Neutrino quantum kinetics in core-collapse supernova

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自己紹介

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2002 – 2006:　早稲田大学理工学部（物理学科）

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2018 – 2021:　Princeton University

2021 – 現在:　国立天文台

研究テーマ:　高エネルギー天体現象の理論的研究

手法:　数値シミュレーション

Recent review for the theory of core-collapse supernova

解　説

重力崩壊型超新星の物理――研究の現状と今後の課題


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重力崩壊型超新星爆発は，宇宙で起こる**大質量星**の爆発現象である．爆発を駆動している星の中心付近では，高密度（核密度）かつ高温（10 MeV以上）環境が実現され，強い力・弱い力・電磁気力・重力という自然界で働く4つの基本的な力全てが爆発機構に関わっており，理論物理学の観点からも興味深い．爆発によって重元素の生成と宇宙空間への放出が起こるため，宇宙の化学組成を決める重要な天体現象である．また，爆発後には**中性子星**やブラックホールなどの高密度天体を残すことから，宇宙で起こる様々な他の高エネルギー天体現象とも密接に関連する．このように，重力崩壊型超新星爆発の研究は非常に学際的な分野であり，素・核・宇宙・天文学などの幅広い分野の研究者らによって，実験・観測・理論・シミュレーションなどの様々なアプローチにより研究が行われている．

超新星爆発を駆動している中心エンジンは，複数の物理過程が非線形に絡まった系である．その爆発機構は複雑で，理論宇宙物理学の難題の一つとして位置づけられてきた．しかし，ここ10年ほどの間に，超新星爆発の理論は著しく進展した．特に，理論計算（数値シミュレーション）においては，それまで爆発の再現に失敗していたのに対し，近年ではこれに成功するモデルが多く報告されている．こうした進展の一つの理由は，計算機能力の向上と数値計算手法の発展のおかげで，より正確に詳細な物理過程を取り込んだ多次元**ニュートリノ輻射流体計算**が実行可能になったことである．例えば，第一原理計算に最も近いとされる，ボルツマン方程式を直接解く多次元輻射流体計算が「富岳」などのスパコンで少数のモデルに対して実行されている一方，近似的なニュートリノ輸送法を用いた多次元計算がより多くのモデルに対して系統的に行われている．また，ニュートリノと物質との弱い相互作用の扱いについても精密化が進み，例えば核子のweak currentにおける形状因子やストレンジネスの寄与，さらには多体効果なども，シミュレーションでは既に取り込まれている．

シミュレーションが，長時間かつ様々なタイプの大質量星に対して系統的に行えるようになり，観測量の定量的な推定が行えるようになってきたことも，近年の重要な進歩である．実際，過去の超新星理論モデルとは違い，最終的な爆発エネルギーの値や形成される中性子星の質量や半径などが定量的に議論できるようになってきた．電磁波・重力波・ニュートリノに関する理論モデルの精度も格段に上がり，**マルチメッセンジャー天文学**の発展にも貢献している．

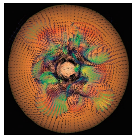
このように超新星爆発の研究は，近年著しく発展したが，それでも超新星爆発機構が完全に解明されたわけではない．実際，現在考慮されているニュートリノ反応の取り扱いには不定性が大きく，それが爆発可否に影響する可能性がある．また，**ニュートリノ集団振動**に代表される量子運動論的な効果は，現在の最も進んだ超新星爆発計算にも取り込まれておらず，現在の超新星爆発の理論を一変させてしまうかもしれない．ニュートリノ反応計算の精度を上げ，量子運動論的ニュートリノ輻射輸送計算に基づいた超新星モデルの再構築が，今後10年の超新星爆発の理論的研究の主要なターゲットになるだろう．

――用語解説――

大質量星：
恒星は核融合反応によって光り輝いている星．大質量星は太陽のおよそ10倍以上の質量を持つ恒星を指す．

中性子星：
主に中性子から構成されている半径10 kmほどの星．質量は太陽よりも重く，現在のところ，その2倍程度の質量を持つものの存在が明らかになっている．

ニュートリノ輻射流体シミュレーション：
ニュートリノと物質の相互作用を考慮し，物質の流体力学的運動と，ニュートリノ輸送を同時に解く数値計算．以下に，空間3次元超新星爆発シミュレーション結果の例を載せる（岩上わかな・長倉洋樹によって作成）．



マルチメッセンジャー天文学：
ある天体から発せられる様々なシグナル（電磁波，ニュートリノ・重力波・宇宙線など）を同時期に観測し，これら複数の観測量を協調させて，天体現象の起源を探ること．

ニュートリノ集団振動：
ニュートリノの自己相互作用によって駆動されるニュートリノ振動（フレーバー混合）現象．

物理学会誌（第1号 2023年）

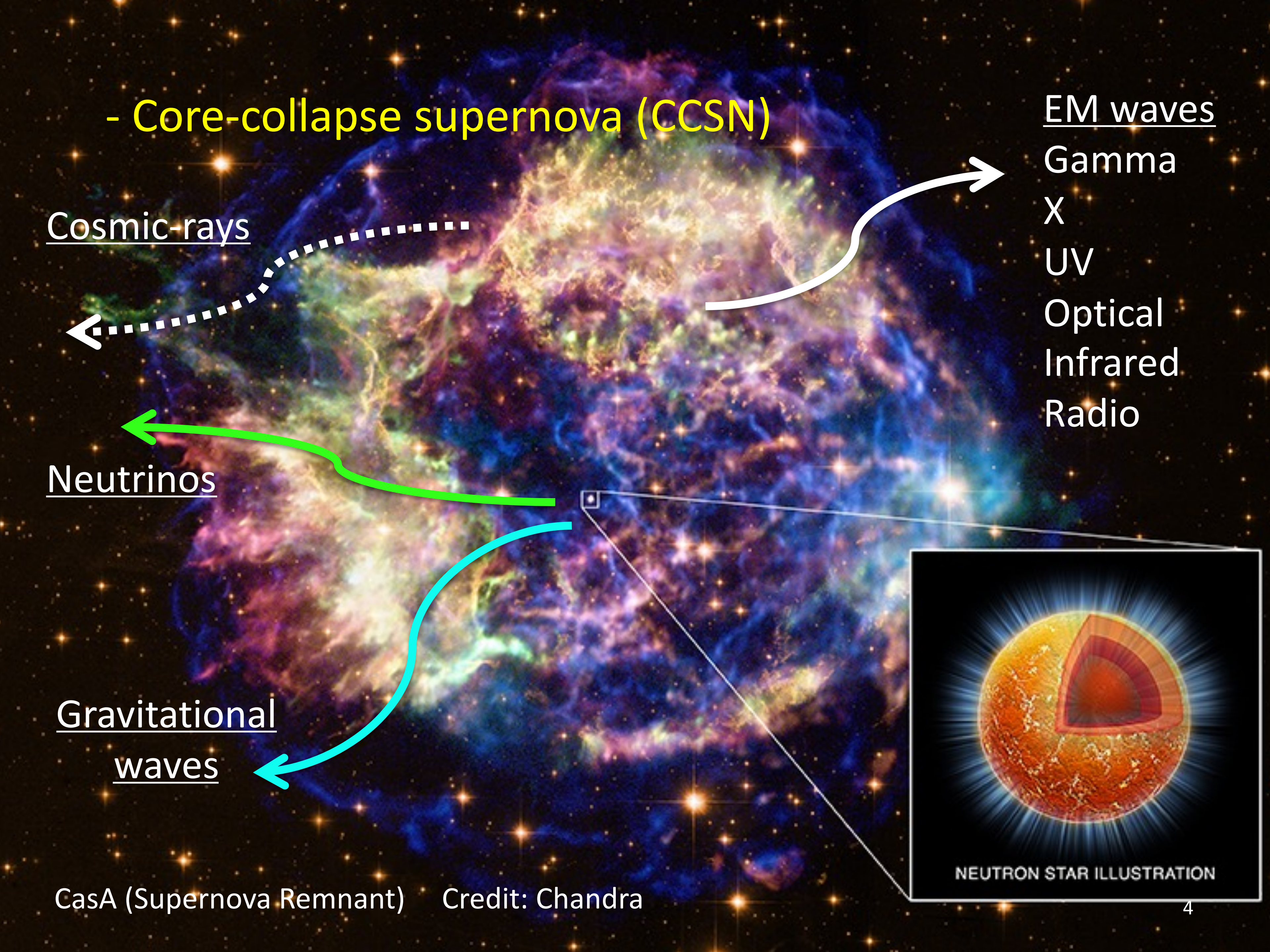
Review

The Physical Mechanism of Core-Collapse Supernovae that Neutrinos Drive

By Shoichi YAMADA,[*1,*2,†] Hiroki NAGAKURA,[*3] Ryuichiro AKAHO,[*1] Akira HARADA,[*4] Shun FURUSAWA,[*5] Wakana IWAKAMI,[*1] Hirotada OKAWA,[*6] Hideo MATSUFURU[*7] and Kohsuke SUMIYOSHI[*8]

Abstract: The current understanding of the mechanism of core-collapse supernovae (CCSNe), the death of massive stars and the main formation channel of compact objects such as neutron stars and black holes, is reviewed for broad readers who may not be familiar with the subject. We hence put an emphasis more on the physical aspects than on the summary of simulations although there is no doubt that the large-scale high-fidelity simulations have played the most important roles in the progresses we have witnessed in the last few decades. There is no doubt, either, that neutrinos are the single most important agent in producing one of the most energetic events in the universe. The so-called neutrino-heating mechanism will be the focus in this review and its crucial ingredients in micro- and macro-physics as well as in numerics will be explained one by one. We will also try to elucidate what are the remaining issues.

