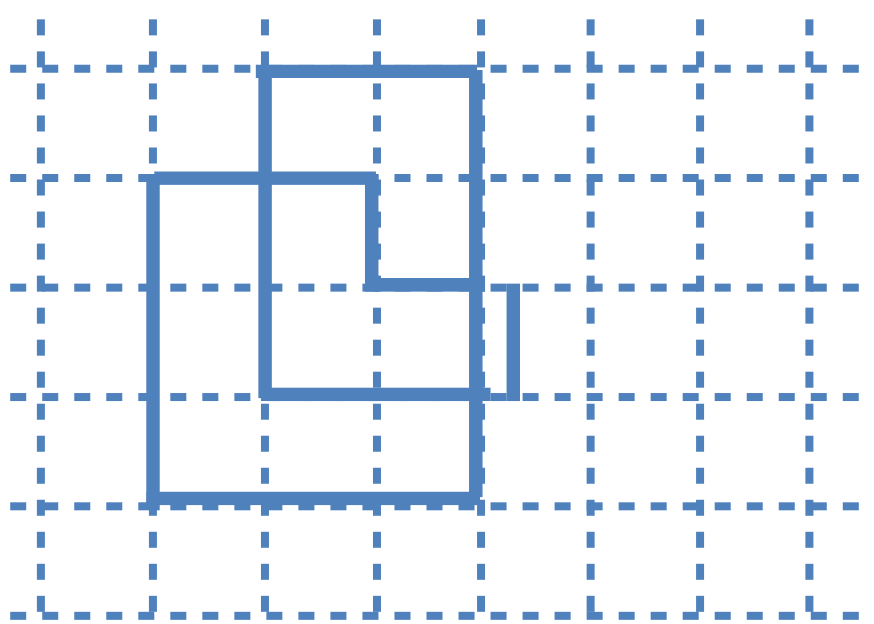


Consider a long, winding string with length L.

energy = tension $\times$ L

entropy $\sim$ L

when L >> 1, huge energy and entropy are packed in a small region → *black hole*



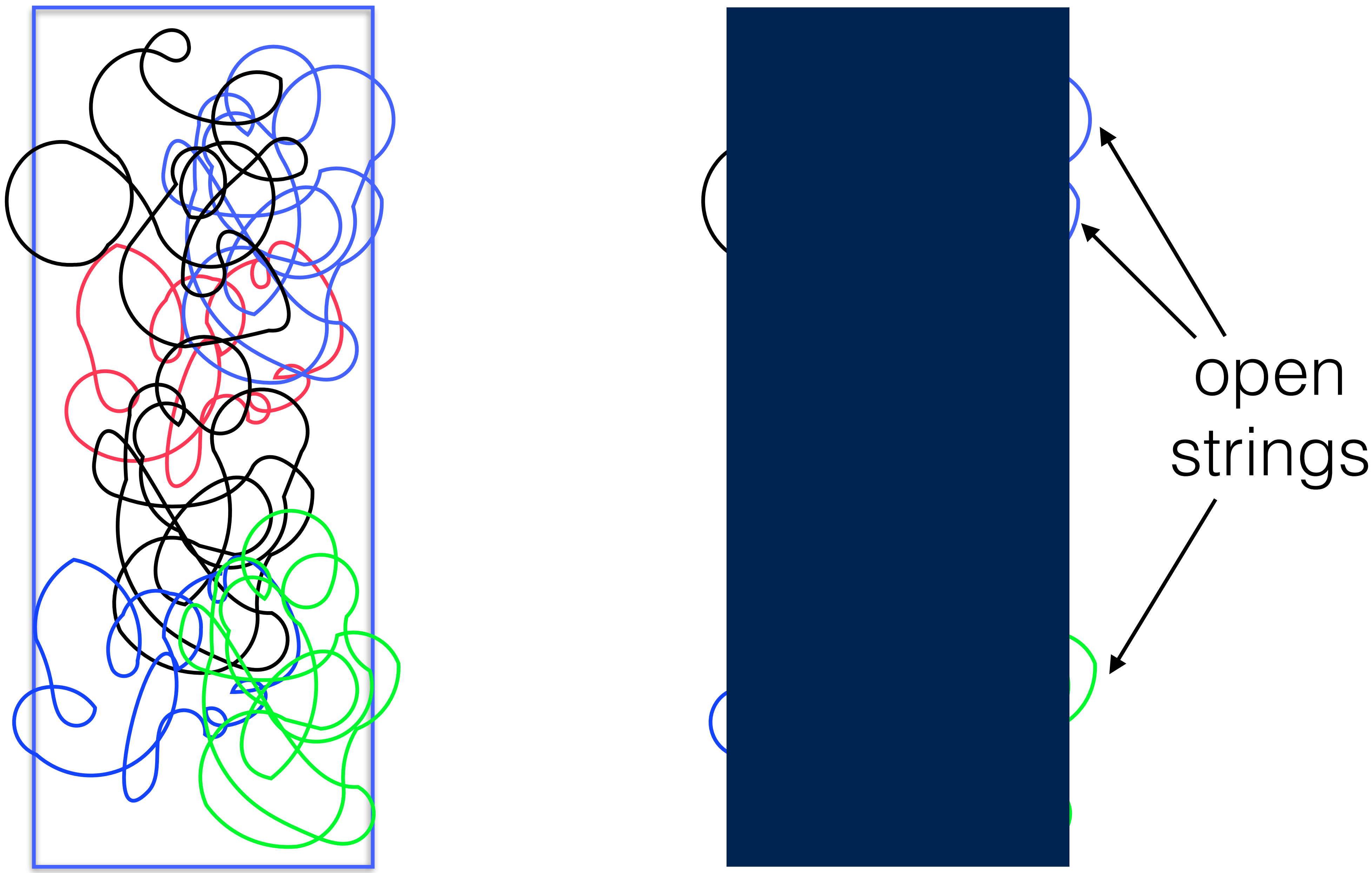
On D-dim square lattice,

# of possible shapes $\sim (2D\text{-}1)^L$

entropy $\sim L\times\log(2D\text{-}1)$

How are they related?

long, winding strings = black brane + open strings



The meaning of N (# of D-branes) becomes clear later.

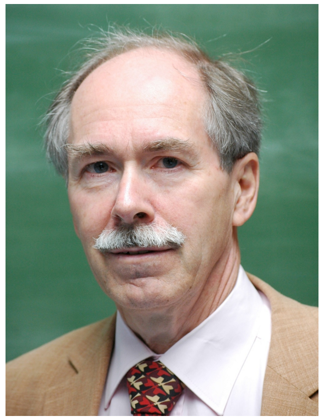
Gauge theory description

confining phase: ’t Hooft, 1974

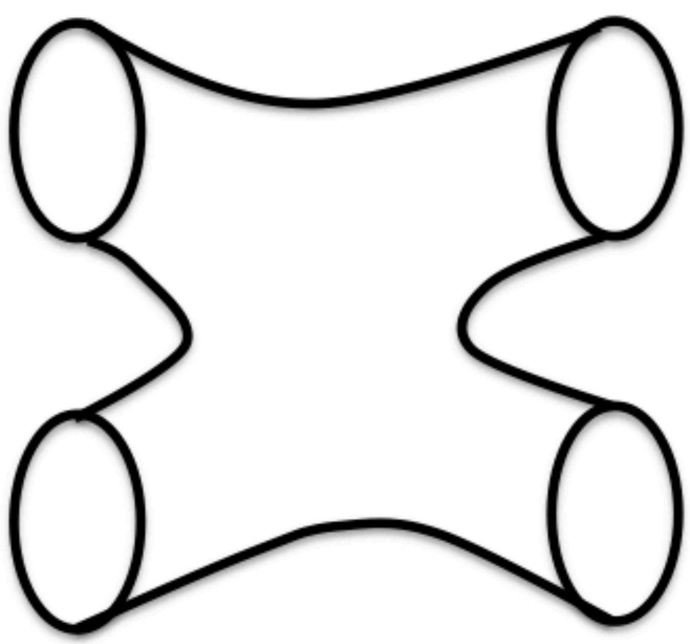
deconfining phase: M.H.-Maltz-Susskind, 2014

Strings out of YM:

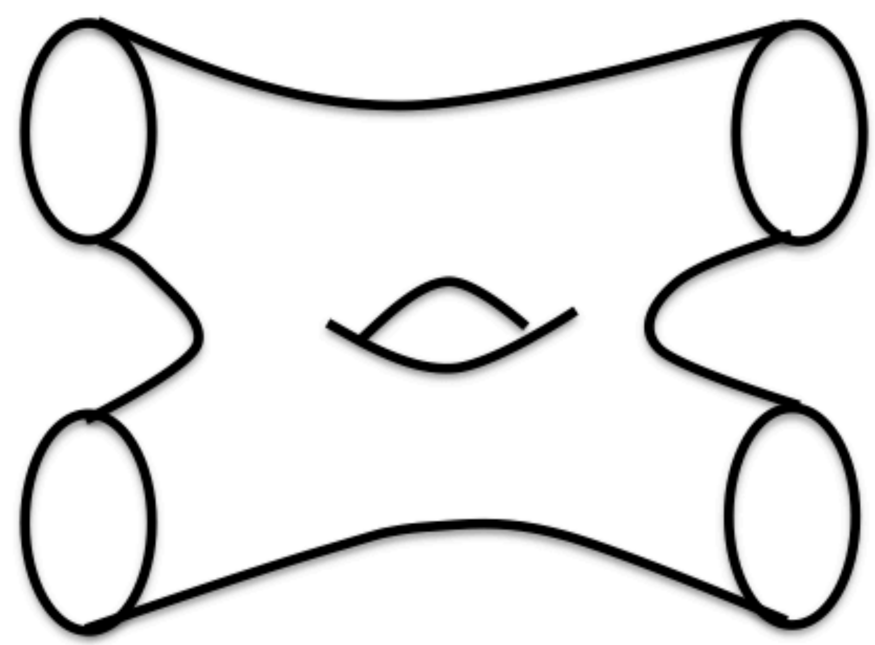
’t Hooft’s argument for the <u>confining</u> phase



scattering of strings



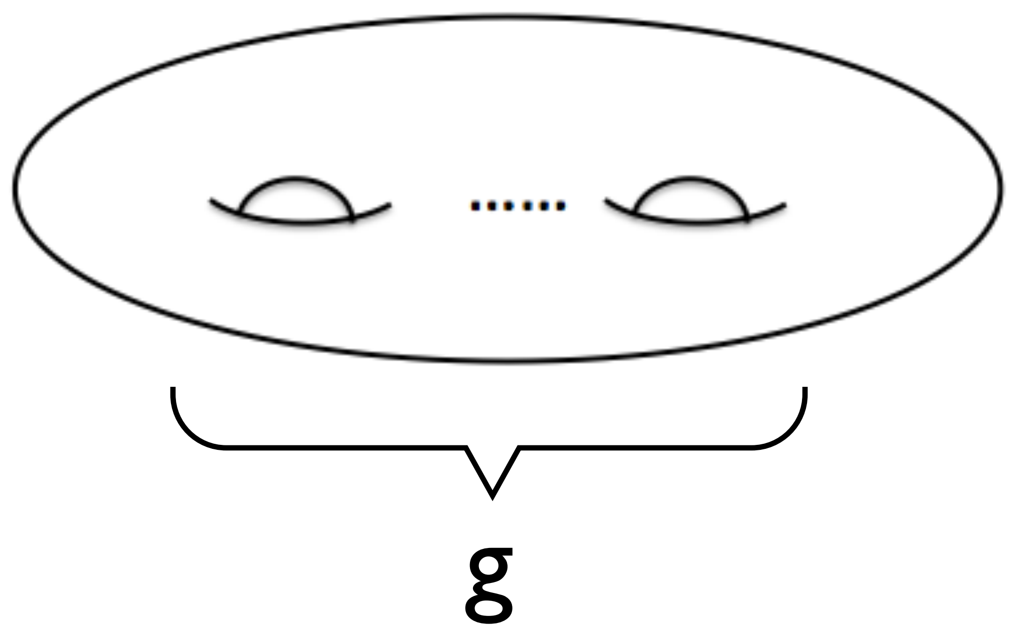
tree



one-loop

$\sim g_s^2$

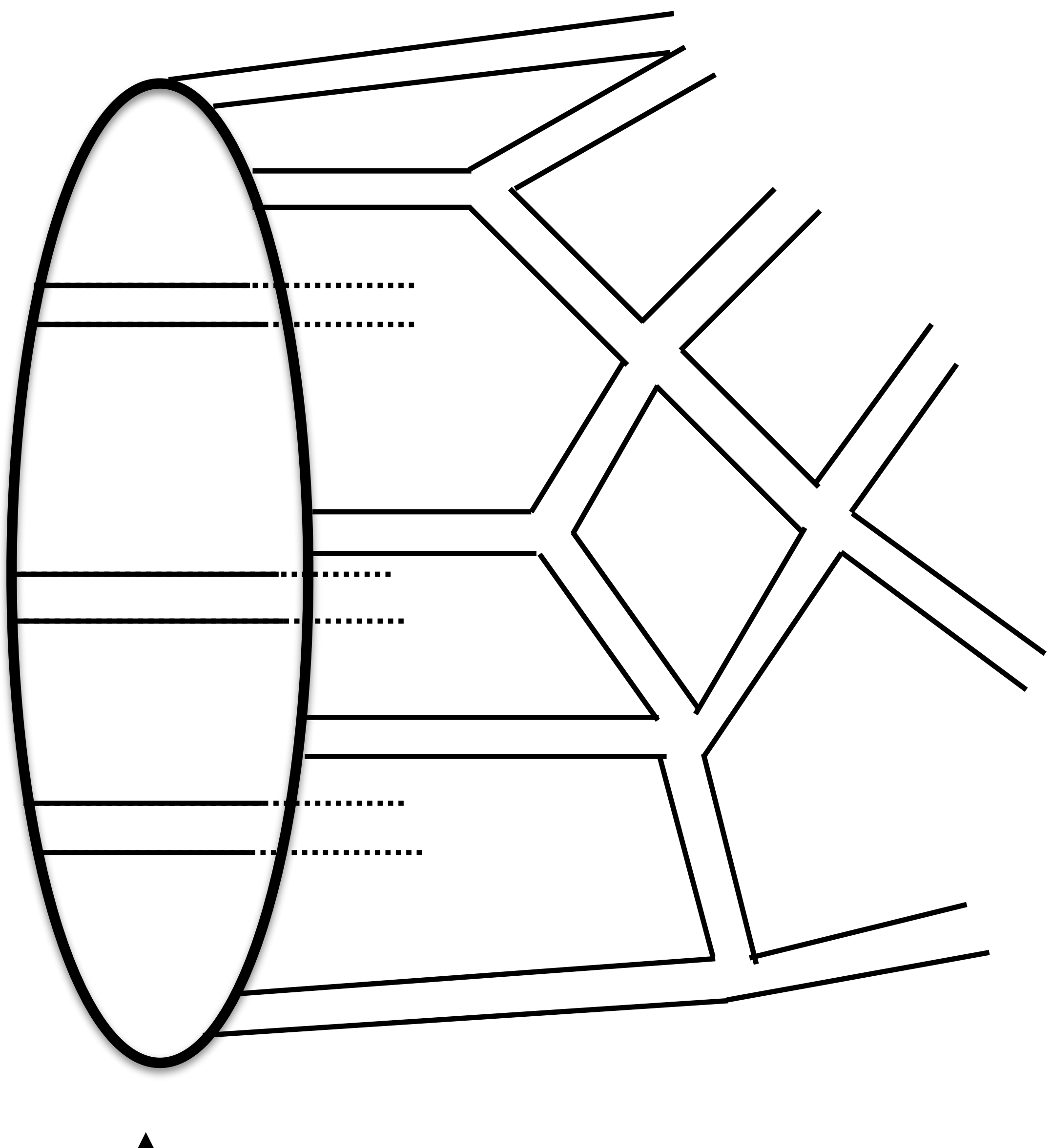
$\boxed{\text{g closed string loops} \rightarrow \text{genus g surface}}$



