中間子のボース・アインシュタイン凝縮と冷却原子気体



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Building blocks of Matter





Heavy quarks

m_c ~ 1.3 GeV m_b ~ 4.2 GeV m_t ~ 171 GeV

Fundamental theory of strong int. = Quantum Chromo Dynamics (QCD) Characteristic strong int. scale ~ $(1 \text{fm})^{-1} \sim 200 \text{ MeV}$

Phase Diagram @ 1980



G. Baym (1980)

Phase Diagram @ 2011







K. Fukushima and T. Hatsuda, "The Phase Diagram of Dense QCD" Rep. Prog. Phys. 74 (2011) 014001

Neutron Star



- nuclei
- neutrons & protons
- mesons (π, K)
- hyperons (Λ , Σ^{-} , Ξ^{-})
- quarks (u,d,s)
- + leptons (e, μ)

Possible phases inside neutron stars

- Nuclei nuclear pasta
- Neutrons & Protons superfluidity, superconductivity
- mesons (π, K)
 Bose-Einstein condensate
- Hyperons
 superfluidity
- Quarks (u,d,s) color superconductivity



 Theoretically sound
 Quantitative predictions still difficult **Recent developments**

- 1. Ab initio calculations : QMD, Lattice QCD etc
- 2. New observations : M, R, T, B, P, ...
- 3. New experiments : RIBF, J-PARC etc
- 4. Proposals to relate theories and observations

examples	
$M = (1.97 \pm 0.04) M_{\odot}$	⇔ cold EOS
X-ray burst	⇔ cold EOS
GW from N $_{\bigstar}$ merger	⇔ hot EOS
Seismology	⇔ crust structure
Cooling of CAS-A $\Leftrightarrow {}^{3}P_{2}$ superfluid	
Magnetars ⇔ fe	rromagnetic core



Cassiopeia A Cooling, 4% decrease in 9 years (Heinke & Ho, ApJ 2010)







Magnetars (from Enoto, 2012) Bs=3.2x10¹⁹V(PPdot) [G]





Another massive Neutron Star !

PSR J0348+0432 : M= 2.01(4) M $_{\odot}$

RESEARCH ARTICLE

A Massive Pulsar in a Compact Relativistic Binary

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ABSTRACT STRUCTURED ABSTRACT EDITOR'S SUMMARY

Many physically motivated extensions to general relativity (GR) predict substantial deviations in the properties of spacetime surrounding massive neutron stars. We report the measurement of a 2.01 \pm 0.04 solar mass (M_{\odot}) pulsar in a 2.46-hour orbit with a 0.172 \pm 0.003 M_{\odot} white dwarf. The high pulsar mass and the compact orbit make this system a sensitive laboratory of a previously untested strong-field gravity regime. Thus far, the observed orbital decay agrees with GR, supporting its validity even for the extreme conditions present in the system. The resulting constraints on deviations support the use of GR-based templates for ground-based gravitational wave detectors. Additionally, the system strengthens recent constraints on the properties of dense matter and provides insight to binary stellar astrophysics and pulsar recycling.



Neutron Star



- nuclei
- neutrons & protons
- mesons (π, K)
- hyperons (Λ , Σ^{-} , Ξ^{-})
- quarks (u,d,s)
- + leptons (e, μ)

Energy per nucleon in pure neutron matter

Morales, (Pandharipande) & Ravenhall, in progress



AV-18 + UIV 3-body (IL 3-body too attractive) Improved FHNC algorithms. Two minima! E/A slightly higher than *Akmal, Pandharipande and Ravenhall, Phys. Rev. C58 (1998) 1804*

G.Baym, GCOE Lecture at Univ. Tokyo (2009)

Attractive configuration for neutrons (pion-exchange)

$$\begin{split} & V_{\text{OPEP}}(r) \\ &= \frac{f_{\pi N}^2}{4\pi} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) (\boldsymbol{\sigma}_1 \cdot \nabla_1) (\boldsymbol{\sigma}_2 \cdot \nabla_2) \frac{e^{-m_{\pi}r}}{r} \\ &= \frac{g_{\pi N}^2}{4\pi} \left(\frac{m_{\pi}}{2M_N}\right)^2 \frac{(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)}{3} \left[(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + S_{12} \left(1 + \frac{3}{m_{\pi}r} + \frac{3}{m_{\pi}^2 r^2}\right) \right] \frac{e^{-m_{\pi}r}}{r} \\ &\xrightarrow{\text{chiral limit}} \frac{g_A^2}{16\pi F_{\pi}^2} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \frac{S_{12}}{r^3} \end{split}$$

$$S_{12} = 3(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{r}}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2,$$