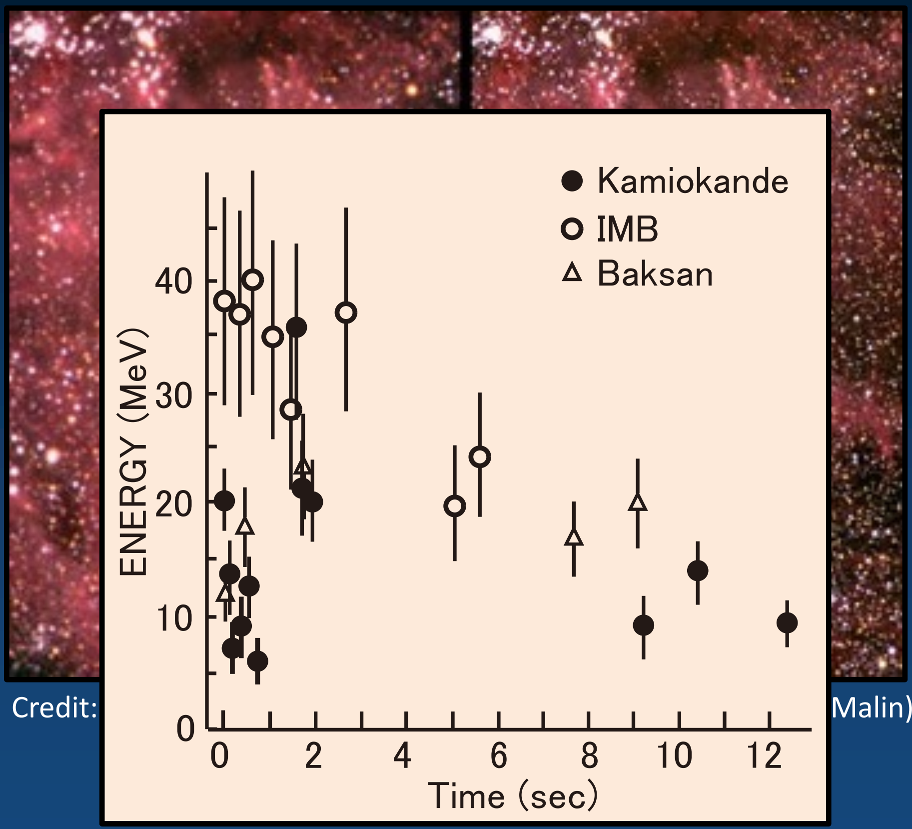
Proceedings of the Japan Academy, Ser. B, Physical and Biological Sciences


- Core-collapse supernova (CCSN)
EM waves
Gamma
X
UV
Optical
Infrared
Radio
Cosmic-rays
Neutrinos
Gravitational waves
NEUTRON STAR ILLUSTRATION
CasA (Supernova Remnant)
Credit: Chandra

- Observational Phenomenology

- Progenitor mass: > 10 Msun
- 1 event /galaxy/ 100 yrs
- Explosion energy: 10^{51} erg
- Nickel mass: 0.1Msun
- Neutron star remnant
- Neutrino emission: 10^{53} erg
- Long GRB – CCSN association

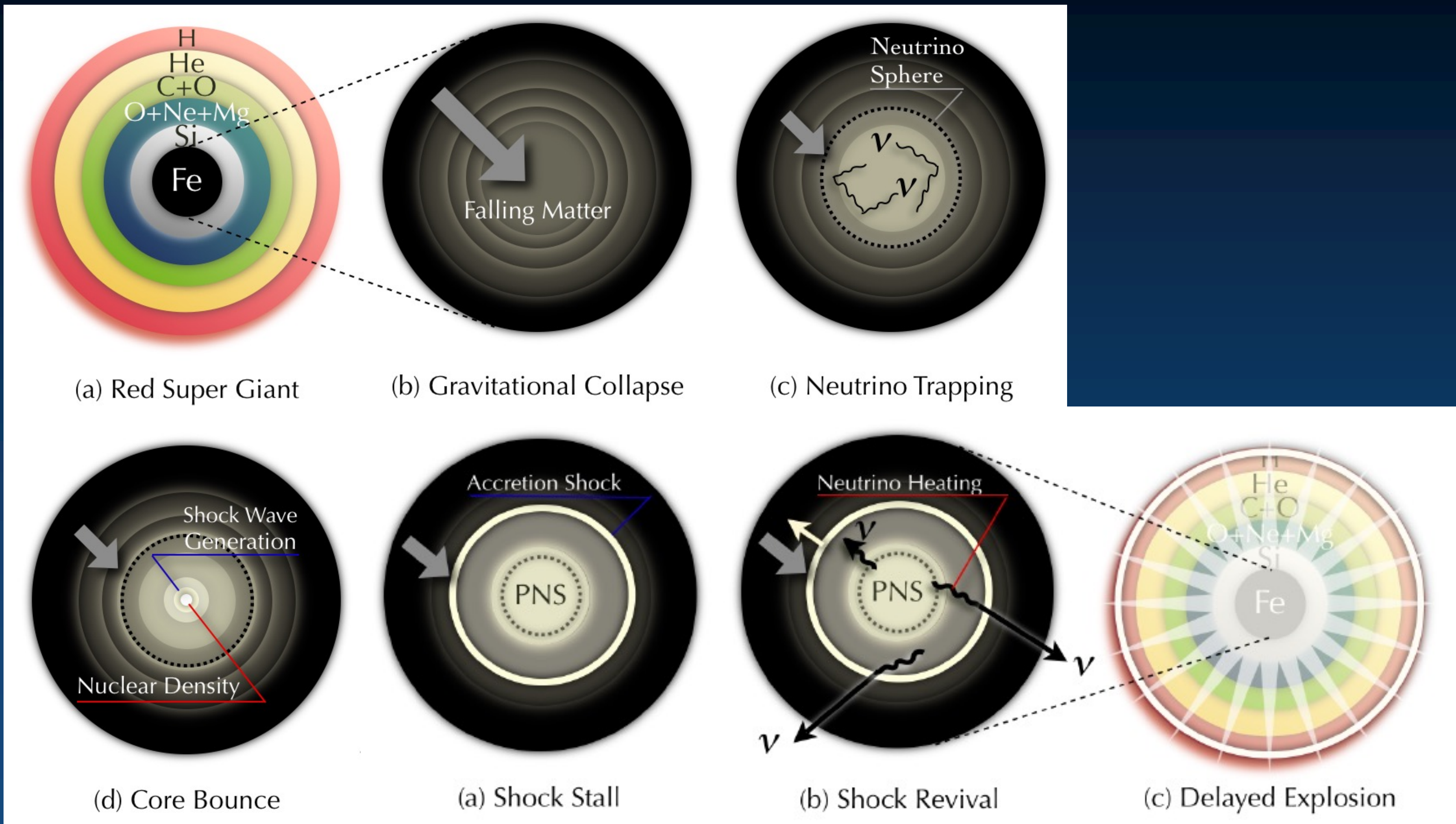
1987A: Optical image



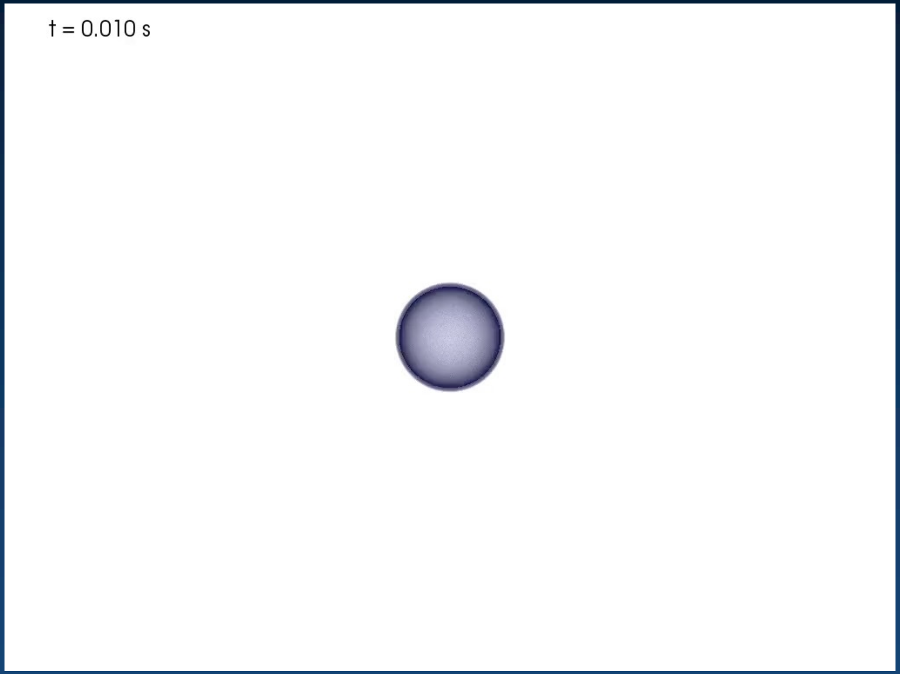
Credit: Malin)

Adapted from J-Features (M. Nakahata)