$\sim g_s^{2g}$

One takes into account the quantum effect order by order, by increasing g one by one.
→ *perturbative* formulation

Main idea



Feynman diagram
= "fishnet" made of gluons
= string worldsheet

How can they be related
without ambiguity?

Wilson loop = creation operator of closed string

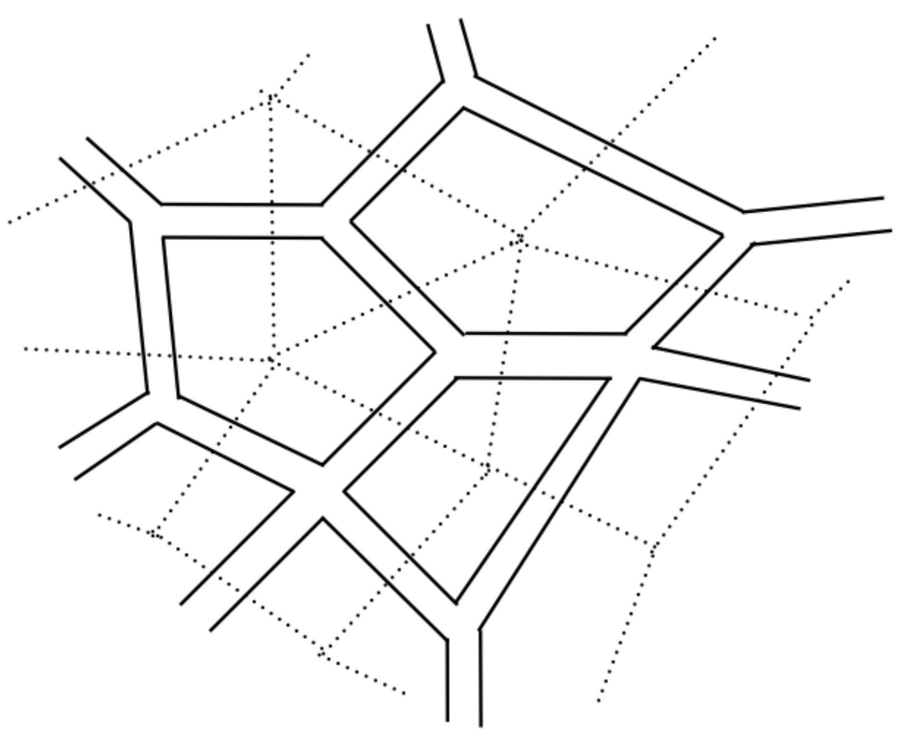
Main idea

Feynman diagram

=

triangulation/quadrangulation of string worldsheet

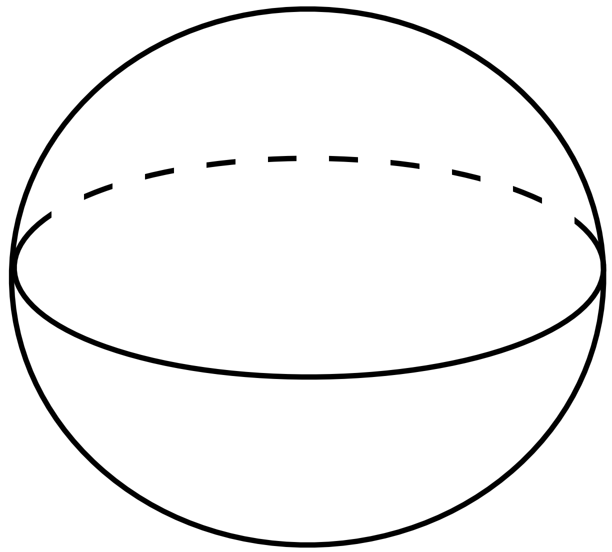
"fish net"



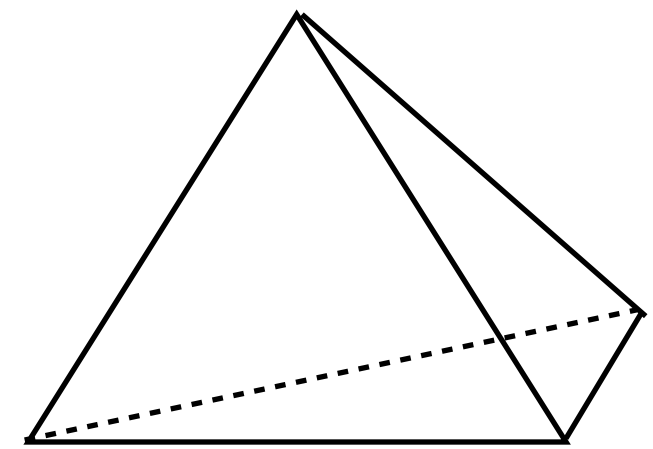
1/N expansion

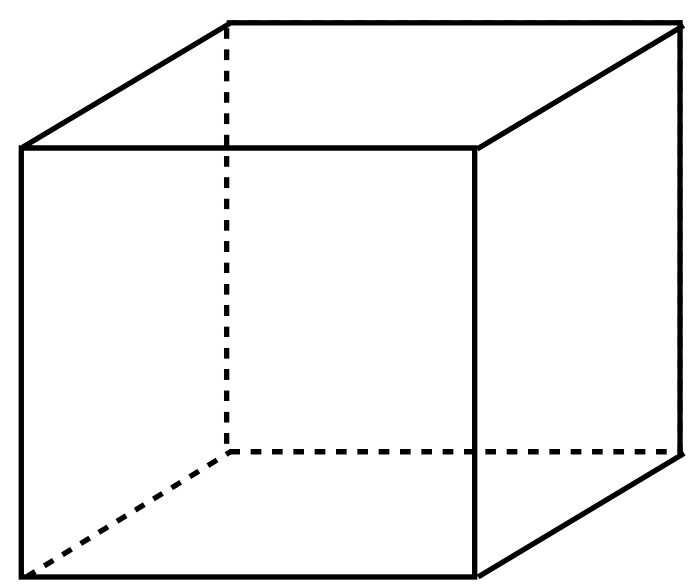
=

genus expansion



two-sphere (g=0)





$$S = \frac{N}{4\lambda}\int d^4x Tr F_{\mu\nu}^2$$

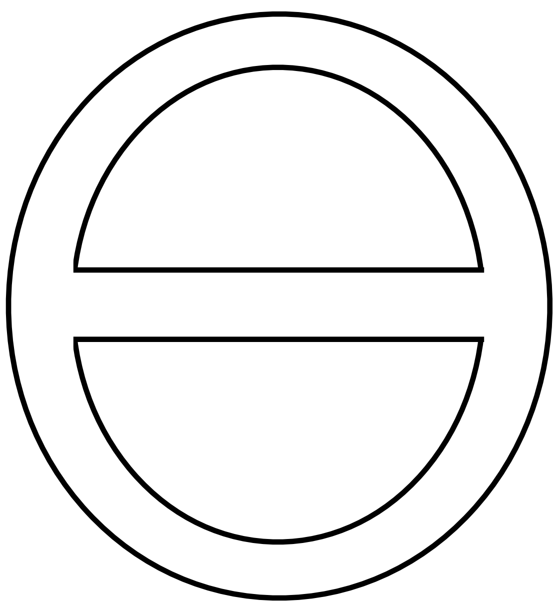
(U(N) gauge group)

vertex $\sim$ N

index loop $\sim$ N

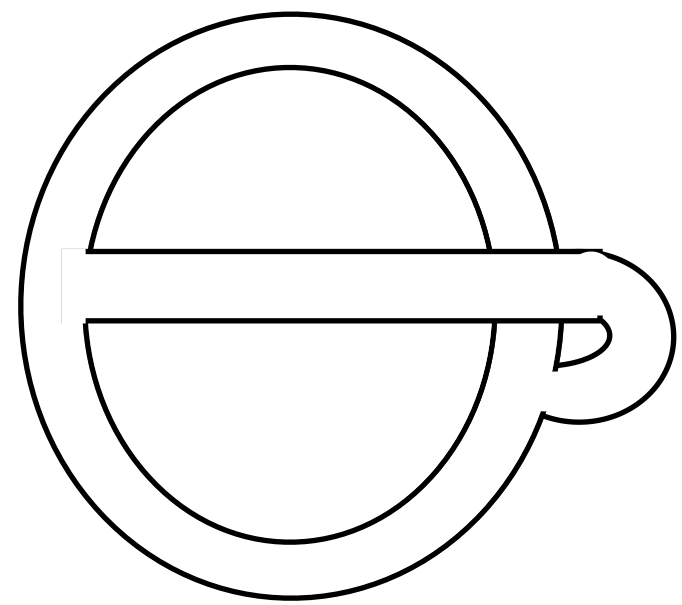
propagator $\sim$ 1/N

planar diagram

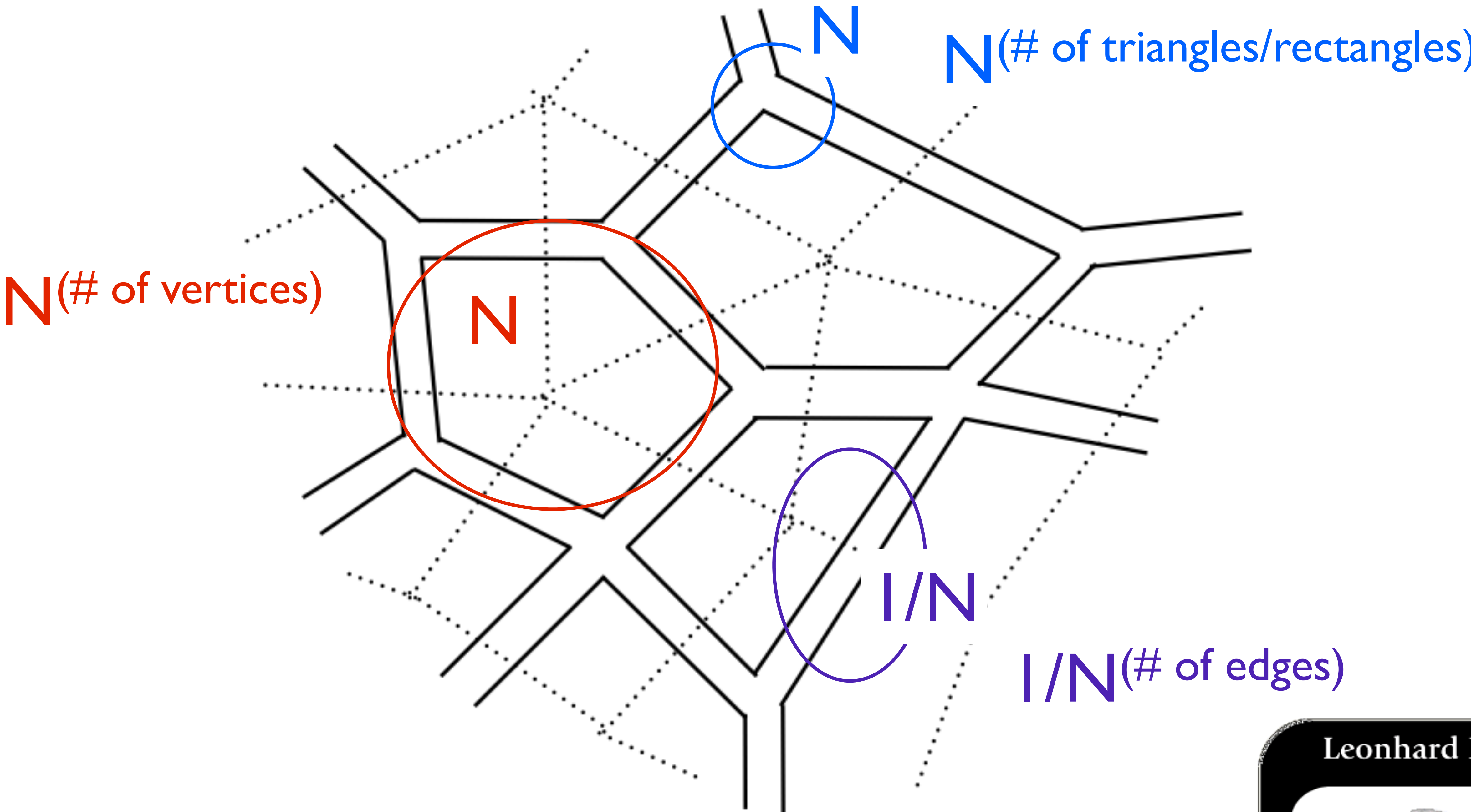


$N^2 \times N^{-3} \times N^3 = N^2$

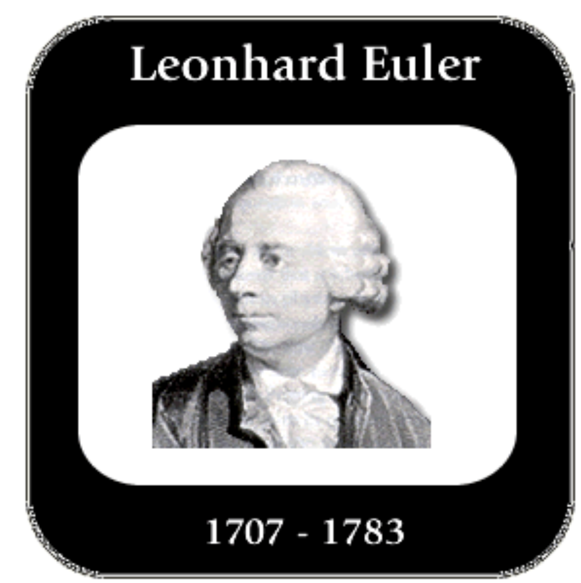
nonplanar diagram
(genus one)