π^0 condensation in neutron matter



$$(-\nabla^2 + m_{\pi}^2) \varphi_c(\mathbf{r})$$
$$= (f/m_{\pi}) \nabla \cdot \langle \psi^{\dagger} \boldsymbol{\sigma} \psi \rangle$$

A. B. Migdal, NPA (1972) Takatsuka, Tamagaki & Tatsumi, Prog. Theor. Phys. Suppl. 112 ('93) 67

π^0 condensation = SDW



Fig. from Takatsuka, Tamagaki & Tatsumi, , Prog. Theor. Phys. Suppl. 112 ('83) 67

Attractive configuration for neutrons (rho-exchange)

 $V_{\rm OREP}(r)$

$$= \frac{f_{\rho N}}{4\pi} (\tau_1 \cdot \tau_2) (\sigma_1 \times \nabla_1) (\sigma_2 \times \nabla_2) \frac{e^{-m_\rho r}}{r}$$

$$= \frac{g_{\rho N}^2}{4\pi} \left(\frac{m_\rho}{2M_N}\right)^2 \frac{\tau_1 \cdot \tau_2}{3} \left[2(\sigma_1 \cdot \sigma_2) - S_{12} \left(1 + \frac{3}{m_\rho r} + \frac{3}{m_\rho^2 r^2}\right)\right] \frac{e^{-m_\rho r}}{r}$$

$$\to -\frac{g_{\rho N}^2}{16\pi M_N^2} (\tau_1 \cdot \tau_2) \frac{S_{12}}{r^3}$$

$$S_{12} = 3(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{r}}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2,$$





B condensations in dipolar atoms ?



$$(-\nabla^2 + m_{\pi}^2) \varphi_c(\mathbf{r})$$
$$= (f/m_{\pi}) \nabla \cdot \langle \psi^{\dagger} \sigma \psi \rangle$$



 $(-\nabla^2 + m_{\rho}^2) \rho_c(\mathbf{r})$

 $\Leftrightarrow A$ $\nabla x \rho$ B S μ

Neutron Star Structure by Tabletop Expt.?



- Hadron-quark crossover ⇔ Bose-Fermi mixture Maeda, Baym & Hatsuda, Phys. Rev. Lett. 103 (2009) 085301
- Meson condensation ⇔ Dipolar atoms Meada, Baym & Hatsuda, Phys. Rev. A 87 (2013) 021604(R)

Ultra-cold atomic Gasses



Figure from Pascal Naidon (RIKEN)

Bose-Einstein Condensate 1995

Fermi superfluid 2003





QUANTUM FLUIDS (SUPERFLUIDS)



Superfluid helium

Superconducting electrons

Superfluid nucleons

Superconducting quarks





10⁻¹ K 1 K

10 K

10⁹ K 10¹⁰K

Two-level toy model

$$\begin{split} H &= \frac{1}{2m} \int \mathrm{d}\vec{r} \, \nabla \Psi^{\dagger}(\vec{r}) \cdot \nabla \Psi(\vec{r}) + \frac{1}{8\pi} \int \mathrm{d}\vec{r} \, \vec{\mathcal{H}}(\vec{r})^2 \\ &- \mu \int \mathrm{d}\vec{r} \, \Psi^{\dagger}\vec{\sigma} \, \Psi \cdot \vec{\mathcal{H}}(\vec{r}) + g' \int \mathrm{d}\vec{r} \, \psi_1^{\dagger} \psi_2^{\dagger} \psi_2 \psi_1 \\ & \int \Psi \equiv (\psi_1, \psi_2)^T \; : \text{two-component fermions} \\ &\vec{\mathcal{H}}(\vec{r}) \; : \text{local magnetic field produced by the dipolars} \\ & \vec{\sigma} \; : \text{Pauli matrices} \qquad \vec{\mathcal{H}} = \nabla \times \vec{\mathcal{A}} \end{split}$$