Standard Model of Core-Collapse Supernovae

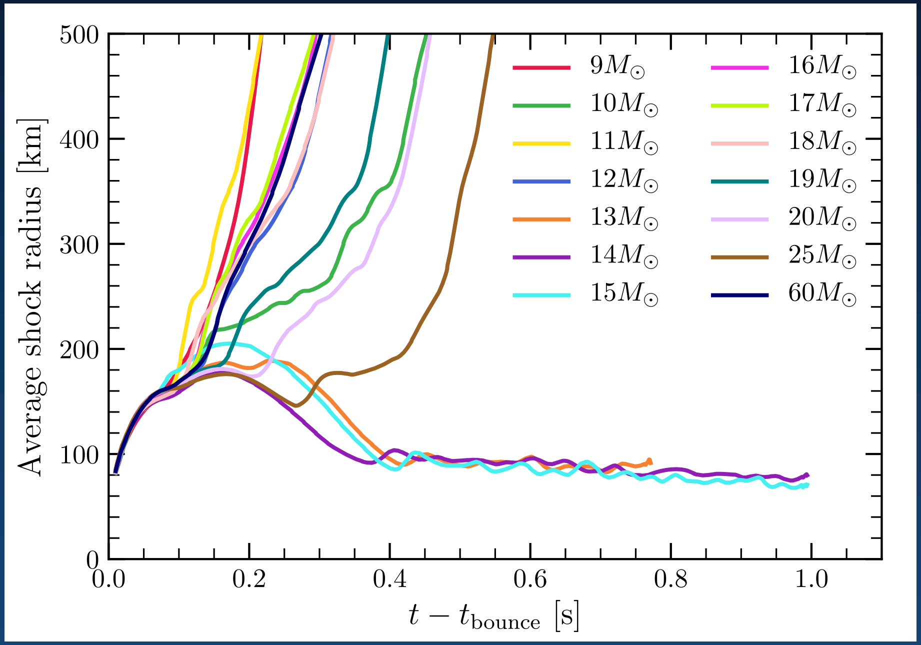


Explosions occur across a wide progenitor mass



Nagakura et al. 2019

Shock evolution for different progenitors



Burrows et al. 2020

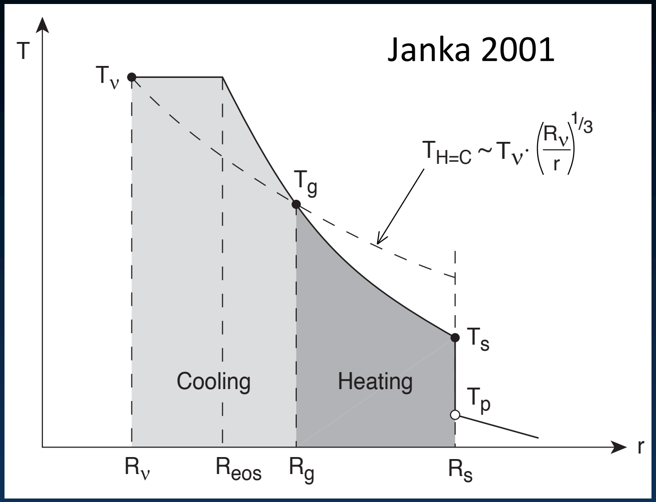
Neutrino-heating mechanism

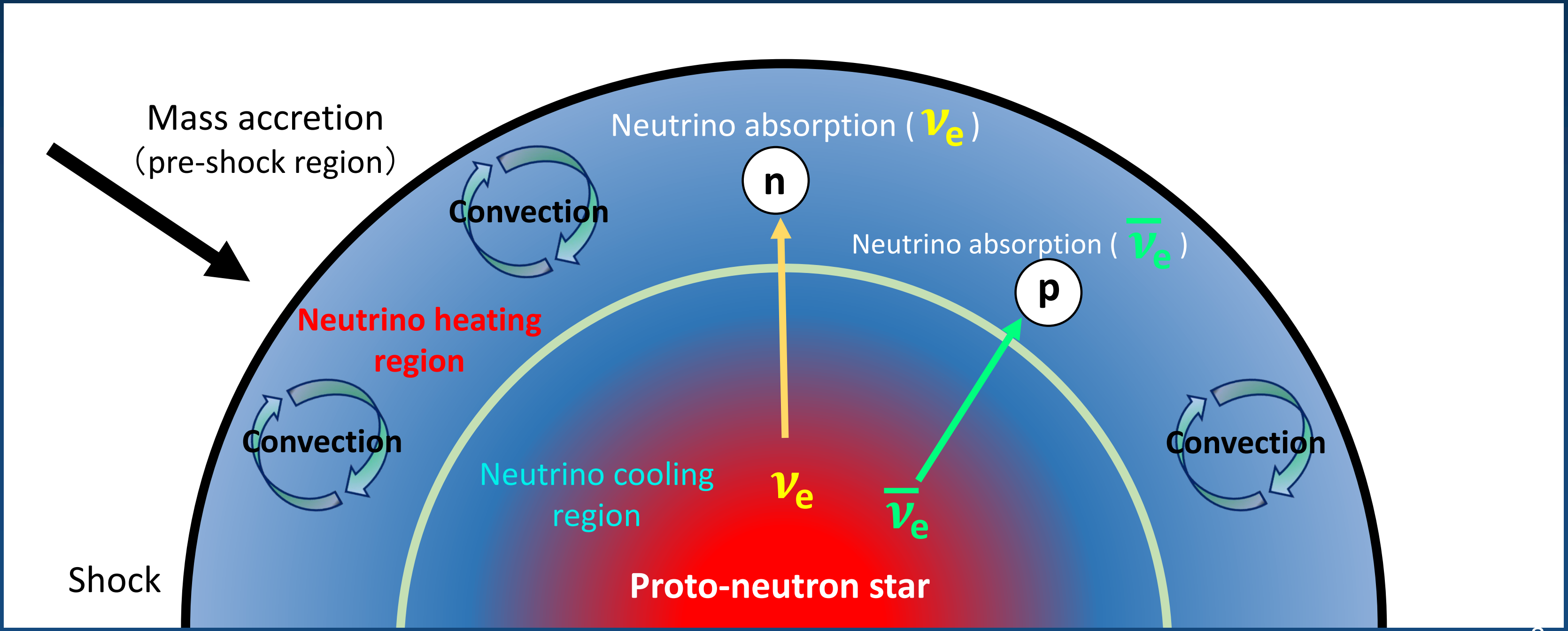
✓ Neutrino Heating Rate

$$Q_\nu^+ \approx 160\mathrm{Mev/s}\frac{\rho}{m_a}\frac{L_{\nu_e,52}}{r_7^2\langle\mu_\nu\rangle}\left(\frac{T_{\nu_e}}{4\mathrm{MeV}}\right)^2$$

✓ Neutrino Cooling Rate

$$Q_\nu^- \approx 145\mathrm{Mev/s}\frac{\rho}{m_a}\left(\frac{T}{2\mathrm{MeV}}\right)^6$$