$N^2 \times N^{-3} \times N^1 = N^0$



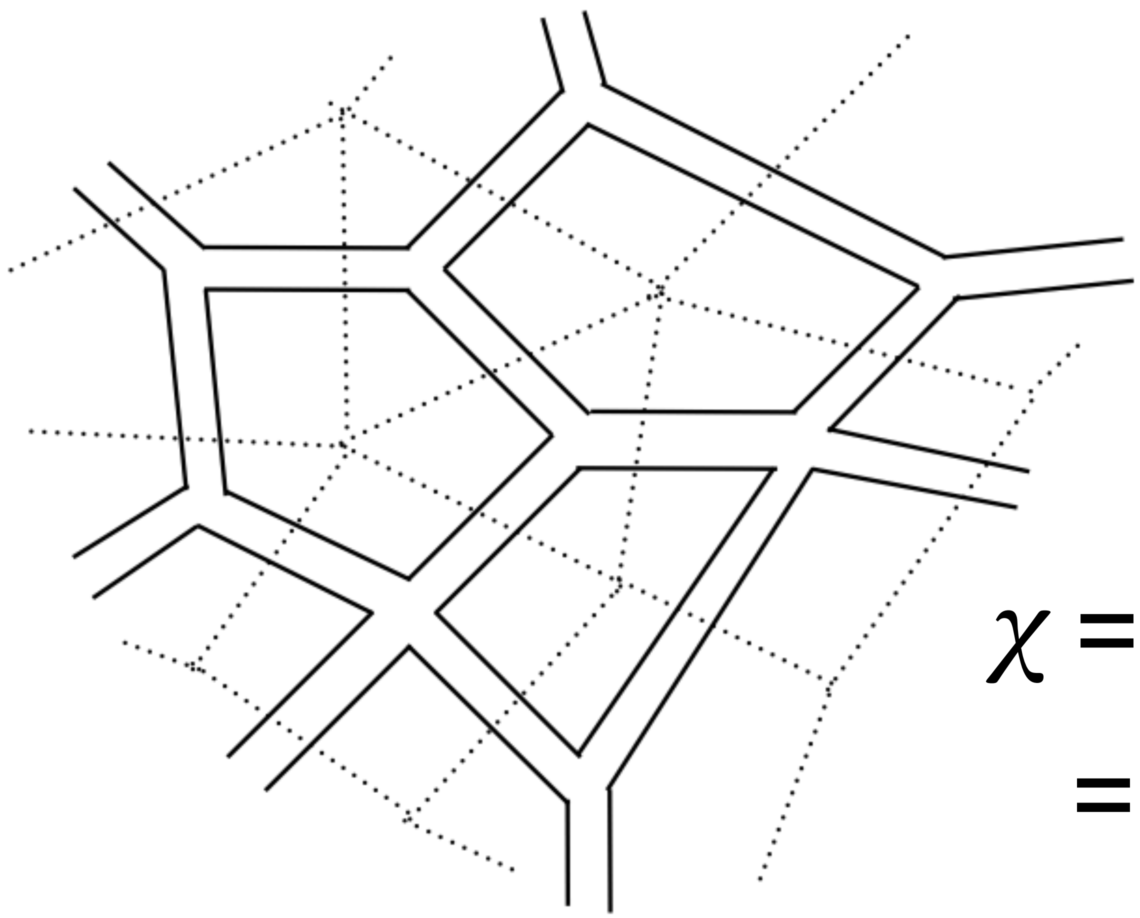
$$N^{(\#\text{ of vertices})} \times 1/N^{(\#\text{ of edges})} \times N^{(\#\text{ of triangles/rectangles})} = N^{\chi}$$



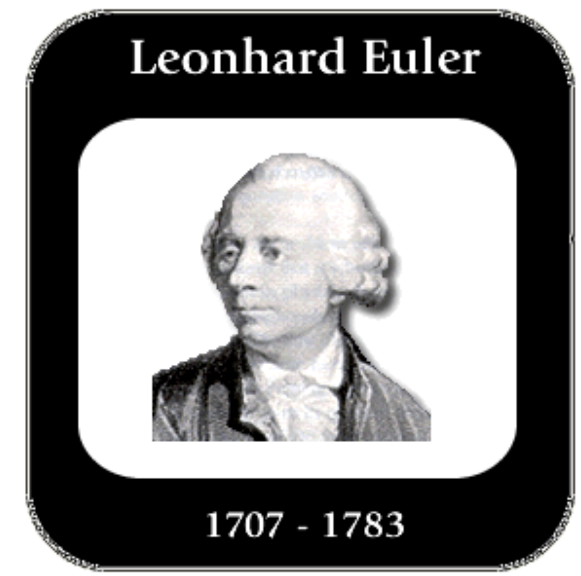
vertex $\sim$ N $\sim$ triangle/rectangle

index loop $\sim$ N $\sim$ vertex

propagator $\sim$ 1/N $\sim$ edges



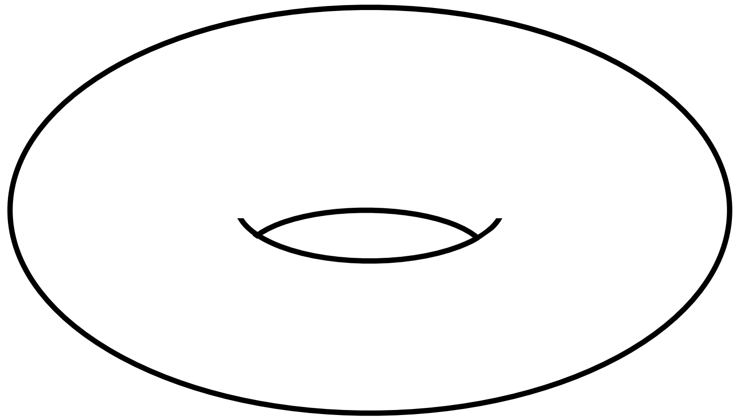
$\sim N^{\chi}$



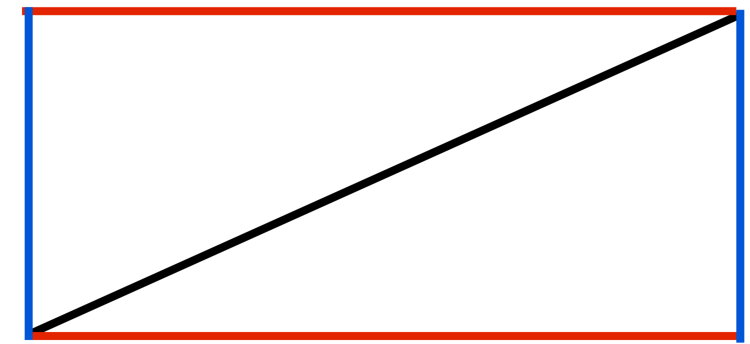
χ = Euler number

= (# vertices) − (# propagators)
+ (# index loops)

= (# triagnles/quadrangles)
− (# edges)+ (# vertices)



torus



triangulation of torus

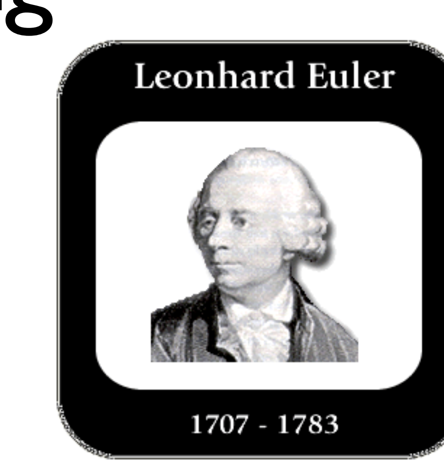
Euler number

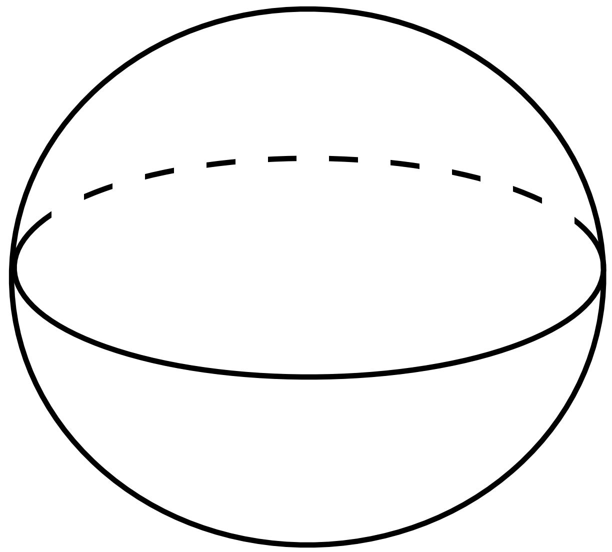
$$\chi = (\#\text{triangles}) - (\#\text{edges}) + (\#\text{vertices}) = 2 - 3 + 1 = 0$$

more generally,

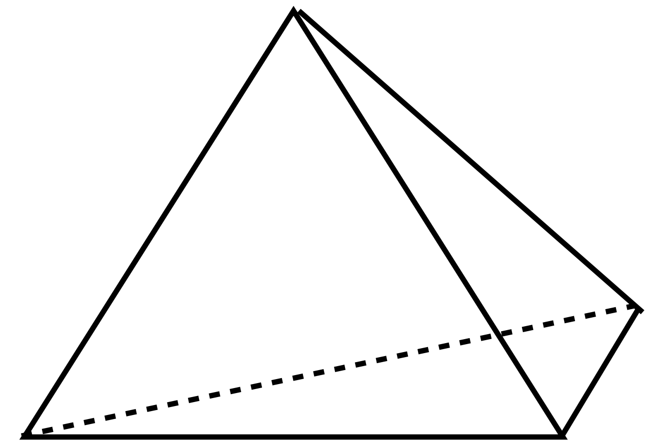
$$\chi = (\#\text{triangles}) - (\#\text{edges}) + (\#\text{vertices}) = 2 - 2g$$

where $g = (\#\text{genus})$



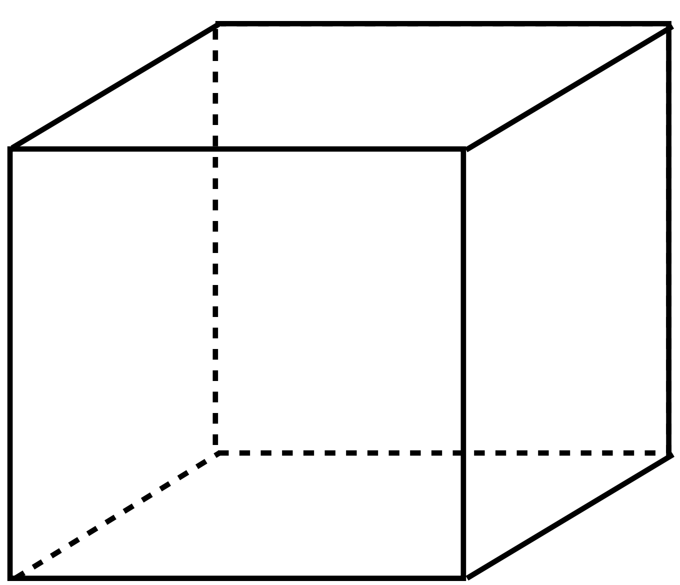


two-sphere (g=0)



4 triangles
6 edges
4 vertices

$$\boxed{4-6+4 = 2 = 2-2g}$$

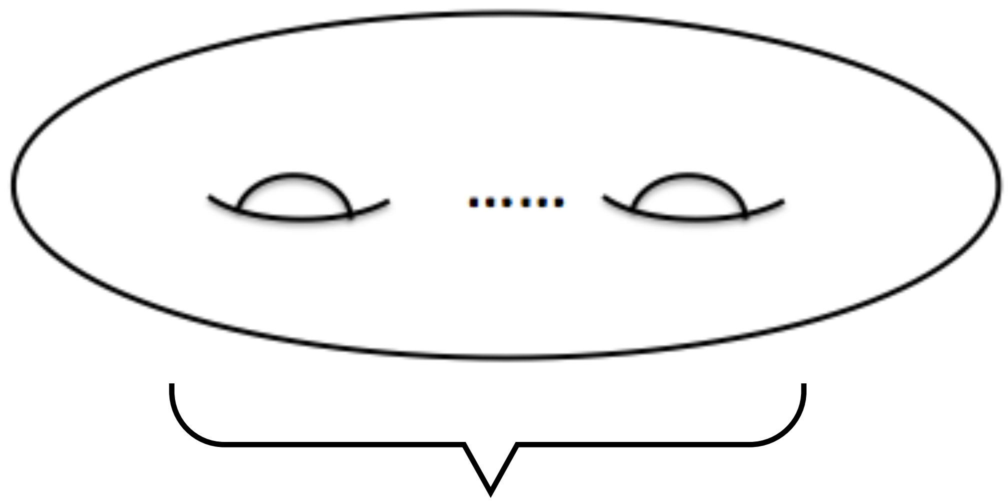


6 squares
12 edges
8 vetices

$$\boxed{6-12+8 = 2 = 2-2g}$$

genus-g diagram = diagram which can be drawn on genus-g surface

$\Rightarrow$ $\boxed{\text{g closed string loops}}$



g

$$(1/N)^{2g-2} = g_s^{\,2g-2}$$

$$1/N = g_s$$

Yang-Mills gives *nonperturbative* formulation of string theory.

large-N limit is free string theory.