 $\frac{Physical \ parameters}{m} : {\rm mass}\,, \quad \mu \ : {\rm magnetic \ moment}\,, \quad n \ : {\rm density} \ {\rm of} \ {\rm atoms}\,$

$$n(\vec{r}) = \langle \Psi^{\dagger}\Psi \rangle, \ \vec{M}(\vec{r}) = \langle \Psi^{\dagger}\vec{\sigma}\Psi \rangle,$$

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冷却原子・分子気体に相応しい現象論的ポテンシャル:

$$U = \frac{\mu^2}{r^3} \{ \vec{\sigma}_1 \cdot \vec{\sigma}_2 - 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) \} + g \,\delta(\vec{r}_1 - \vec{r}_2)$$

長距離: 双極子-双極子ポテンシャル 短距離: 接触型斥カポテンシャル

物理パラメータ(無次元量)

双極子–双極子相互作用の強さ: $\lambda_d = n \mu^2 / \epsilon_F$

接触型斥力の強さ: $\lambda_s = gn/\epsilon_F$

 λ_s - λ_d 平面上の相構造を明らかにする。

(2013) 021604(R)

<u>エネルギー密度--- FG状態</u>

以下、エネルギー密度の期待値を 二成分自由フェルミ気体のエネルギー密度で無次元化して表記。

1.フェルミ気体状態

$$\tilde{\mathcal{E}}_{\mathrm{FG}} = 1 + \frac{5}{12}\lambda_{\mathrm{s}}$$

双極子ポテンシャルの寄与はゼロ

エネルギー密度--- FN状態
$$p_{\mathrm{F},\uparrow} = (6\pi^2 n)^{1/3}$$
 $\tilde{\mathcal{E}}_{\mathrm{FN}} = \frac{2^{2/3}}{3} \left(2\gamma + \frac{1}{\gamma^2} \right) - \frac{5\pi}{9} \lambda_d I(\gamma)$ $\gamma^{-1} p_{\mathrm{F},\uparrow} z$ $\tilde{\mathcal{E}}_{\mathrm{FN}} = \frac{1}{3} \left(2\gamma + \frac{1}{\gamma^2} \right) - \frac{5\pi}{9} \lambda_d I(\gamma)$ χ $\Xi \bar{\kappa} \beta \nabla z \mu z z \lambda \mu z - \mu z$ $\chi \bar{\psi} z \bar{\psi} z$ $\Xi \bar{\kappa} \beta \nabla z \mu z z \lambda \mu z - \mu z \bar{\psi} \bar{\psi}$ $\chi \bar{\psi} z \bar{\psi} z$ $\Xi \bar{\kappa} \beta \nabla z \mu z z \lambda \mu z - \mu z \bar{\psi} z$ $\chi \bar{\psi} z \bar{\psi} z \bar{\psi} z$

変形関数

$$I(\gamma) = -2 - \frac{6}{\gamma^3 - 1} - \frac{6 \arccos \gamma^{3/2}}{(\gamma^{-1} - \gamma^2)^{3/2}}$$

B.M. Fregoso, E. Fradkin, PRL 103, 205301 (2009)

FG状態 v.s. FN状態

"Ferronematic Ground State of the Dilute Dipolar Fermi Gas" B.M. Fregoso, E. Fradkin, PRL **103**, 205301 (2009)

Maeda, Hatsuda, Baym, Phys. Rev. A 87 (2013) 021604(R)

Antiferrosmectic –C (AFSC)

O Variational wave function

$$\phi_{\ell,\vec{q}_{\perp}}(x,y,z) = \frac{e^{-(z-d\ell)^2/2b^2}}{(\pi b^2)^{1/4}} \frac{e^{i\vec{q}_{\perp}\cdot\vec{r}_{\perp}}}{\sqrt{V_{\perp}}} \chi_{\ell}$$
$$\chi_{\ell} = (1,(-1)^{\ell+1})/\sqrt{2}$$

We assume that the fermions

are well localized in the z-direction, with $d \gtrsim \sqrt{2}b$, and in the AFSC ground state fill these states with all ℓ , and q_{\perp} up to the transverse Fermi momentum, $q_{F\perp} = \sqrt{4\pi nd}$, where n is the average density.