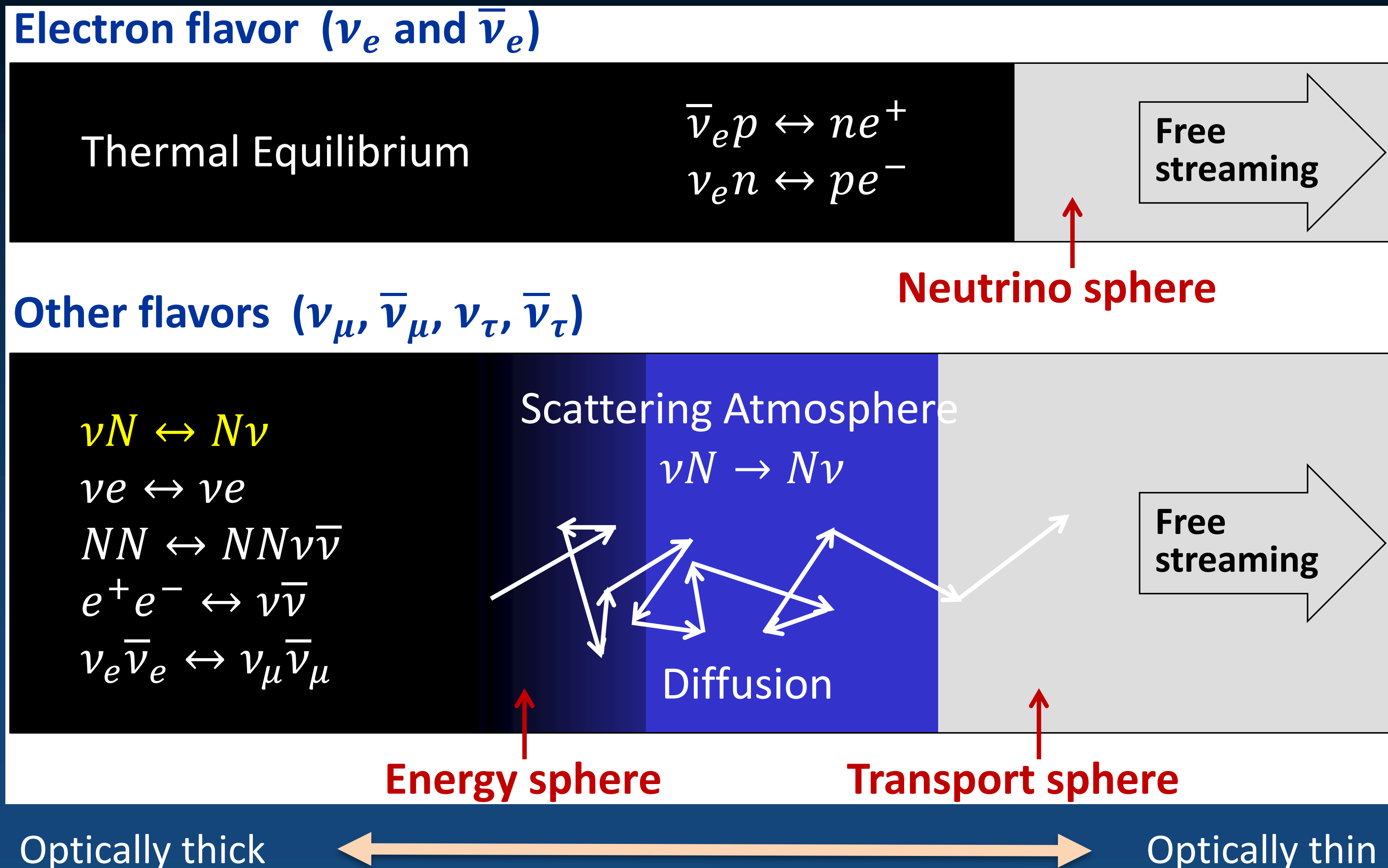


Basic Equations

- $(\rho_0 u^\mu)_{;\mu} = 0$: Continuity Equation
- $T^{\mu\nu}_{(\mathrm{hd});\nu} + (T^{\mu\nu}_{(\mathrm{em});\nu}) = G^\mu$: Energy Momentum Conservation
- $(n_e u^\mu)_{;\mu} = \Gamma$: Lepton number Conservation
- $(F^{\mu\nu}{}_{;\nu} = 4\pi J^\mu)$: Maxwell Equation
- $G_{\mu\nu} = 8\pi T_{\mu\nu}$: Einstein Equation
- $\boxed{p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = \left(\frac{\delta f}{\delta \tau}\right)_{\mathrm{col}}}$: Boltzmann Equation (Neutrino Transfer)

Modeling of neutrino radiation field: kinetic treatment is necessary

Figure by Janka 2017

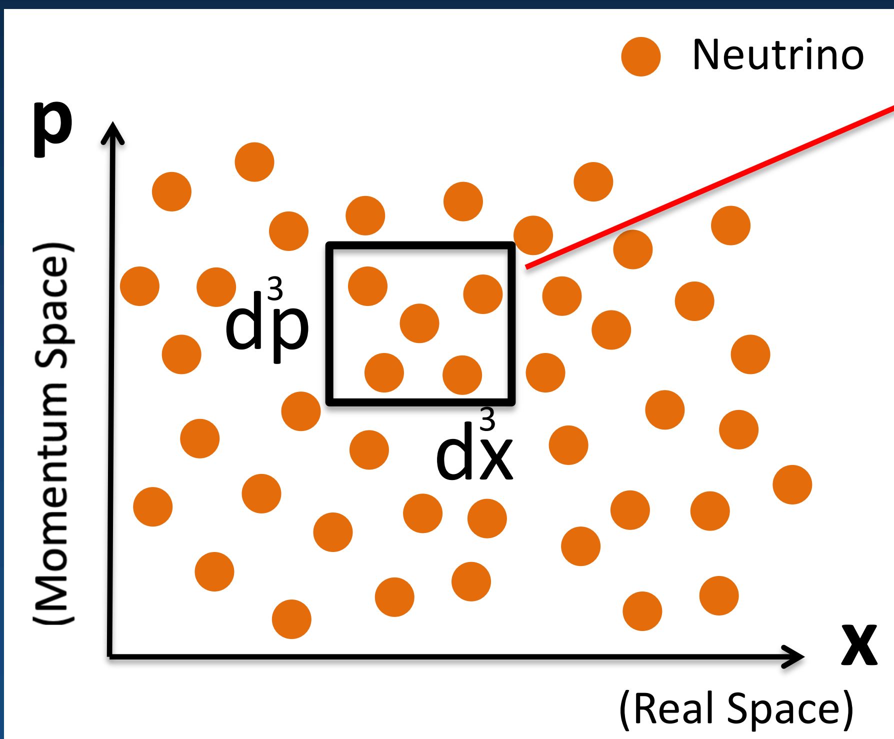


Boltzmann neutrino transport

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = \left(\frac{\delta f}{\delta \tau}\right)_{\mathrm{col}},$$

(Time evolution + Advection Term) (Collision Term)

6 dimensional phase space



$$dN = f(t, \boldsymbol{p}, \boldsymbol{x}) d^3p d^3x$$

Conservative form of GR Boltzmann eq.

$$\begin{aligned}
&\frac{1}{\sqrt{-g}} \frac{\partial(\sqrt{-g}\,\nu^{-1} p^\alpha f)}{\partial x^\alpha}\bigg|_{q_{(i)}} + \frac{1}{\nu^2}\frac{\partial}{\partial \nu}\left(-\nu f p^\alpha p_\beta \nabla_\alpha e^\beta_{(0)}\right) \\
&+ \frac{1}{\sin\bar\theta}\frac{\partial}{\partial\bar\theta}\left(\nu^{-2}\sin\bar\theta f \sum_{j=1}^{3} p^\alpha p_\beta \nabla_\alpha e^\beta_{(j)} \frac{\partial \ell_{(j)}}{\partial \bar\theta}\right) \\
&+ \frac{1}{\sin^2\bar\theta}\frac{\partial}{\partial\bar\varphi}\left(\nu^{-2} f \sum_{j=2}^{3} p^\alpha p_\beta \nabla_\alpha e^\beta_{(j)} \frac{\partial \ell_{(j)}}{\partial \bar\varphi}\right) = S_{\mathrm{rad}},
\end{aligned}$$

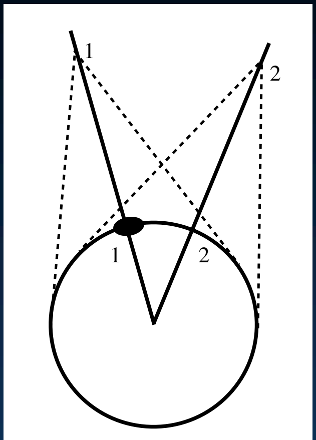
Shibata, Nagakura, Sekiguchi, and Yamada (2014)

Various Approximations for Multi-D Neutrino Transfer

See Mezzacappa et al. 2020

- **Ray-by-Ray Approach** (UTK-Oak Ridge, MPA)

  Neutrino-transport is essentially same as spherical symmetry.



Schematic picture of ray-by-ray approach (Lentz et al. 2012)

- **Isotropic Diffusion Source Approximation (IDSA)** (Basel, Japan)

  Neutrinos are decomposed into trapped and streaming parts.

  Two reduced equations are coupled by each source term, which is approximately described under diffusion treatment.
  (See e.g., Berninger et al. 2013)

- **Moment method** (UTK-Oak Ridge, Princeton, Michigan)

$$M_{(\nu)}^{\alpha_1 \alpha_2 \cdots \alpha_k}(x^\beta) = \int \frac{f(p'^\alpha, x^\beta)\delta(\nu - \nu')}{\nu'^{k-2}} p'^{\alpha_1} p'^{\alpha_2} \cdots p'^{\alpha_k} dV'_p,$$

Shibata et al. 2011

  Neutrino angular direction is integrated. The so-called "closure relation" is imposed in the higher moment.

- **Multi-Group Flux-Limited-Diffusion (MGFLD)** (UTK-Oak Ridge)

  Neutrino Transports are treated as the Energy-Dependent Diffusion Equation.

Numerical methods for Boltzmann solver

Boltzmann equation in flat space time

$$\frac{\partial f}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(f\cos\bar{\theta} r^2) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(f\sin\theta\sin\bar{\theta}\cos\bar{\varphi})$$
$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(f\sin\bar{\theta}\sin\bar{\varphi}) - \frac{1}{r}\frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}(f\sin^2\bar{\theta})$$
$$- \frac{\partial}{\partial\bar{\varphi}}\left(f\frac{\cot\theta}{r}\sin\bar{\theta}\sin\bar{\varphi}\right) = S_{\mathrm{rad}}.$$

Distribution function (7D)

$$f = f(t, r, \theta, \phi, \nu, \bar{\theta}, \bar{\phi})$$

1. t: time
2. r: radius
3. θ: zenith angle (real)
4. ϕ: azimuthal angle (real)
5. ν: energy
6. θ: zenith angle (momentum)
7. ϕ: azimuthal angle (momentum)

1. Discrete-ordinate Sn method

- Finite difference discretization of 7 dimensional Boltzmann equation.