Hamilton formulation on lattice

Understand it by using the Hamiltonian formulation of lattice gauge theory (Kogut-Susskind, 1974)

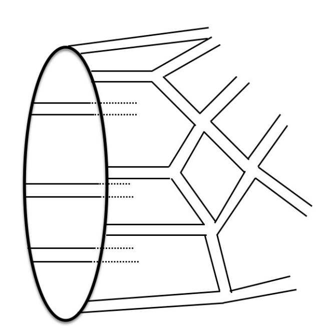
$$H = \frac{\lambda N}{2} \sum_{\vec{x}} \sum_{\mu} \sum_{\alpha=1}^{N^2} \left(E^{\alpha}_{\mu,\vec{x}}\right)^2 + \frac{N}{\lambda} \sum_{\vec{x}} \sum_{\mu<\nu} \left(N - \mathrm{Tr}(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}})\right)$$

$$[E^{\alpha}_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = \delta_{\mu\nu} \delta_{\vec{x}\vec{y}} \cdot \tau^{\alpha} U_{\nu,\vec{y}}$$

$$\sum_{\alpha=1}^{N^2} \tau^{\alpha}_{ij} \tau^{\alpha}_{kl} = \frac{\delta_{il}\delta_{jk}}{N^2}$$

Hilbert space is expressed by <u>Wilson loops.</u>

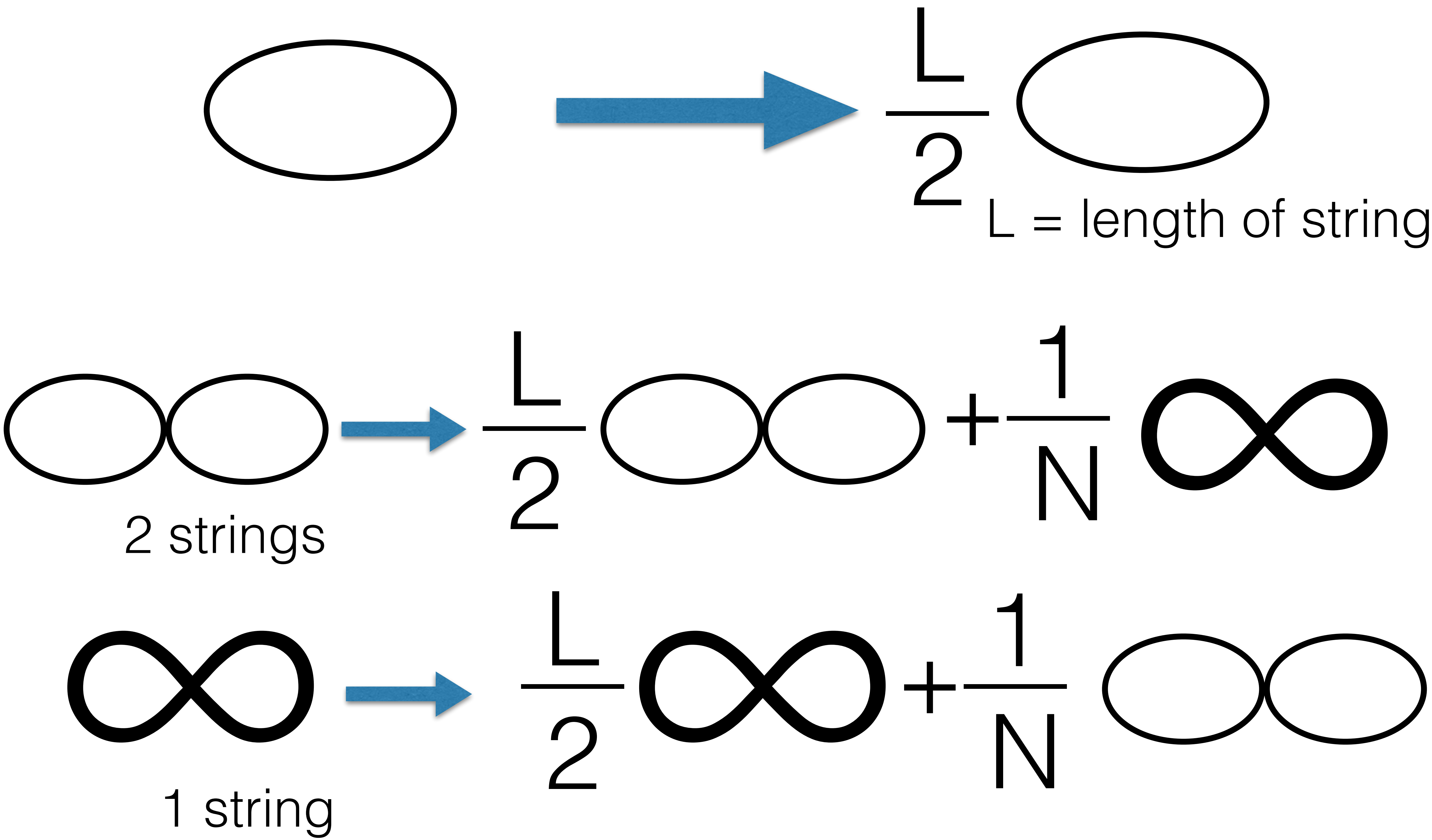
(closed string)



strong coupling limit

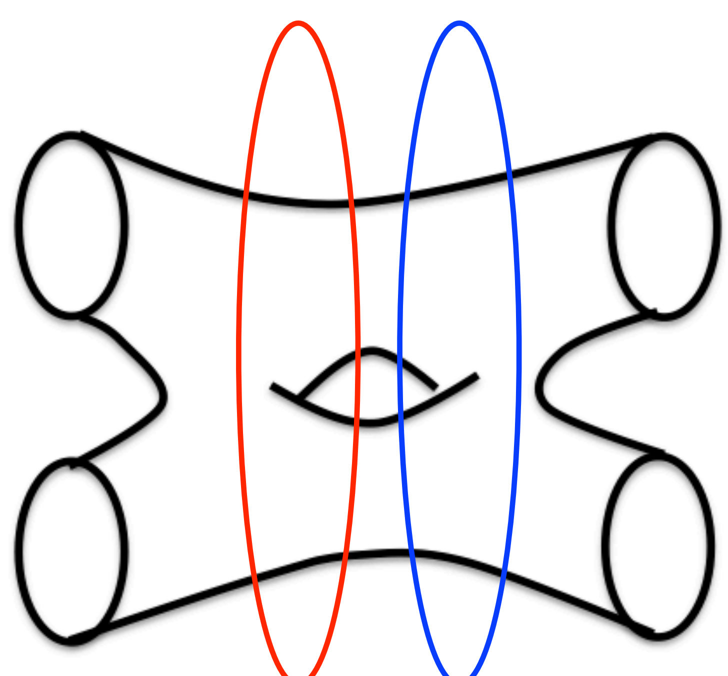
$$H = \frac{N}{2}\sum_{\vec{x}}\sum_{\mu}\sum_{\alpha=1}^{N^2}(E^{\alpha}_{\mu,\vec{x}})^2$$

(λ=1 for simplicity)



$$W_C = \mathrm{Tr}(U_{\mu,\vec{x}} M_C) \qquad W_{C'} = \mathrm{Tr}(U_{\mu,\vec{x}} M_{C'})$$

$$\begin{aligned} & H|W_C, W_{C'}\rangle \\ &= \frac{\lambda(L+L')}{2}|W_C, W_{C'}\rangle \\ &\quad +\lambda N \sum_\alpha \mathrm{Tr}(\tau^\alpha U_{\mu,\vec{x}} M_C) \cdot \mathrm{Tr}(\tau^\alpha U_{\mu,\vec{x}} M_{C'})|0\rangle \\ &= \frac{\lambda(L+L')}{2}|W_C, W_{C'}\rangle + \frac{\lambda}{N}\mathrm{Tr}(U_{\mu,\vec{x}} M_C U_{\mu,\vec{x}} M_{C'})|0\rangle \end{aligned}$$



Lattice gauge theory description at strong coupling

Understand it by using the Hamiltonian formulation of lattice gauge theory (Kogut-Susskind, 1974)

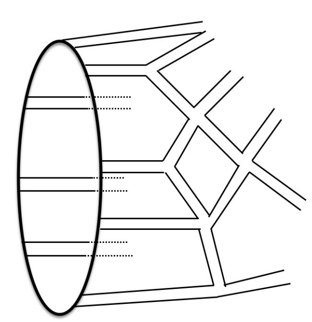
$$H = \frac{\lambda N}{2}\sum_{\vec{x}}\sum_{\mu}\sum_{\alpha=1}^{N^2}\left(E^{\alpha}_{\mu,\vec{x}}\right)^2 + \frac{N}{\lambda}\sum_{\vec{x}}\sum_{\mu<\nu}\left(N - \mathrm{Tr}(U_{\mu,\vec{x}}U_{\nu,\vec{x}+\hat{\mu}}U^{\dagger}_{\mu,\vec{x}+\hat{\nu}}U^{\dagger}_{\nu,\vec{x}})\right)$$

$$[E^{\alpha}_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = \delta_{\mu\nu}\delta_{\vec{x}\vec{y}}\cdot\tau^{\alpha}U_{\nu,\vec{y}}$$

$$\sum_{\alpha=1}^{N^2}\tau^{\alpha}_{ij}\tau^{\alpha}_{kl} = \frac{\delta_{il}\delta_{jk}}{N^2}$$

Hilbert space is expressed by <u>Wilson loops.</u>

(closed string)



strong coupling limit

$$H = \frac{N}{2}\sum_{\vec{x}}\sum_{\mu}\sum_{\alpha=1}^{N^2}(E^{\alpha}_{\mu,\vec{x}})^2$$