<u>AFSC状態での物理量</u>

フェルミオンの数密度

$$n(\vec{r}) = \langle \Psi^{\dagger} \Psi \rangle = \frac{nd}{b\sqrt{\pi}} \sum_{\ell=-\infty}^{\infty} e^{-(z-d\ell)^2/b^2}$$
$$= n + 2n \sum_{j=1}^{\infty} e^{-j^2 \pi^2/\Gamma} \cos\left(2j\pi z/d\right)$$

局所磁化

$$\langle M_x(\vec{r}) \rangle = -\frac{\mu n d}{b\sqrt{\pi}} \sum_{\ell=-\infty}^{\infty} (-1)^{\ell} e^{-(z-d\ell)^2/b^2}$$

$$= -2\mu n \sum_{j=1}^{\infty} e^{-(2j-1)^2 \pi^2/4\Gamma} \cos\left\{(2j-1)\pi z/d\right\}$$

エネルギー密度
AFSC状態変分パラメータ(無次元量)
$$\Gamma = (d/b)^2$$
の $\tilde{\mathcal{E}}_{AFSC}$ $\Gamma = (d/b)^2$ $\alpha = 1/(2q_F^2b^2)$ $\tilde{\mathcal{E}}_{AFSC}$ 運動エネルギー
ス(3\pi)^{2/3}ブパ3 $\alpha^{-1/3}$ 運動エネルギー
ス方向の零点振動
メッ方向の2次元F.E. $-\frac{20\pi}{3}\lambda_d \sum_{j=1}^{\infty} e^{-(2j-1)^2\pi^2/2\Gamma} \left\{ \frac{1}{3} - F(\alpha) \right\}$ 運動エネルギー
ス極子ポテンシャルHartreeFock第合
第合 $+\frac{5}{6}\lambda_s \left\{ \frac{1}{2} - \sum_{j=1}^{\infty} \left[e^{-(2j-1)^2\pi^2/2\Gamma} - e^{-2j^2\pi^2/\Gamma} \right] \right\}$ 短距離斥力

 $Q(\Gamma)$ 僅かな寄与(パウリの排他律による)

$$F(\alpha) = \alpha \int_0^\infty ds J_1 \left(\sqrt{2s/\alpha}\right)^2 e^s \left\{ (2+s^{-1})K_0(s) - 2K_1(s) \right\}$$

$$\frac{ \overline{g} \beta \sqrt[n]{5} \overline{J} - \overline{g} (\frac{\pi}{2} \sqrt{2} \overline{g})}{\Gamma = (d/b)^2 \qquad \alpha = 1/(2q_F^2 b^2)}$$

λs=2.5

変分法による相図

○ 結合チャンネル<u>RPA解析(L=0とL=2の</u>結合)

$$\left(1-\frac{3}{4}\lambda_s-2\pi\lambda_d\right)\left(1+\frac{\pi}{2}\right)-\frac{\pi^2}{2}\lambda_d^2 = 0$$

T. Sogo, M. Urban, P. Shuck, T. Miyakawa , PRA **85**, 031601(R) (2012)

<u>RPA解析も考慮した相図</u>

<u>実験との関連</u>

Dy原子、5µB、散乱長10nm

<u>トイモデルの問題点とその克服可能性</u>

1. ¹⁶¹Dyのスピンは 21/2

2-hyperfine stateをとる。厳密にはPauli-spinで書けない。

→ mult-level model への拡張必要。

2. ¹⁶¹Dyでも、AFSCにはMagnetic moment がまだ小さい。

$$\lambda_d = n \mu^2 / \epsilon_F = 0.3 \times 10^{-2} m_{100} n_{12}^{1/3} \mu_{10}^2$$
ターゲット

→ Electric dipolar molecules の可能性: $\mu_{10}^2 \sim 1(^{161}\text{Dy}), 310(\text{CH}_3\text{F})$

Simulating quantum magnets with symmetric top molecules

Michael L Wall*, Kenji Maeda**, and Lincoln D Carr Department of Physics, Colorado School of Mines, Golden, Colorado 80401, USA arXiv:1305.1236v1 [cond-mat.quant-gas] 6 May 2013

Summary

Neutron star

<u>Meson</u> condensation By tensor force <u>Photon</u> condensation by dipolar interaction

- ➤ Almost the same hamiltonian in both systems ρ⁰ cond. ⇔ magnetic dipolars (smectic C phase) ρ^c cond. ⇔ magnetic dipolars (chiral nematic phase) ?