2. Boltzmann equation is 7D, integral, partial-differential equation

$$S^{(\mathrm{scat})}_{\mathrm{rad}}(\nu,\Omega) = -\frac{(\nu)^2}{(2\pi)^3}\int d\Omega^{'} R_{scat}(\Omega,\Omega^{'}) \times \left(f(\nu,\Omega) - f(\nu,\Omega^{'})\right)$$

(Isoenergy-Scattering)

3. Indispensable to use implicit (or semi-Implicit) time evolution

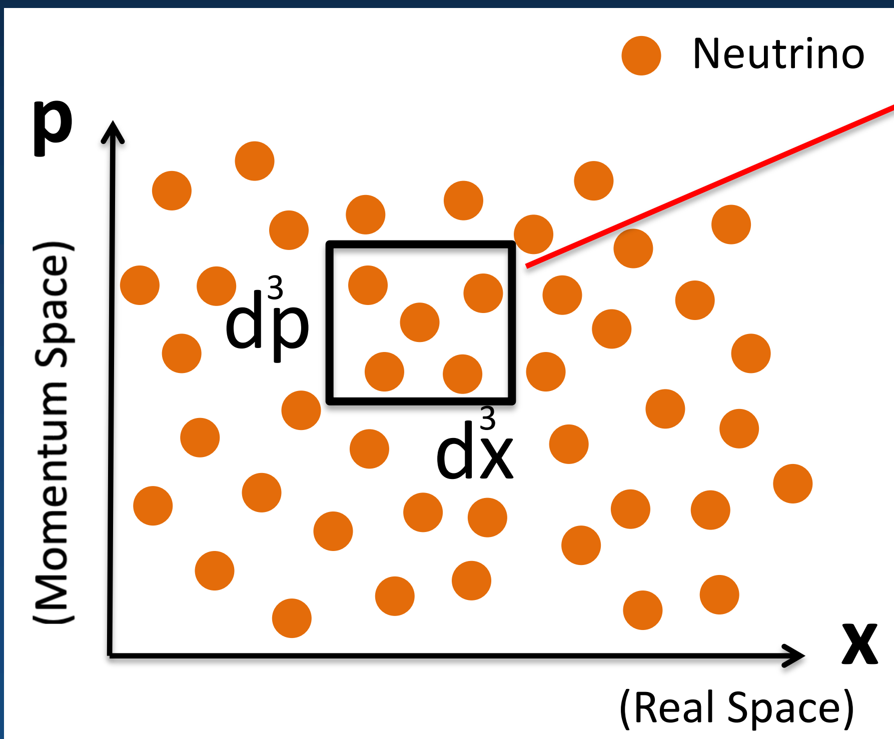
- Boltzmann equation possesses characteristics of “stiff equation”.

Boltzmann neutrino transport

$$p^{\mu}\frac{\partial f}{\partial x^{\mu}}+\frac{dp^{i}}{d\tau}\frac{\partial f}{\partial p^{i}}=\boxed{\left(\frac{\delta f}{\delta \tau}\right)_{\mathrm{col}}},$$

(Time evolution + Advection Term) (Collision Term)

6 dimensional phase space



$$dN = f(t,\boldsymbol{p},\boldsymbol{x})d^3p d^3x$$

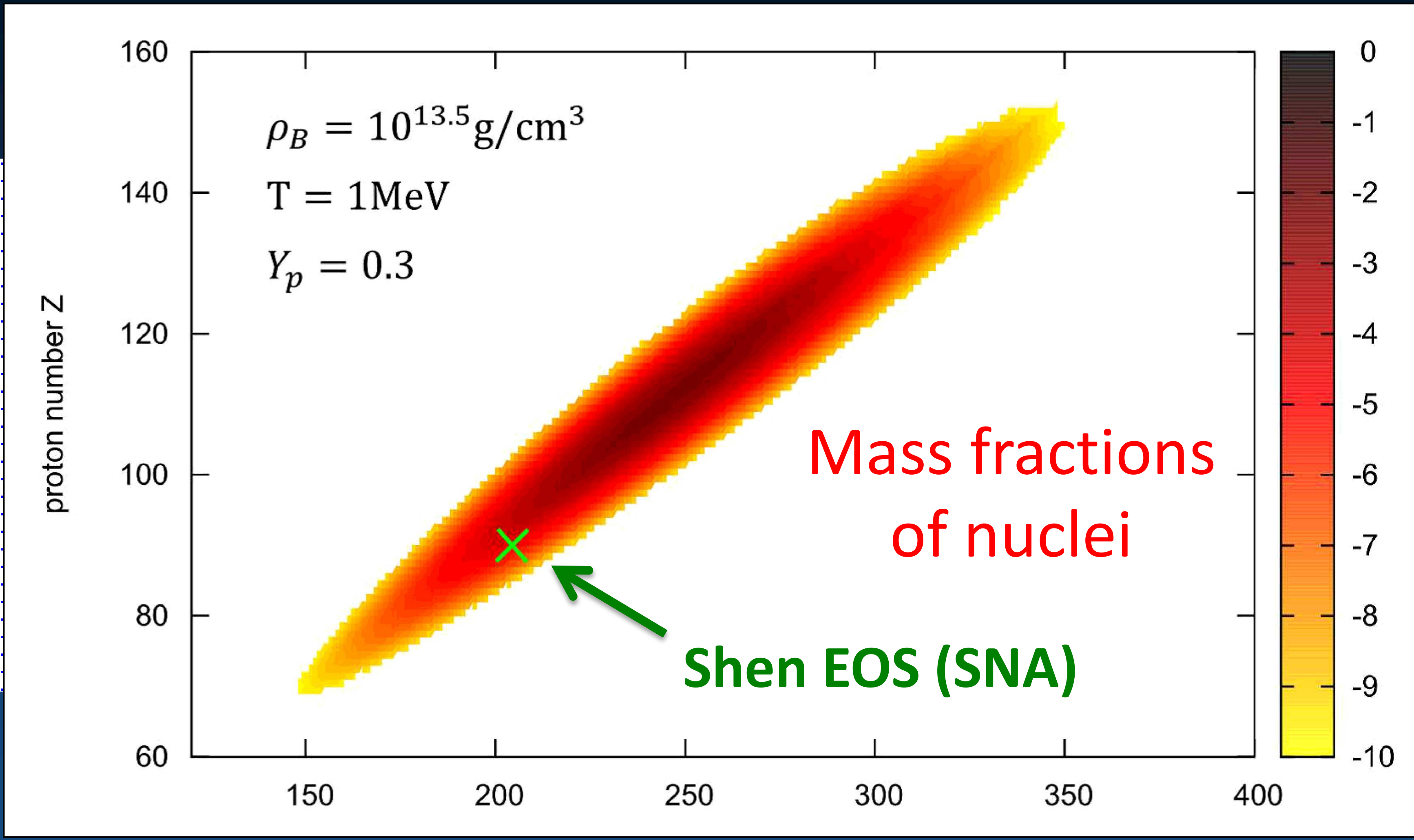
Conservative form of GR Boltzmann eq.

$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\nu^{-1}p^{\alpha}f)}{\partial x^{\alpha}}\bigg|_{q_{(i)}}+\frac{1}{\nu^2}\frac{\partial}{\partial\nu}\left(-\nu f p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(0)}\right)$$
$$+\frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}\left(\nu^{-2}\sin\bar{\theta}f\sum_{j=1}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\theta}}\right)$$
$$+\frac{1}{\sin^2\bar{\theta}}\frac{\partial}{\partial\bar{\varphi}}\left(\nu^{-2}f\sum_{j=2}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\varphi}}\right)=S_{\mathrm{rad}},$$

Shibata, Nagakura, Sekiguchi, and Yamada (2014)

Nuclear-statistical Equilibrium EOS

Hempel et al. 2011, Furusawa et al. 2011, Steiner et al. 2013 and Furusawa, H.N et al. 2017



Mass number

Weak Interactions

Basic Sets:

$\nu_e\, n \rightleftharpoons e^-\, p$	Bruenn (1985)
$\bar{\nu}_e\, p \rightleftharpoons e^+\, n$	Bruenn (1985)
$\nu_e\, A' \rightleftharpoons e^-\, A$	Bruenn (1985)
$\nu\, N \rightleftharpoons \nu\, N$	Bruenn (1985)
$\nu\, A \rightleftharpoons \nu\, A$	Bruenn (1985), Horowitz (1997)
$\nu\, e^\pm \rightleftharpoons \nu\, e^\pm$	Bruenn (1985)
$e^-\, e^+ \rightleftharpoons \nu\, \bar{\nu}$	Bruenn (1985)
$NN \rightleftharpoons \nu\bar{\nu}NN$	Hannestad & Raffelt (1998)

Lentz et al. 2011, Kotake et al. 2018
See also Burrows et al. 2004

Extensions

Lepton Sectors (including muons):

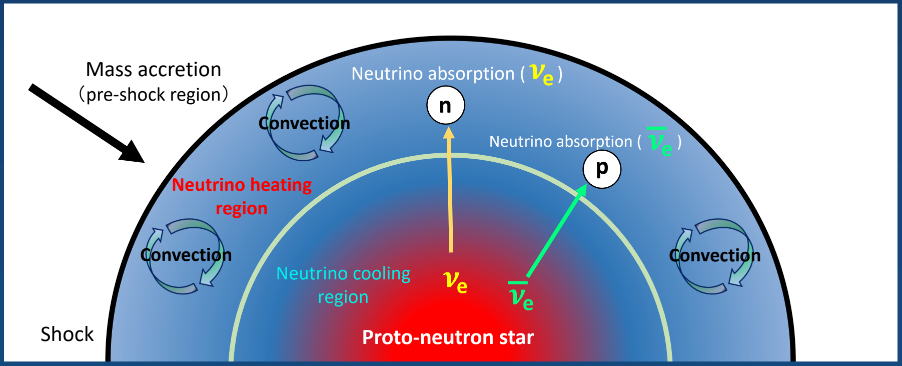
$$\nu_e + \bar{\nu}_e \rightleftharpoons \nu_x + \bar{\nu}_x$$
$$\nu_x + \nu_e(\bar{\nu}_e) \rightleftharpoons \nu_x' + \nu_e'(\bar{\nu}_e')$$