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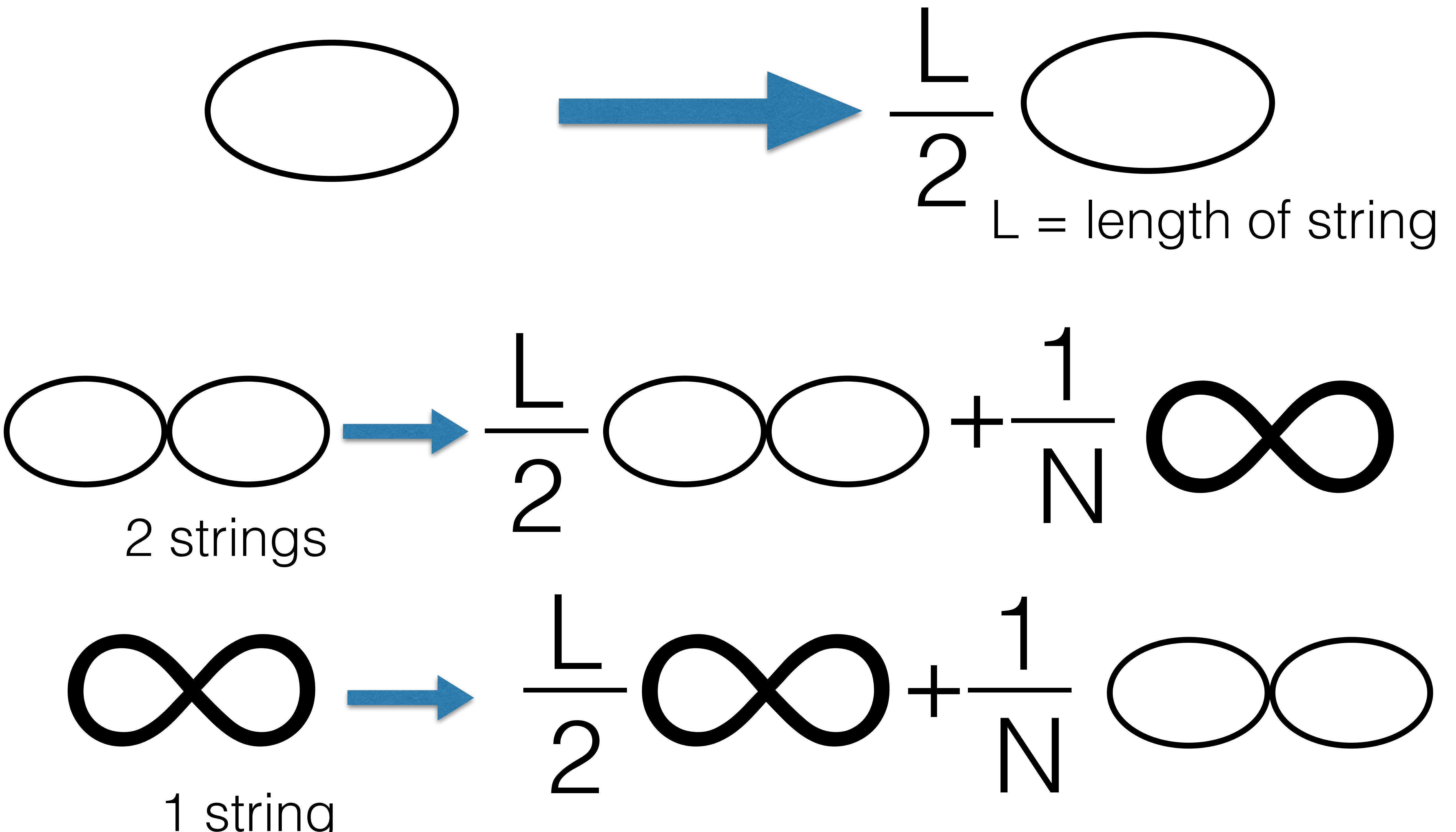
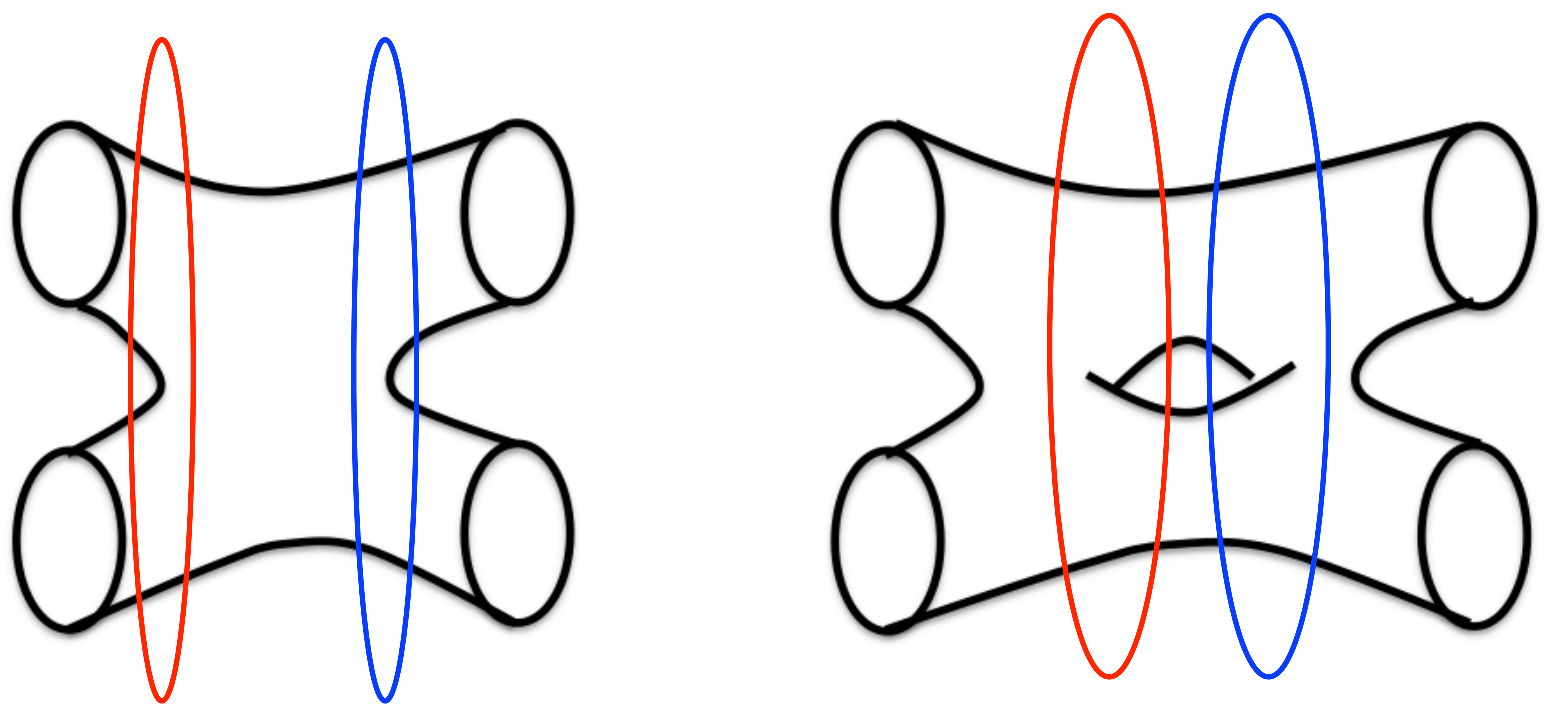




splitting $\sim$ 1/N

joining $\sim$ 1/N

$1/N^2$ for each loop of closed strings

"large-N limit is the theory of free string"

Strings out of YM: deconfining phase

M.H.-Maltz-Susskind, 2014

Hilbert space is always the same. Why don't we express the deconfining phase by using Wilson loops?

- interaction (joining/splitting) is 1/N-suppressed

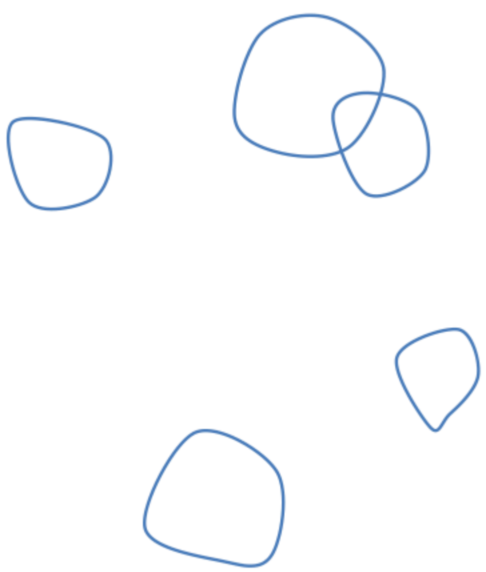
"large-N limit is the theory of free string"

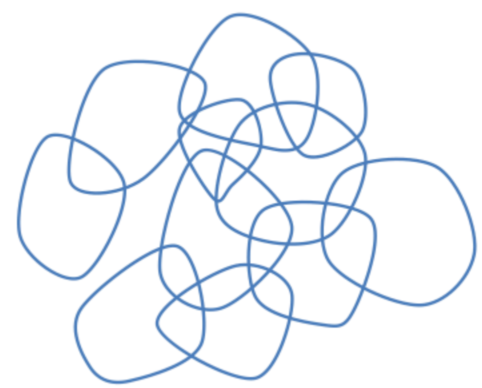
- It is true when L is $O(N^0)$. ($\rightarrow$confining phase)

- In deconfinement phase, total length of the strings is $O(N^2)$ $\rightarrow$ number of intersections is $O(N^2)$ $\rightarrow$interaction is **not** negligible

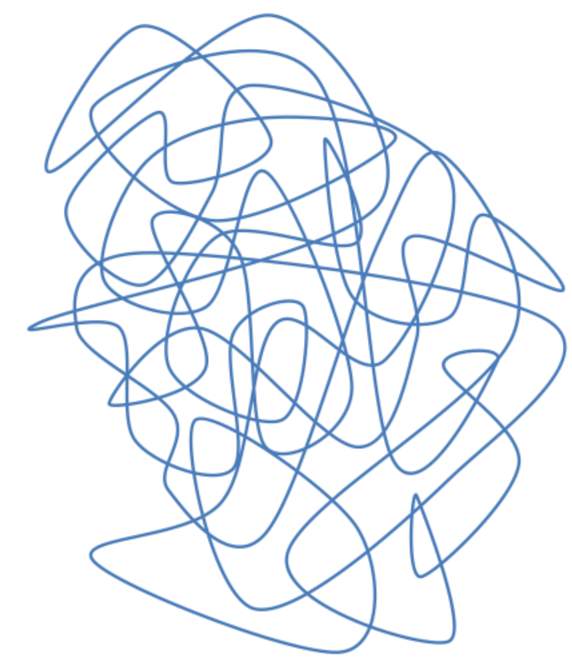
$\boxed{\text{large-N limit is still very dynamical!}}$

confining phase
= gas of short strings





long and winding string,
which is interpreted as BH,
appears



as the density of strings increase,
interaction between strings
becomes important,and…

Why $L \sim N^2$?

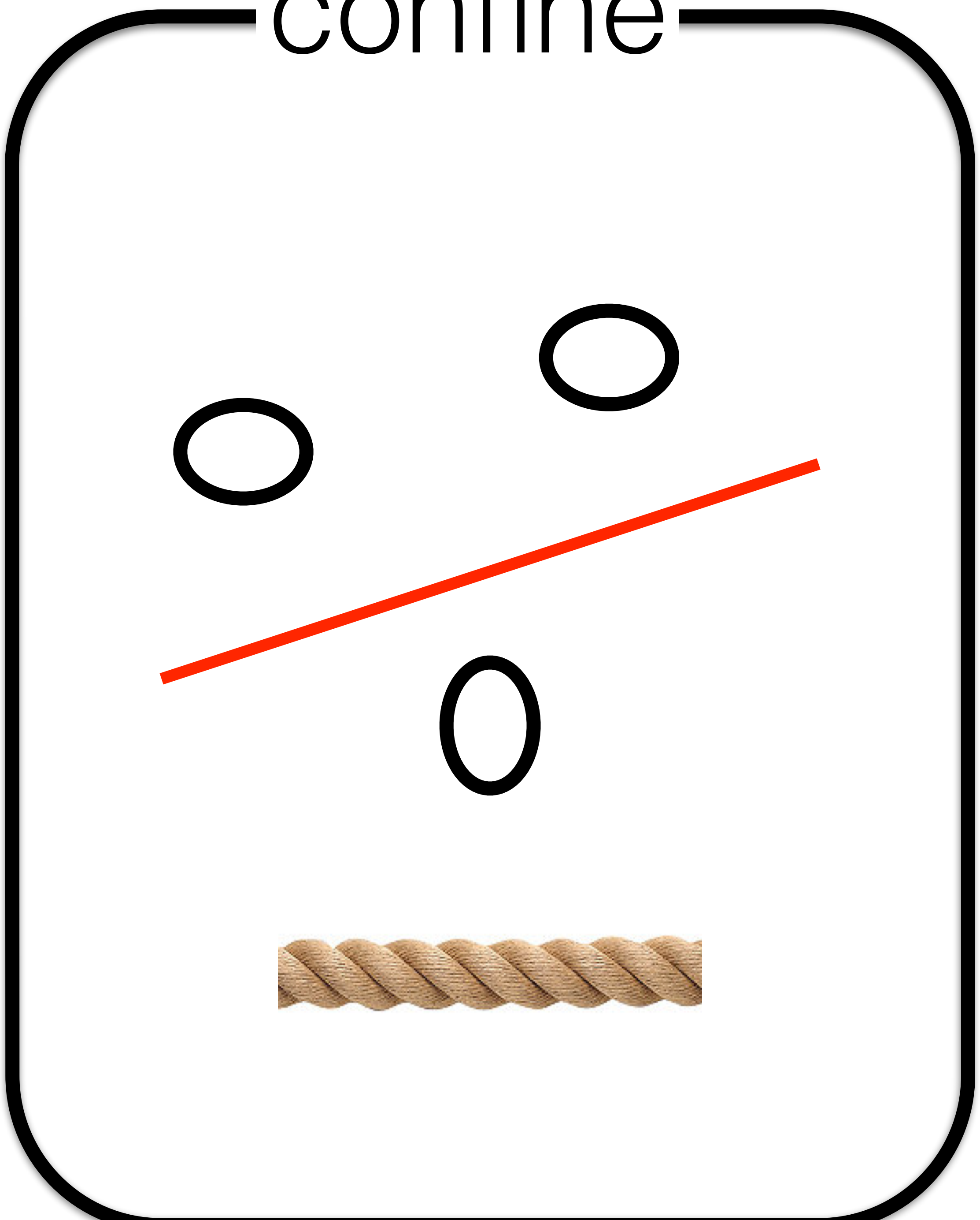
- Tr(UU'U''.....)

length $\gtrsim$ N^2 → factorizes to shorter traces

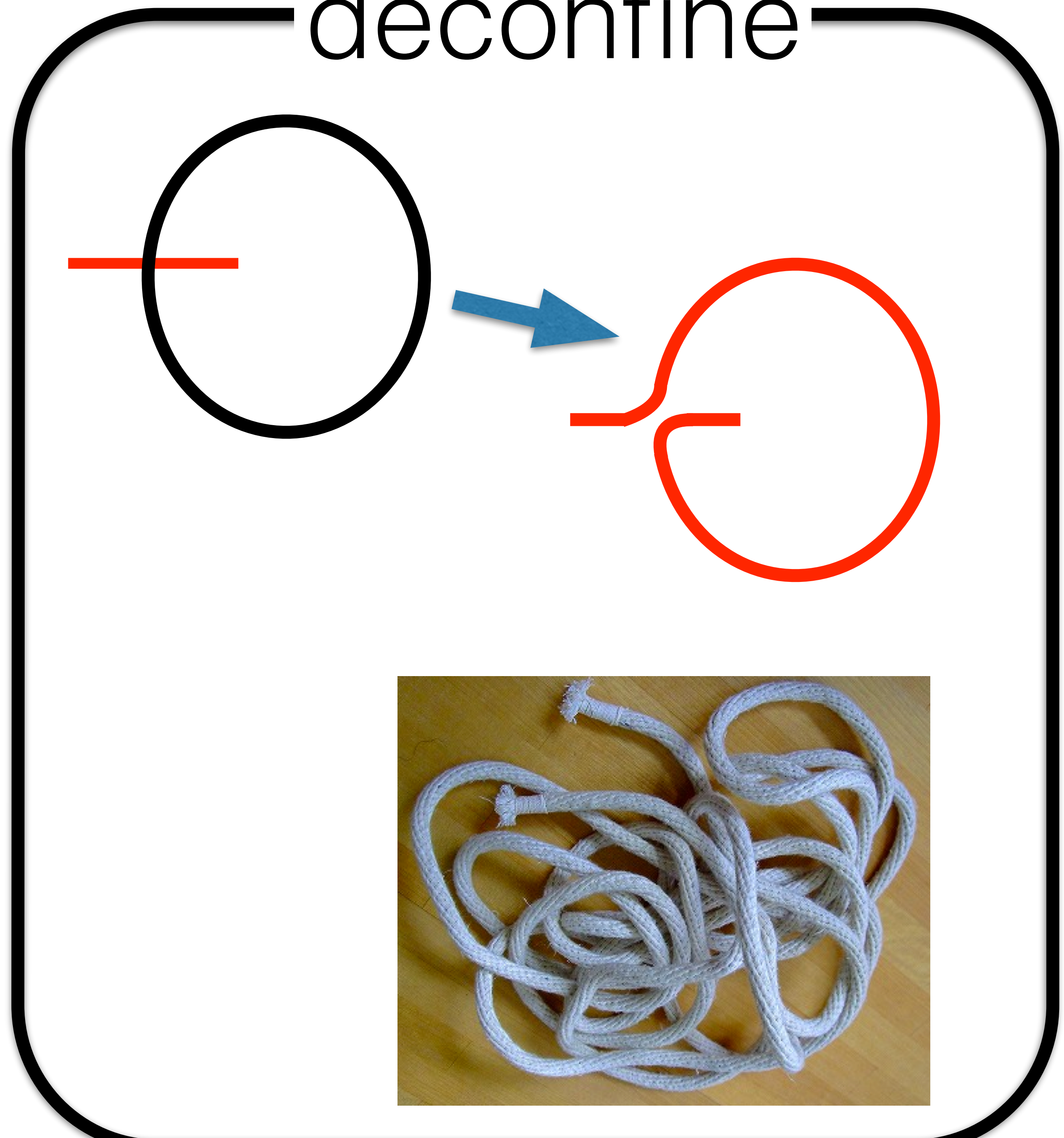
N^2 is the upper bound.
Beyond there, the counting changes;
<u>not much gain for the entropy.</u>