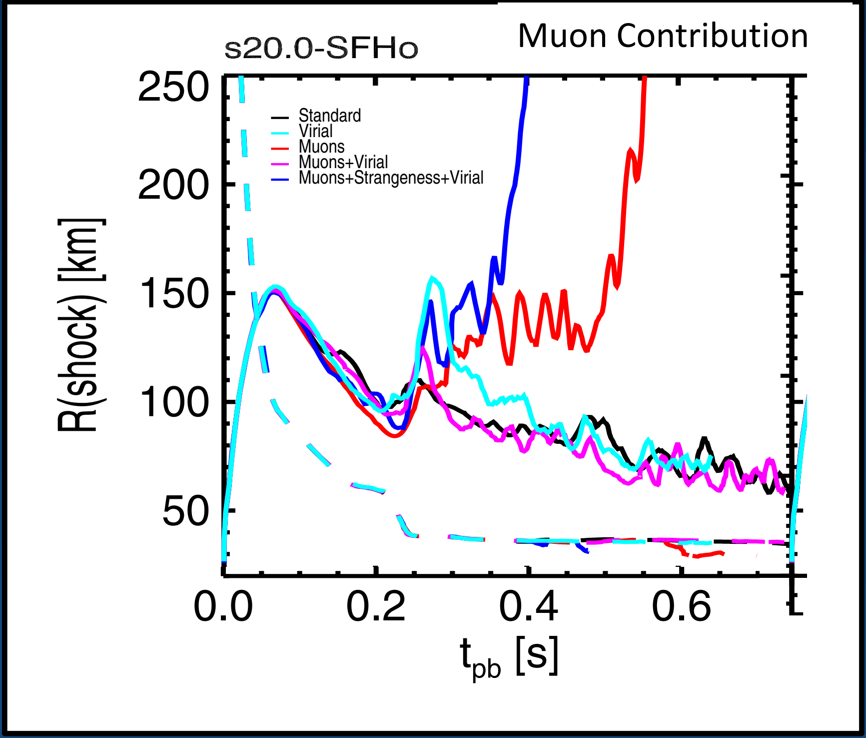
Buras et al. (2003), Fischer et al. (2009)

$\nu + \mu^- \rightleftarrows \nu' + \mu^{-\prime}$	$\nu + \mu^+ \rightleftarrows \nu' + \mu^{+\prime}$
$\nu_\mu + e^- \rightleftarrows \nu_e + \mu^-$	$\overline{\nu}_\mu + e^+ \rightleftarrows \overline{\nu}_e + \mu^+$
$\nu_\mu + \overline{\nu}_e + e^- \rightleftarrows \mu^-$	$\overline{\nu}_\mu + \nu_e + e^+ \rightleftarrows \mu^+$
$\overline{\nu}_e + e^- \rightleftarrows \overline{\nu}_\mu + \mu^-$	$\nu_e + e^+ \rightleftarrows \nu_\mu + \mu^+$

$$\nu_\mu + n \rightleftarrows p + \mu^- \qquad \overline{\nu}_\mu + p \rightleftarrows n + \mu^+$$

Bollig et al. '18





Weak Interactions

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$\nu_e\, A' \rightleftharpoons e^-\, A$	Bruenn (1985)
$\nu\, N \rightleftharpoons \nu\, N$	Bruenn (1985)
$\nu\, A \rightleftharpoons \nu\, A$	Bruenn (1985), Horowitz (1997)
$\nu\, e^\pm \rightleftharpoons \nu\, e^\pm$	Bruenn (1985)
$e^-\, e^+ \rightleftharpoons \nu\, \bar{\nu}$	Bruenn (1985)
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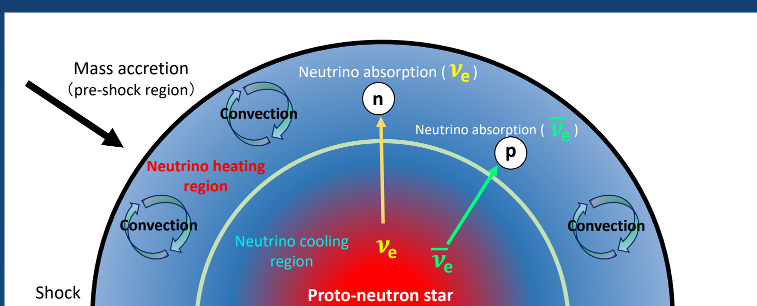


Updates

Hadron Sectors (Nucleon scattering):

Nucleon Neutral Weak Current

$$J_\mu = \langle N(p')|F_1(Q^2)\gamma_\mu + \underline{F_2(Q^2)\sigma_{\mu\nu}q^\nu} + G_A(Q^2)\gamma_\mu\gamma_5|N(p)\rangle$$

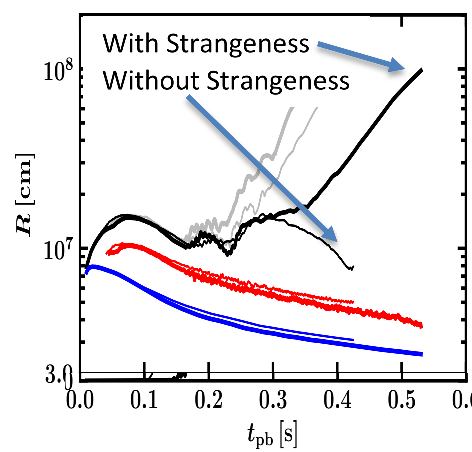
Weak magnetism

$$G_A(Q^2) = \frac{1}{2}\frac{G_A(0)}{(1+Q^2/M_A^2)}\tau_3 + \underline{G_A^s(Q^2)}$$

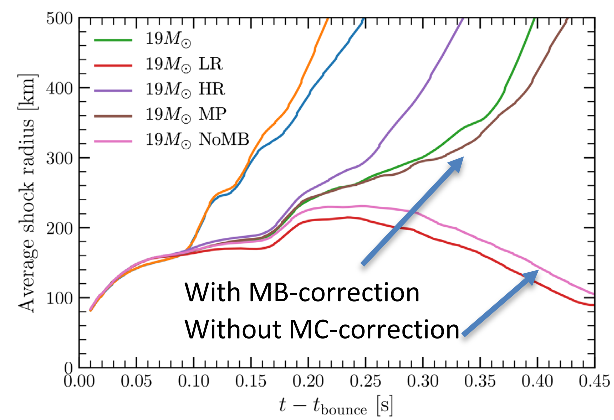
Strangeness contribution

$$\frac{1}{V}\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_\nu^2}{16\pi^2}\left[g_a^2(3-\cos\theta)(n_n+n_p)\underline{S_A} + (1+\cos\theta)n_n\underline{S_V}\right].$$