(de)confinement of probe charges

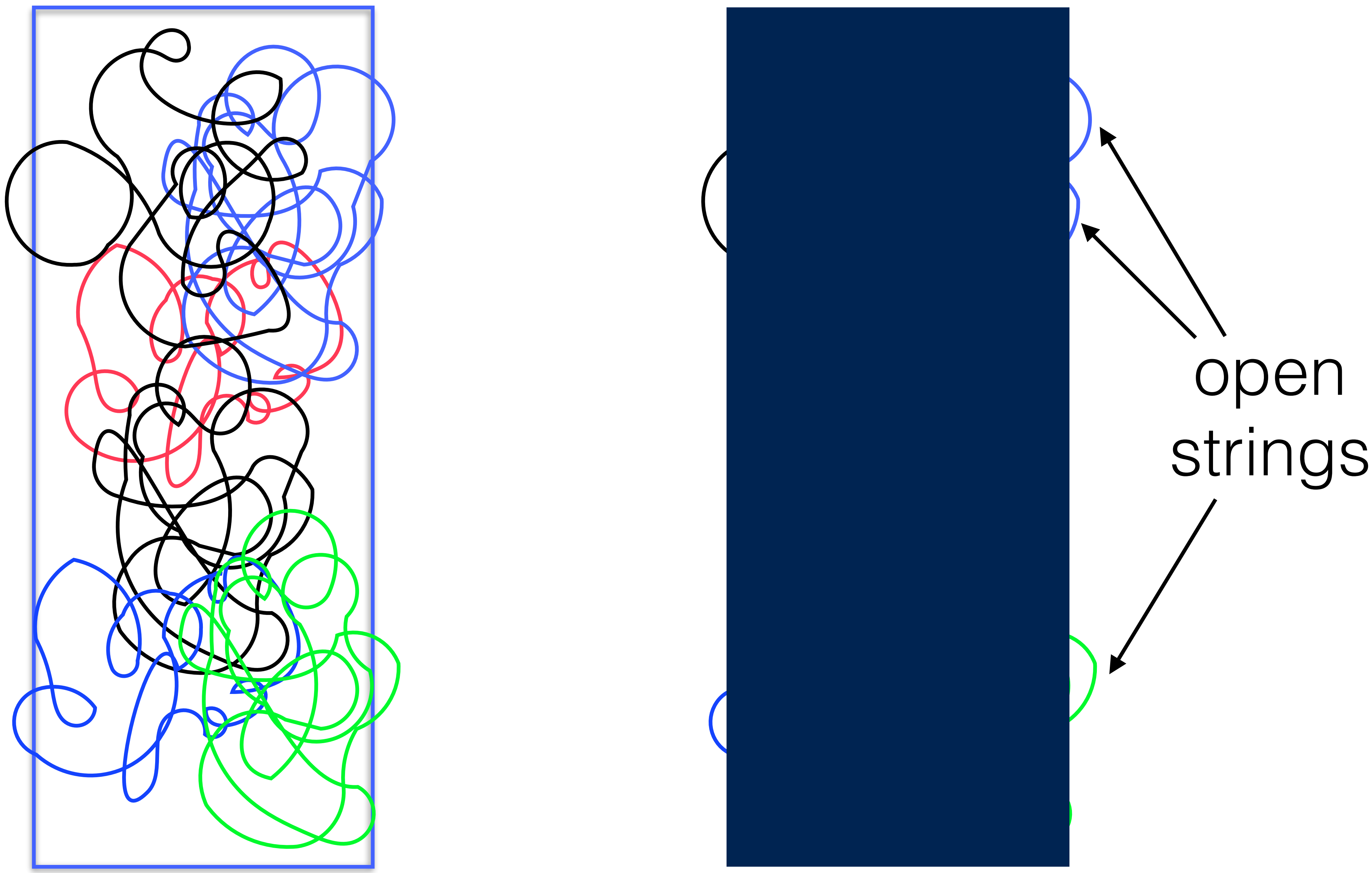
confine



deconfine



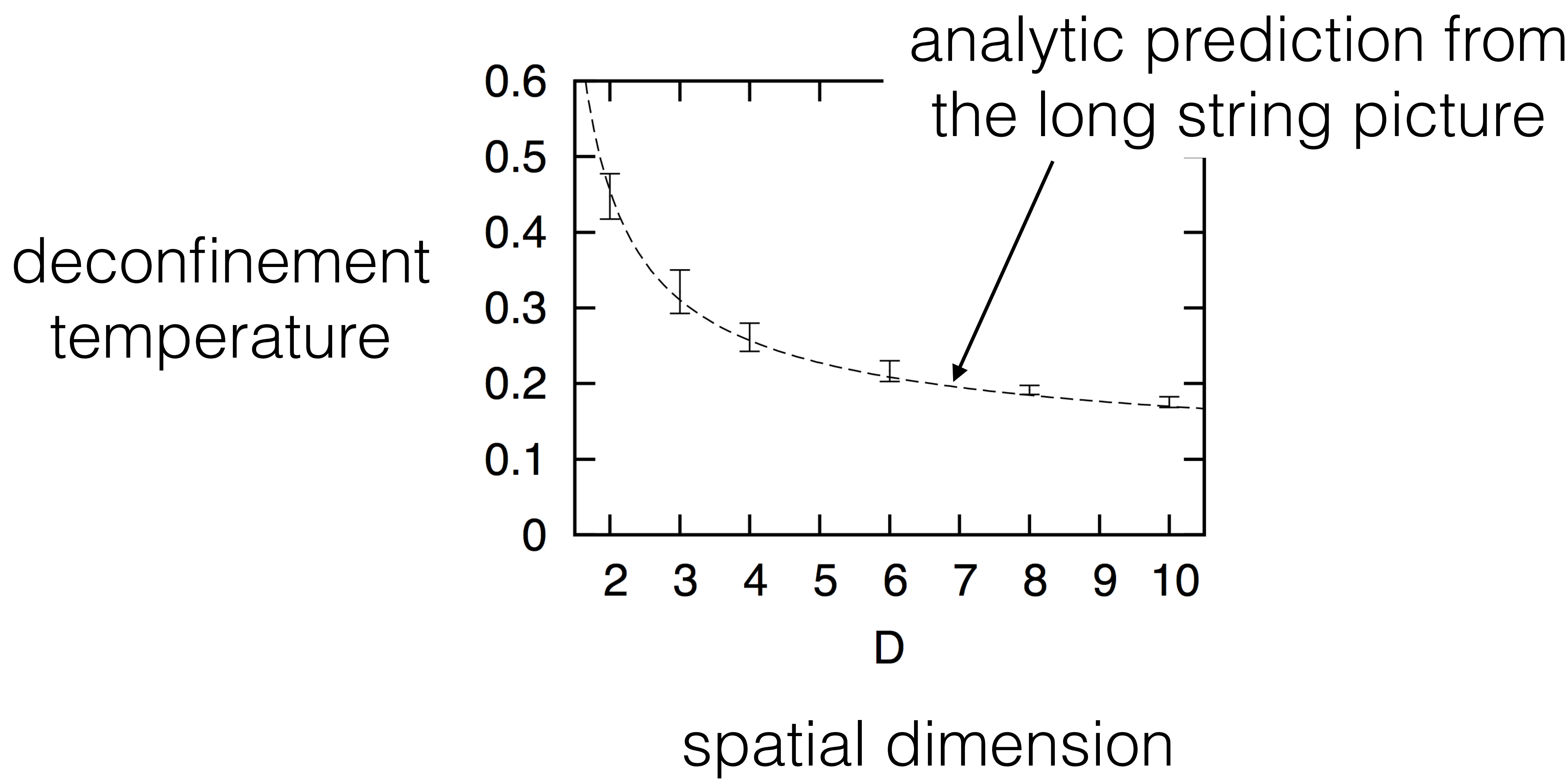
long, winding *QCD*-strings = black brane + open *QCD*-strings



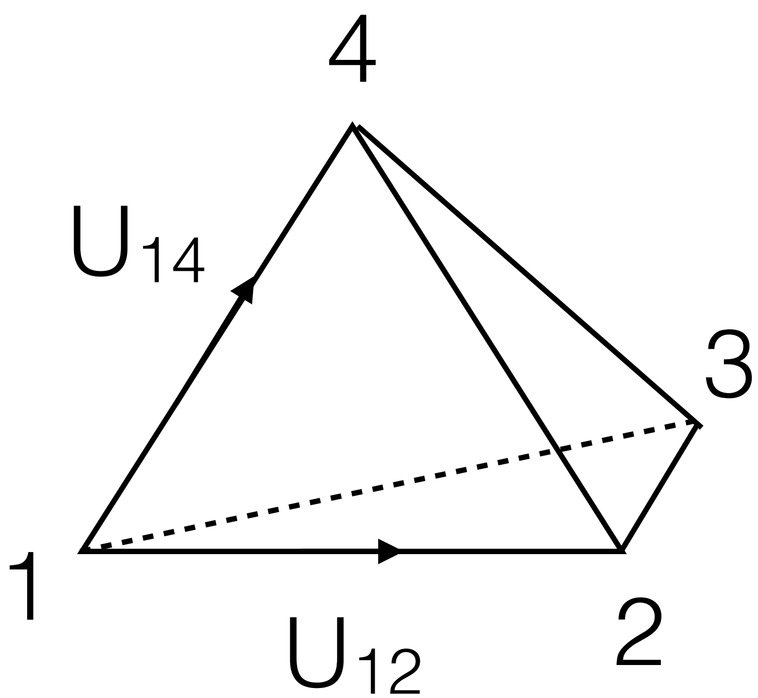
open strings = Wilson lines, which have N color d.o.f at endpoints

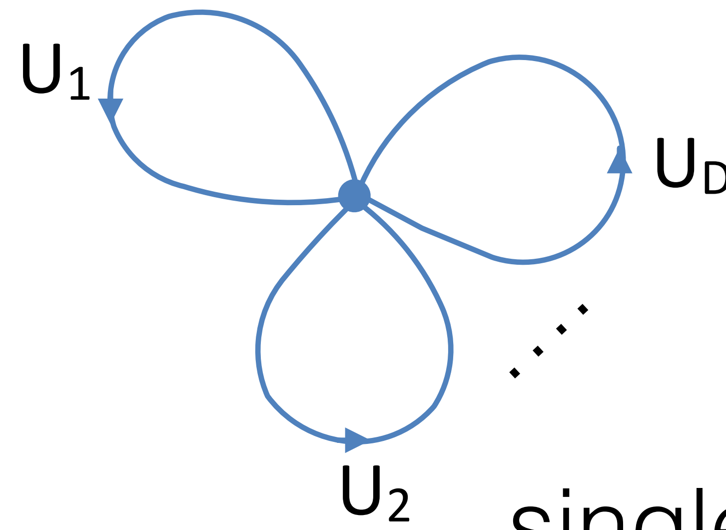
→ black brane is made from N Dp-branes

D-dim square lattice at strong coupling



matrix models at strong coupling



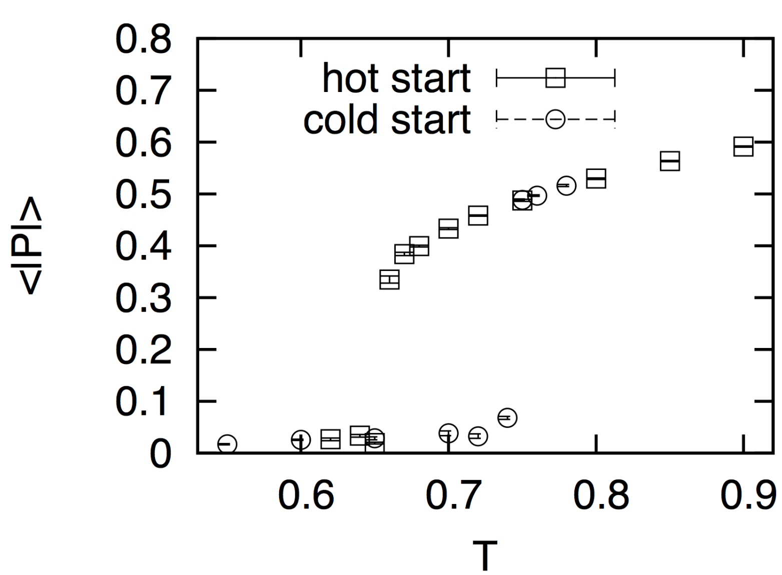


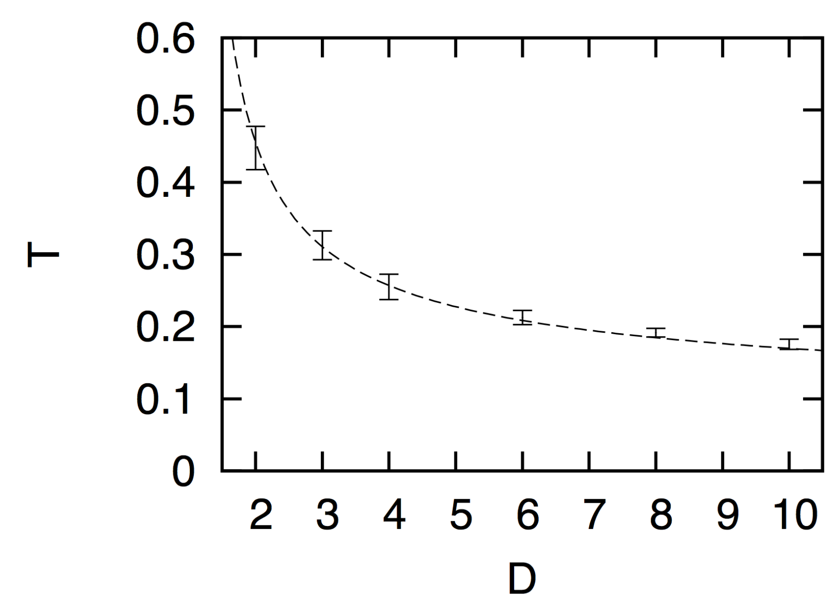
$$T_c = \frac{1}{2\log(2D-1)}$$

single-site with D-links
(Eguchi-Kawai model)

tetrahedron $T_c = \frac{1}{2\log 2} = 0.72\ldots$

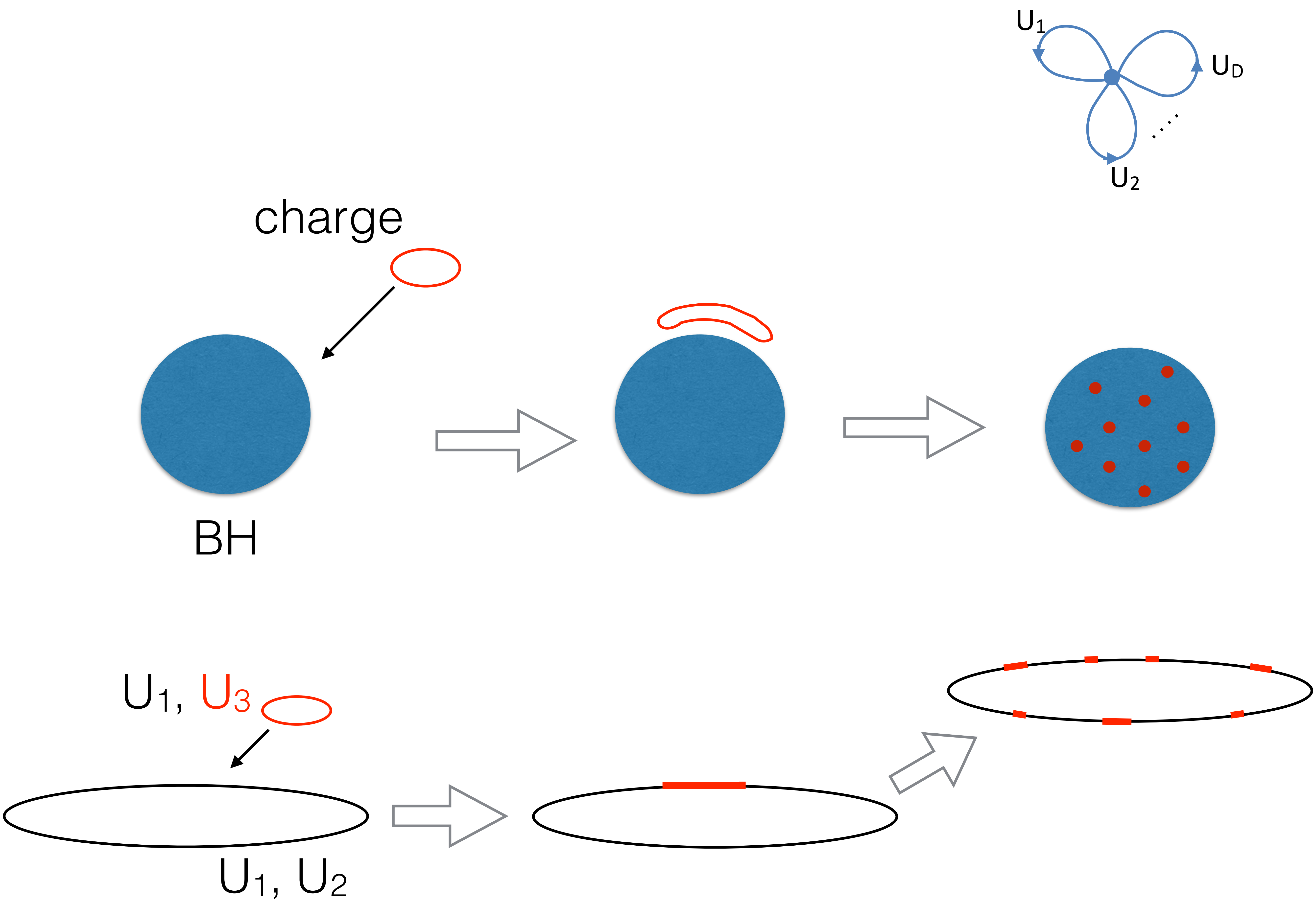
(Equivalent to large-volume lattice via Eguchi-Kawai equivalence)



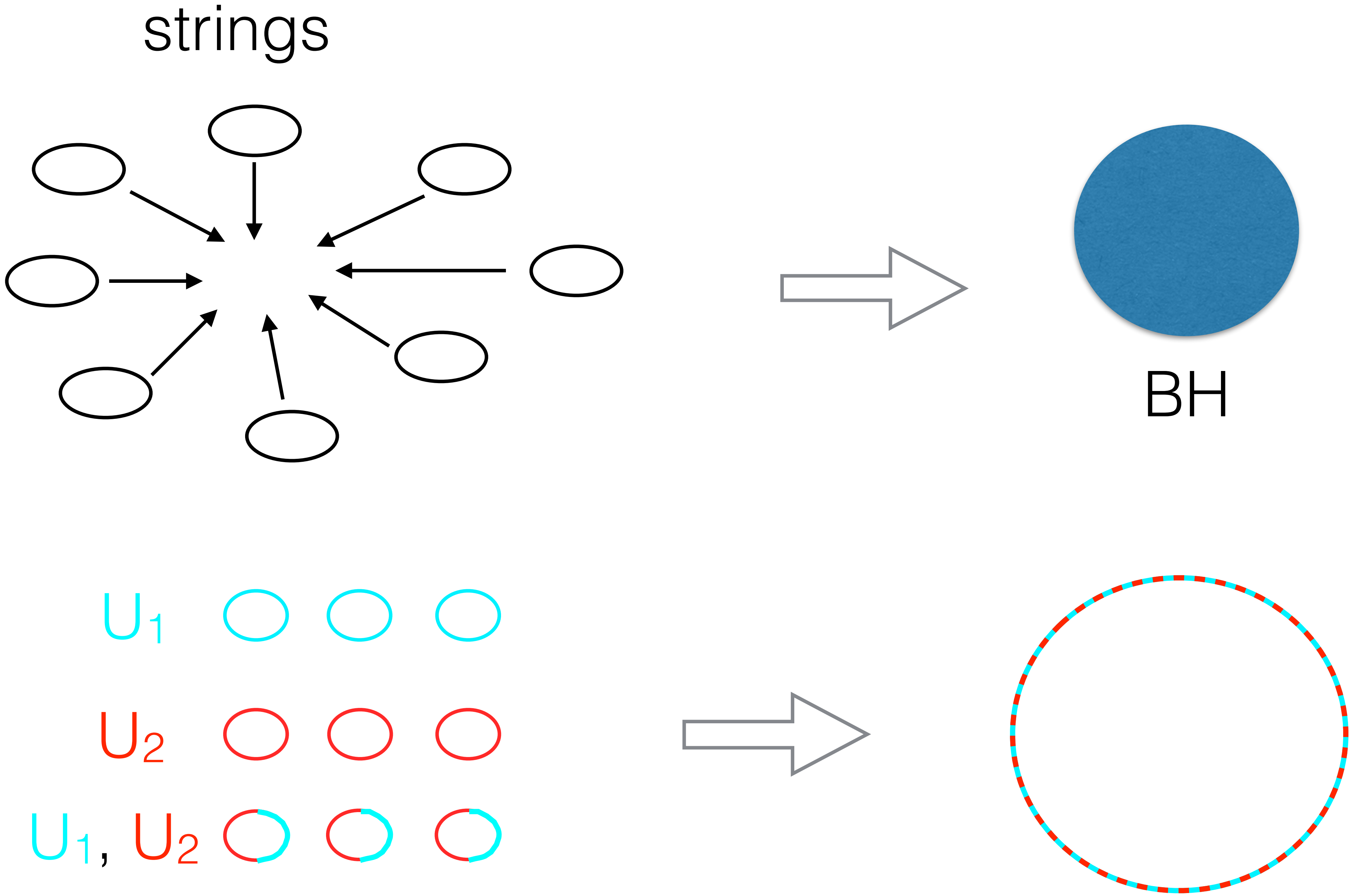


Real-time study of BH thermalization

Berkowitz-M.H.-Hayden-Maltz-Susskind, in progress



U_1
U_D
U_2
charge
BH
U_1, U_3
U_1, U_2

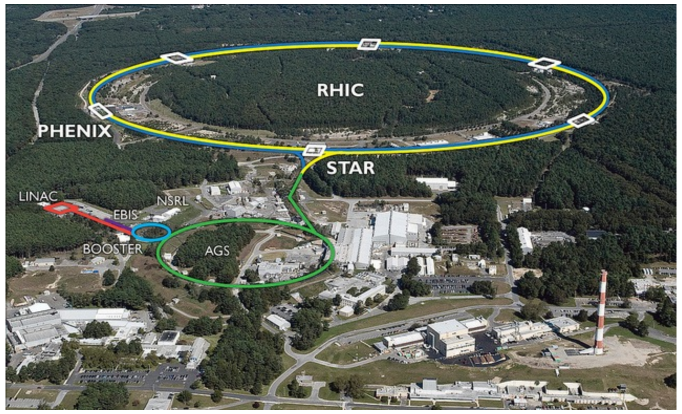


$$\boxed{\text{Does YM thermalize as fast as BH?}}$$

conclusion(1)

Maldacena's conjecture is correct
at finite temperature,
including $1/\lambda$ and $1/N$ corrections,
at least to the next-leading order.

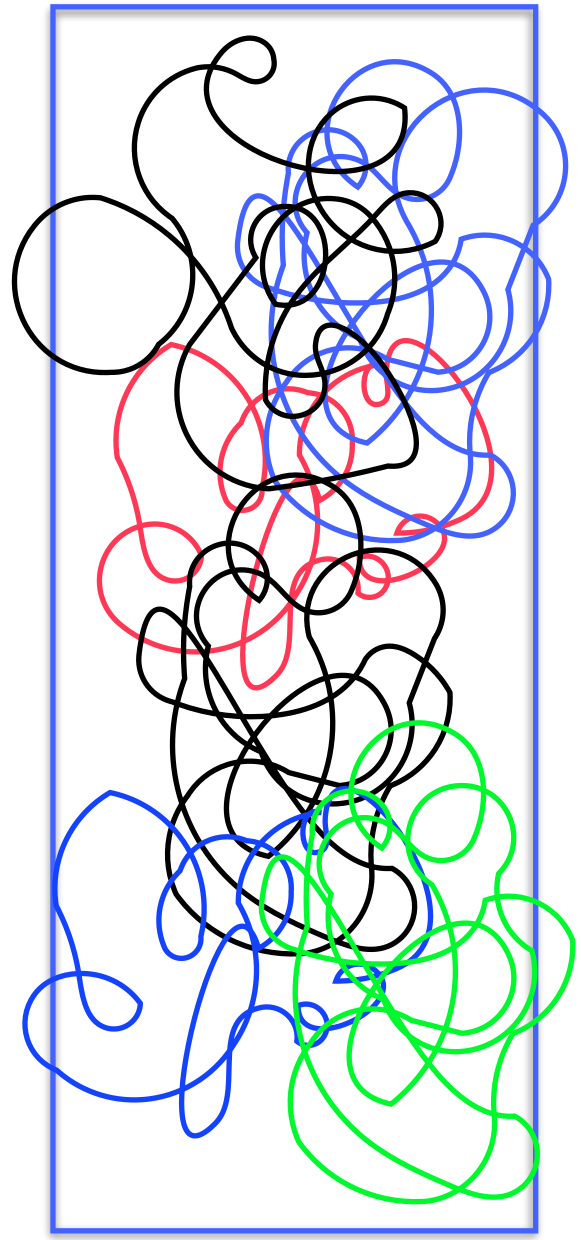
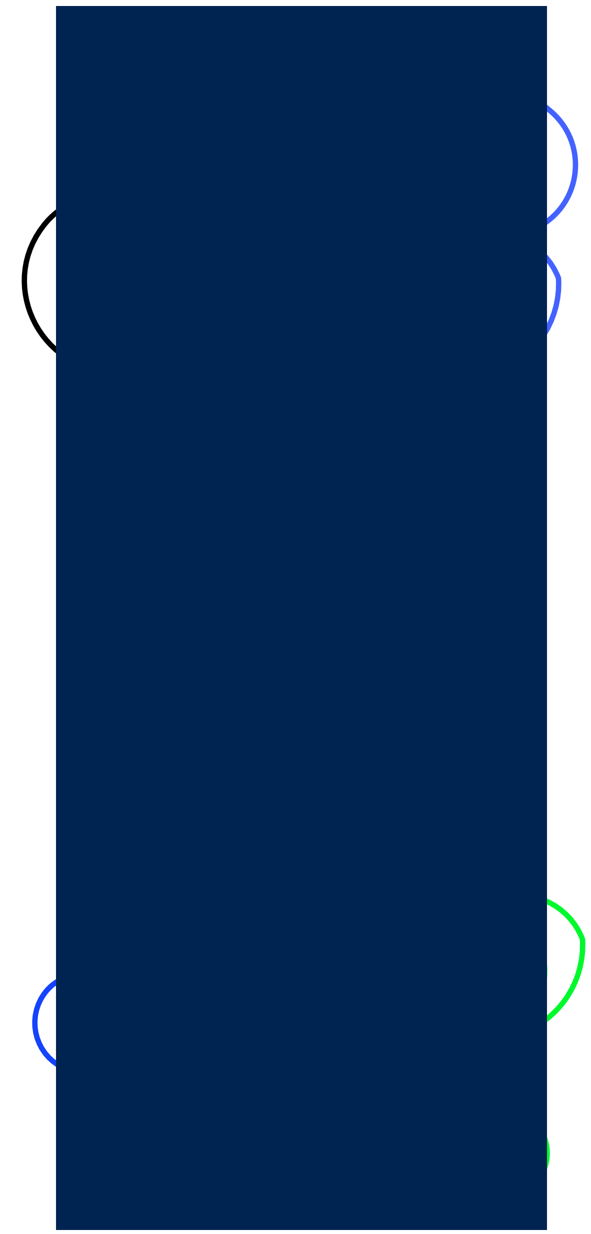
so, lattice/nuclear theorists can study
quantum gravity, by studying field theory.
You can do something string theorists cannot do.