Many-body corrections

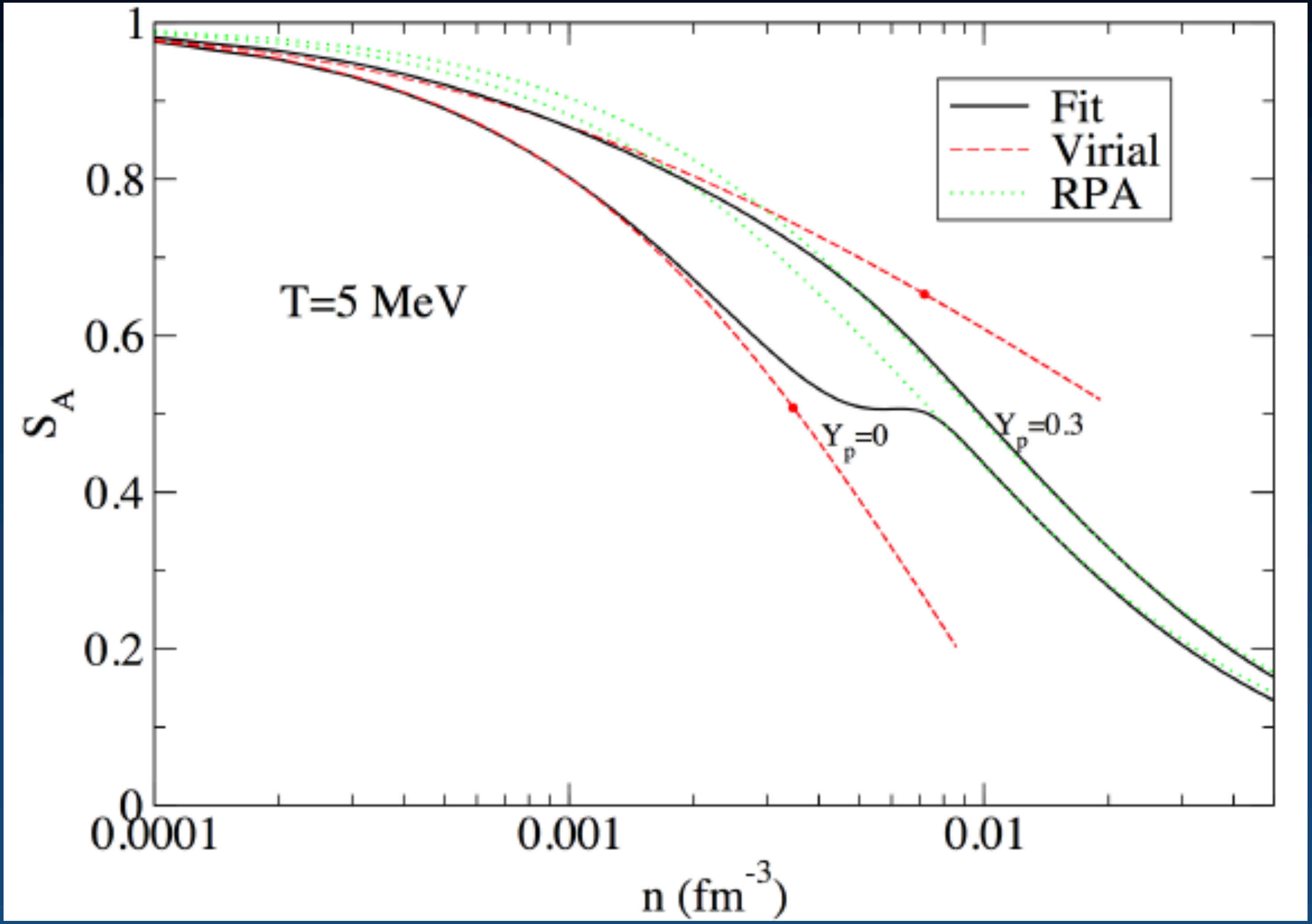


Melson et al. 2015



Burrows et al. 2020

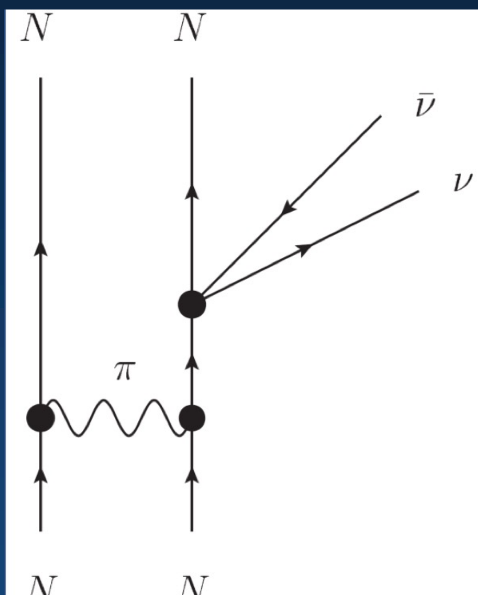
The spin (axial) S_A response

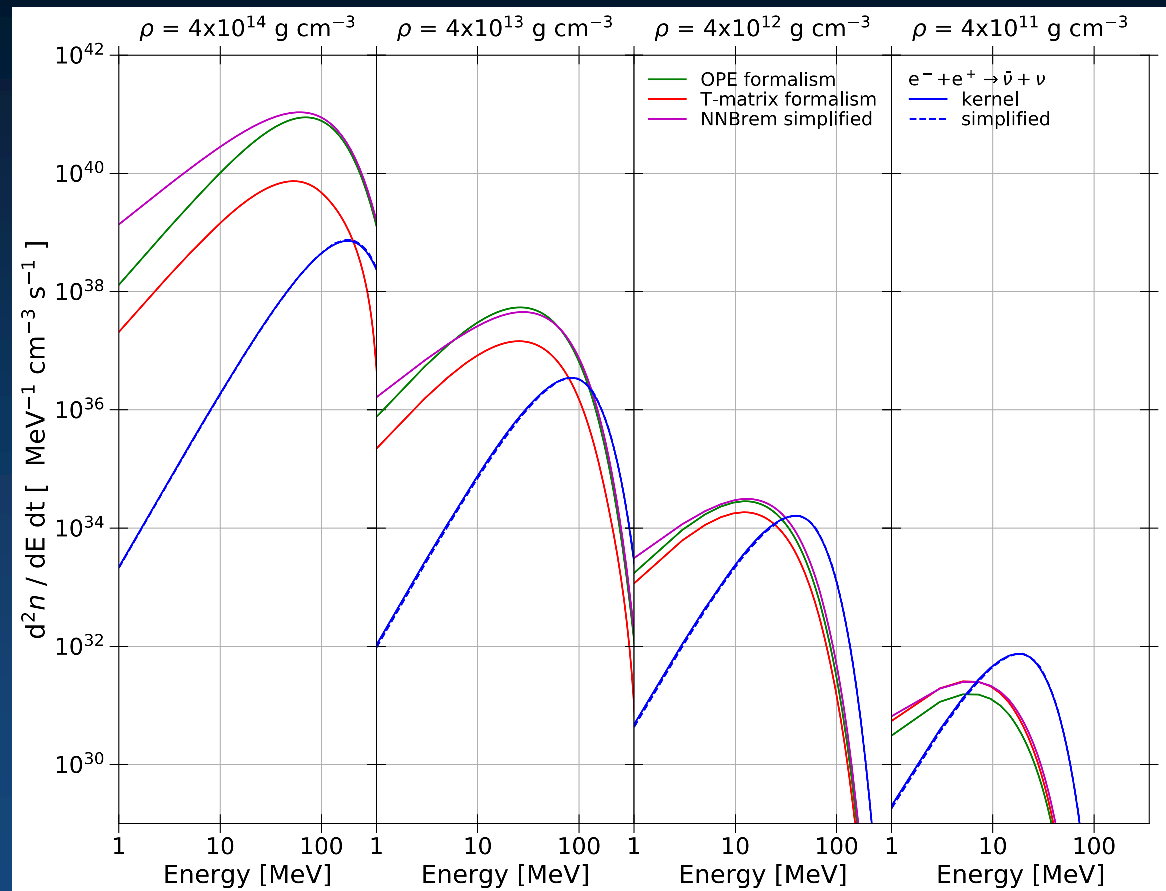


Horowitz et al. 2017

- Nucleon bremsstrahlung of neutrino pairs

- Major production channel of muon- and tau- neutrinos
- Major role in proto-neutron star cooling phase

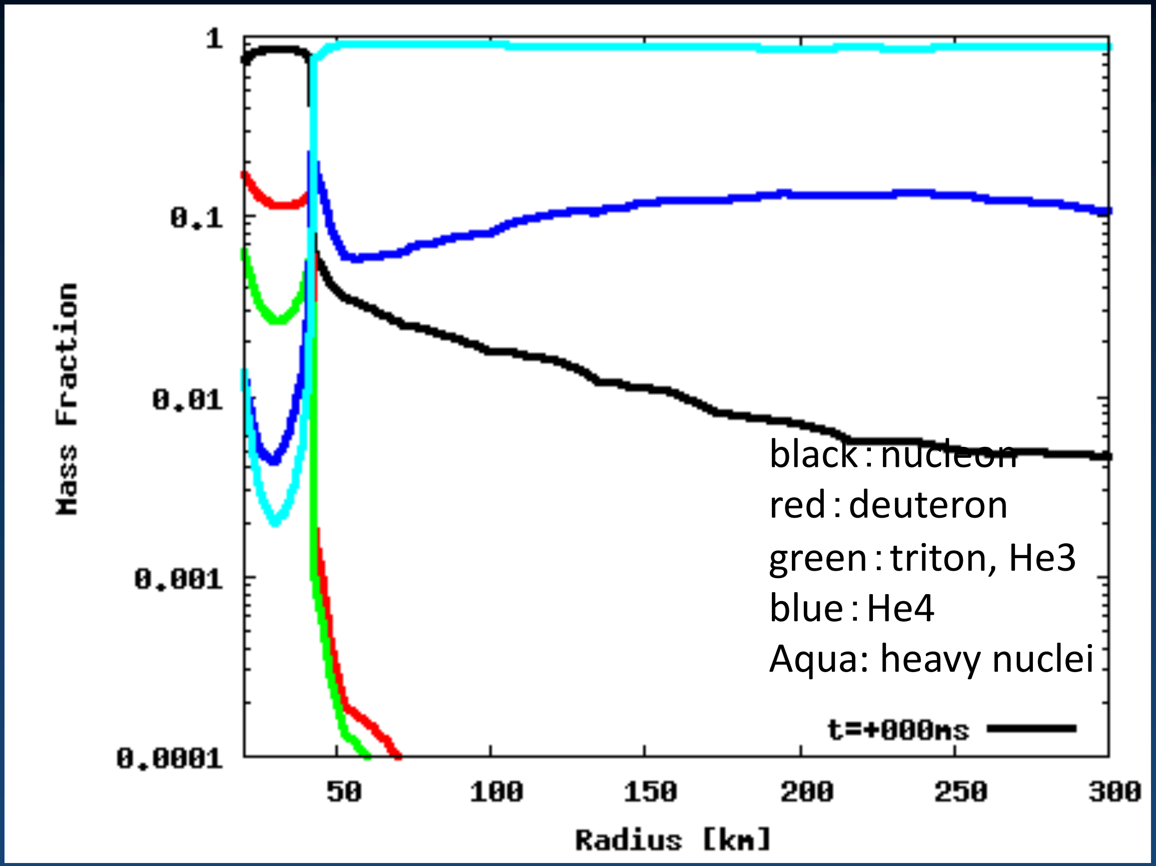




Betranhandy and O'Connor 2020
(see also Guo and Martinez-Pinedo 2019)