RHIC is a machine for quantum gravity!

Occupy Princeton

$\boxed{\text{conclusion(2)}}$

deconfinement phase =  = 

Strong coupling limit contains the essence.

Stringy picture should be useful for learning about QGP.

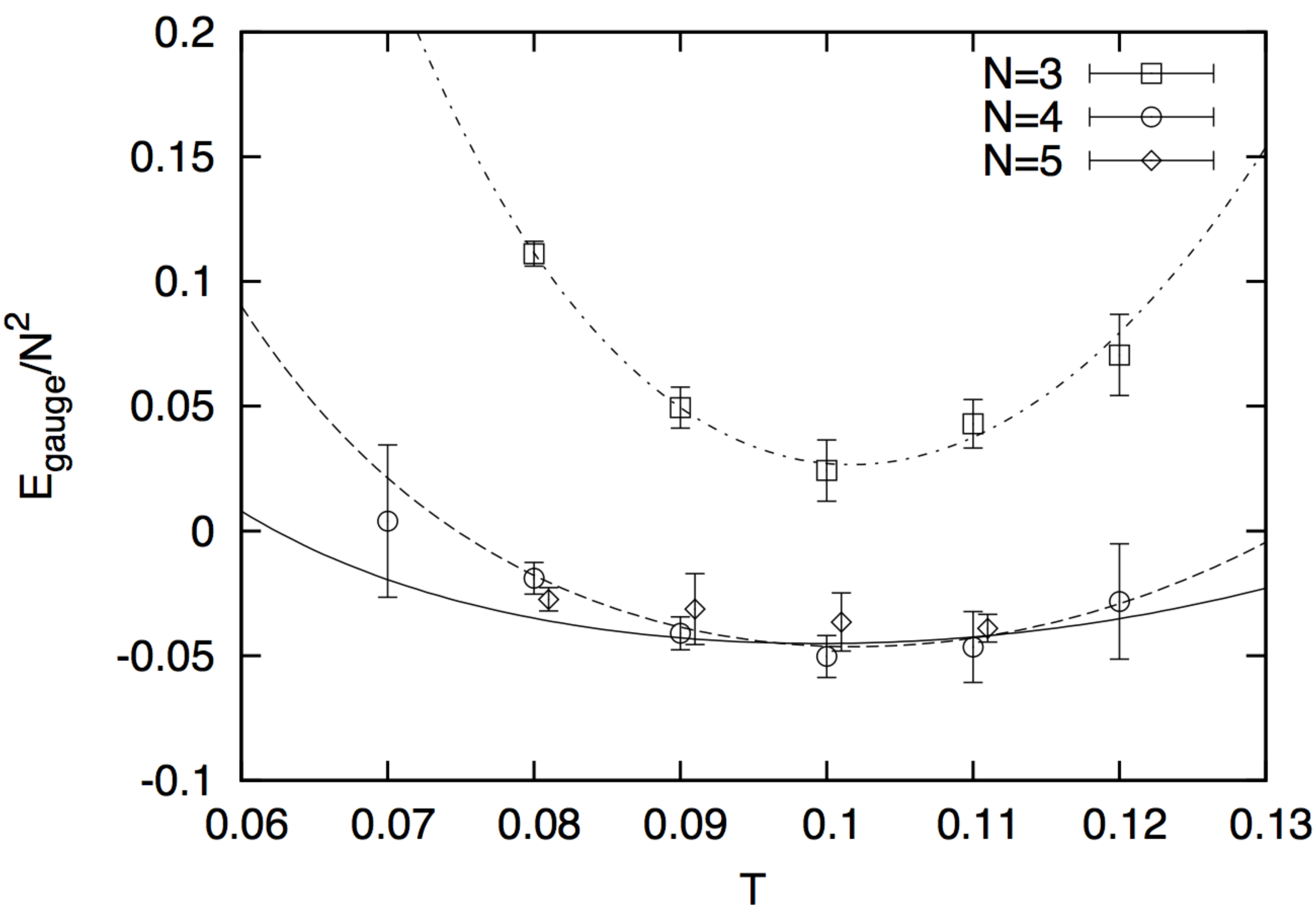
conclusion (for string theorists)

Maldacena's conjecture is correct
at finite temperature,
including $1/\lambda$ and $1/N$ corrections,
at least to the next-leading order.

Let's find good problems in SYM,
which nuclear/lattice theorists can solve,
and at the same time,
tells us about quantum gravity.

Your ideas will be appreciated!

backup



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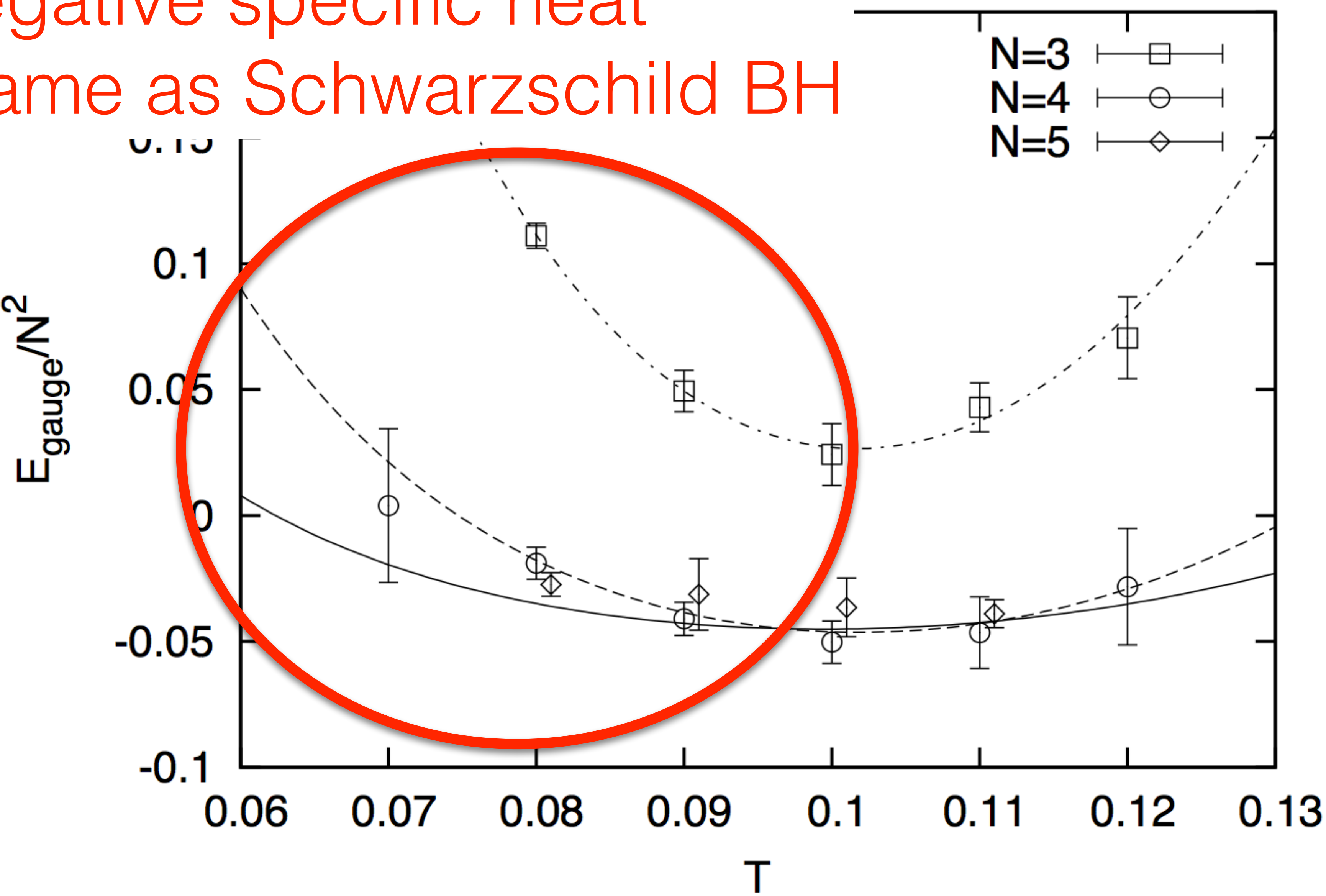
negative specific heat
→ the same as Schwarzschild BH



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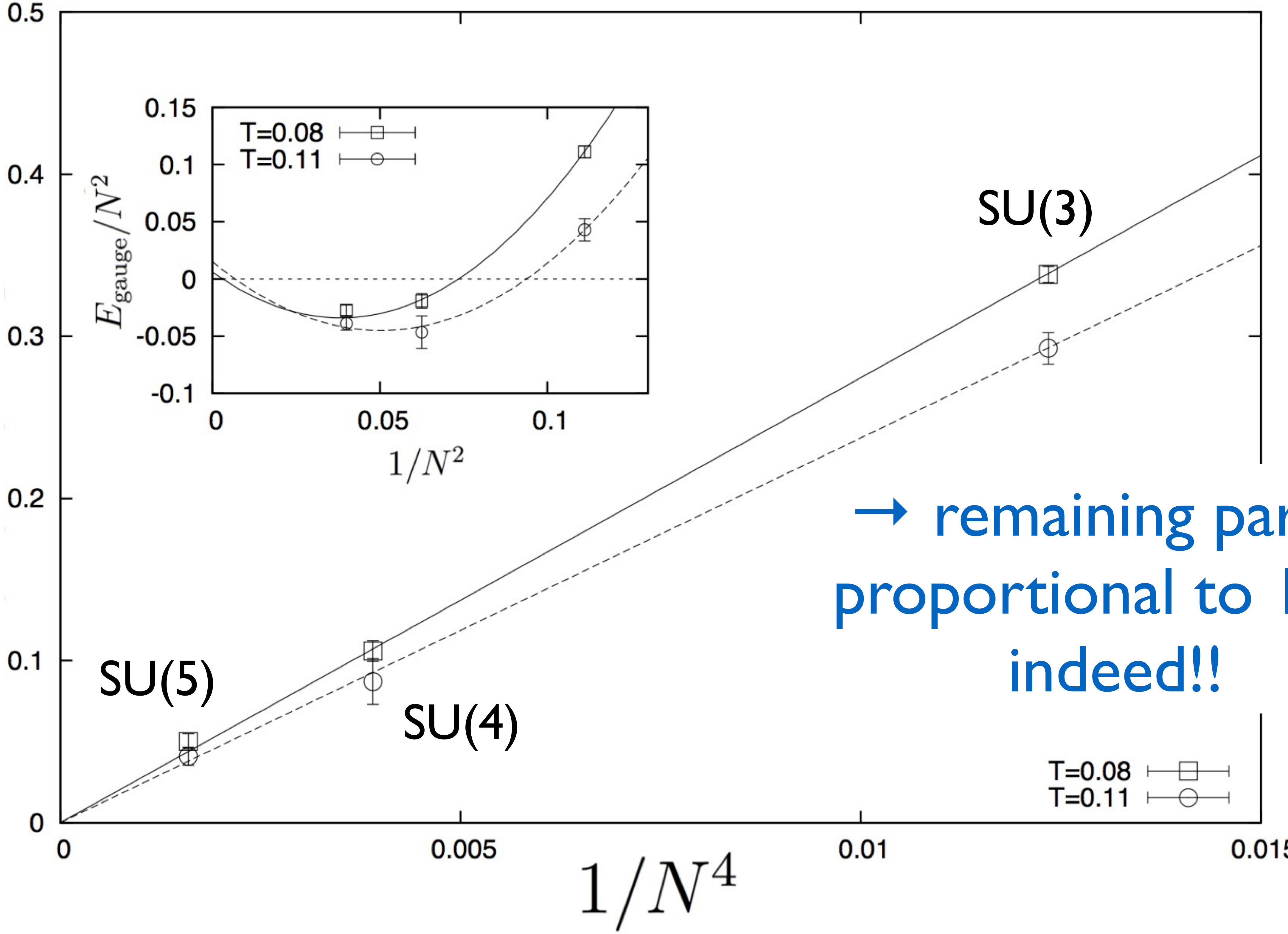
$E/N^2 - (7.41T^{2.8} - 5.77T^{0.4}/N^2)$ vs. $1/N^4$



→ remaining part is proportional to $1/N^4$ indeed!!

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