- Weak reactions with light nuclei

Furusawa et al. 2013, Nagakura et al. 2019, Furusawa and Nagakura 2022



$$
\begin{aligned}
(\mathrm{elpp}) &: \nu_e + {}^2\mathrm{H} \longleftrightarrow e^- + p + p, \\
(\mathrm{ponn}) &: \bar{\nu_e} + {}^2\mathrm{H} \longleftrightarrow e^+ + n + n, \\
(\mathrm{el2h}) &: \nu_e + n + n \longleftrightarrow e^- + {}^2\mathrm{H}, \\
(\mathrm{po2h}) &: \bar{\nu_e} + p + p \longleftrightarrow e^+ + {}^2\mathrm{H}, \\
(\mathrm{el3he}) &: \nu_e + {}^3\mathrm{H} \longleftrightarrow e^- + {}^3\mathrm{He}, \\
(\mathrm{po3h}) &: \bar{\nu_e} + {}^3\mathrm{He} \longleftrightarrow e^+ + {}^3\mathrm{H}.
\end{aligned}
$$

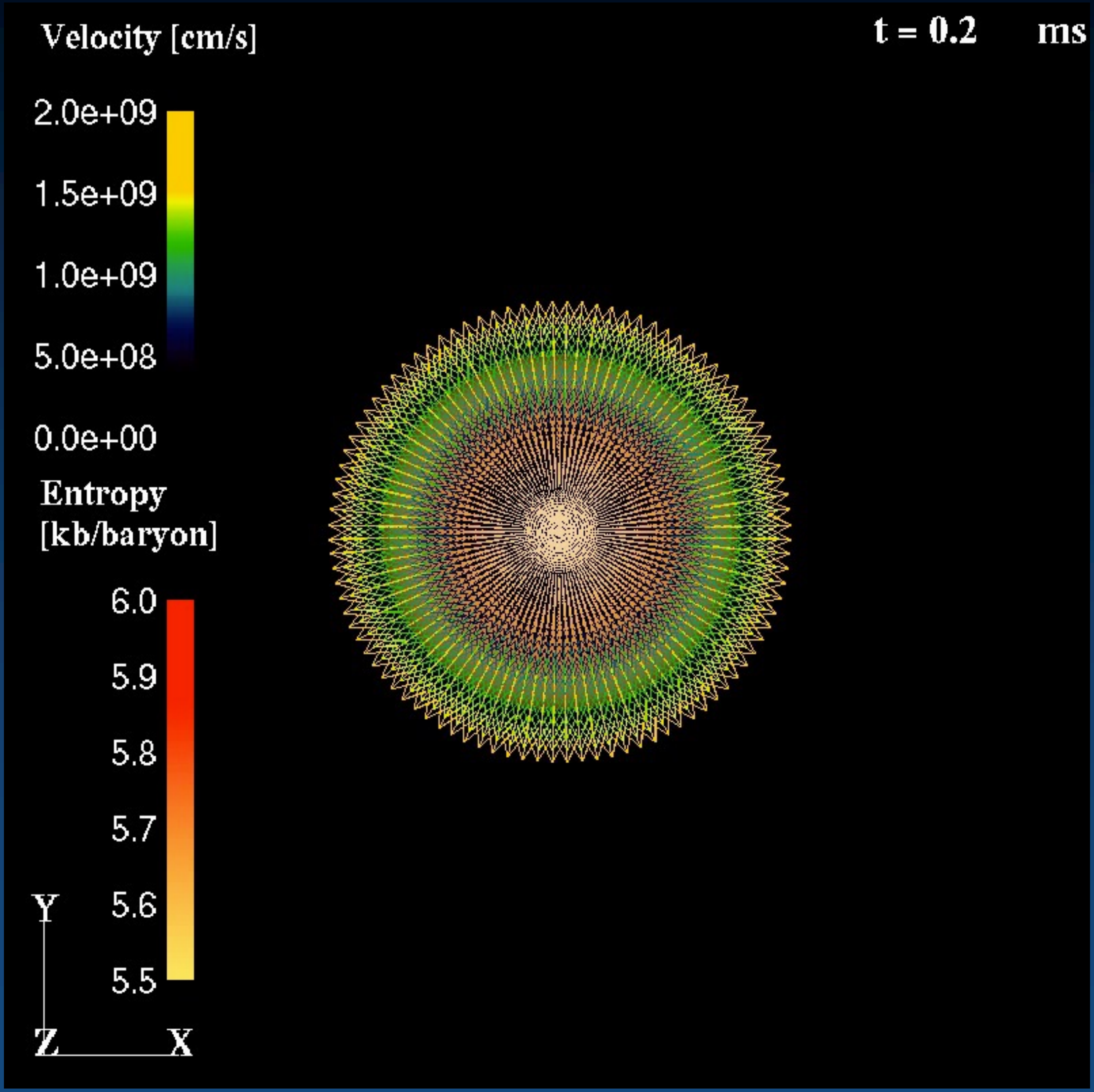
Nagakura et al. 2019

Multi-nuclear treatments of EOS are mandatory for precise computations of nuclear-weak reaction rates

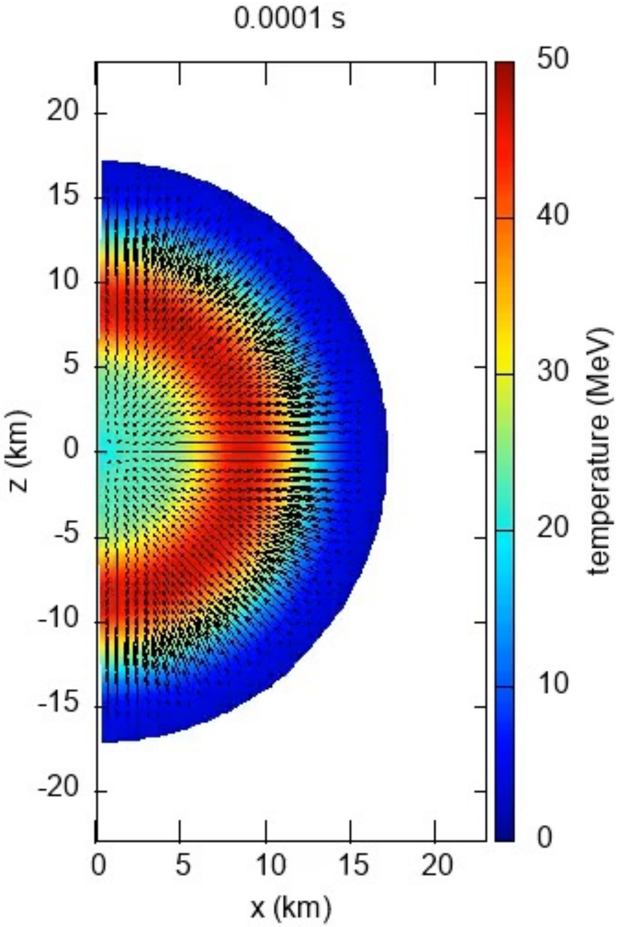
Hempel et al. 2011, Furusawa et al. 2011, Steiner et al. 2013 and Furusawa, H.N et al. 2017

- 3D CCSN simulations with full Boltzmann neutrino transport

Iwakami et al. 2020, 2021

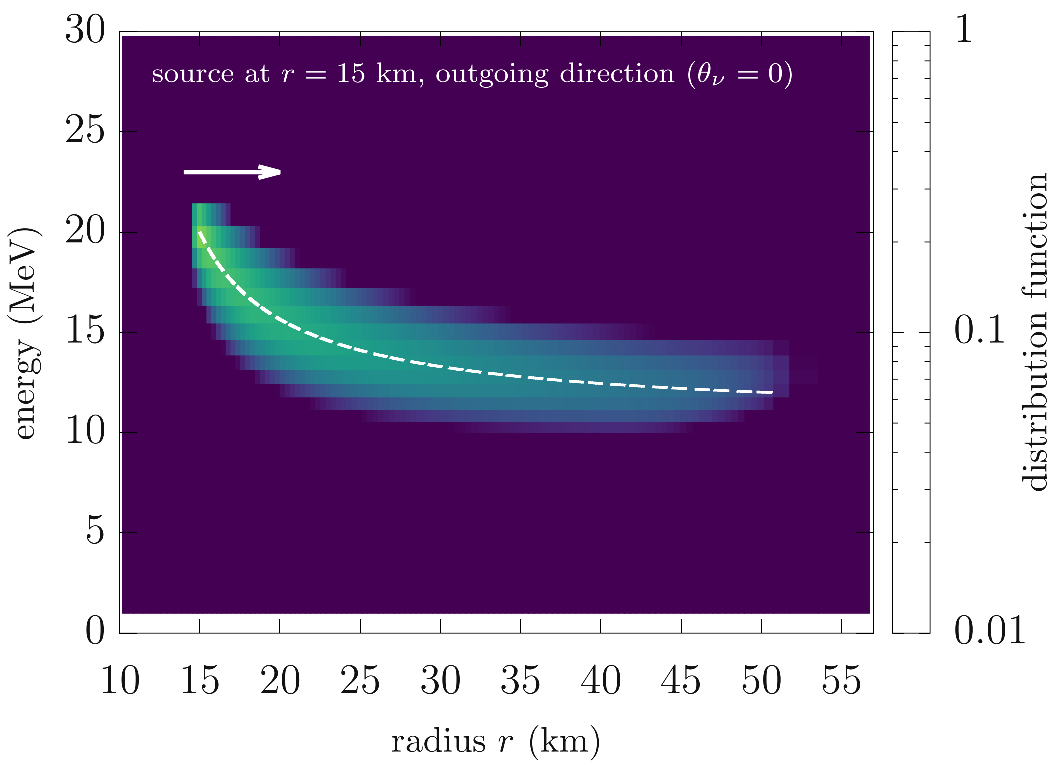


ν GR simulations with full Boltzmann neutrino transport



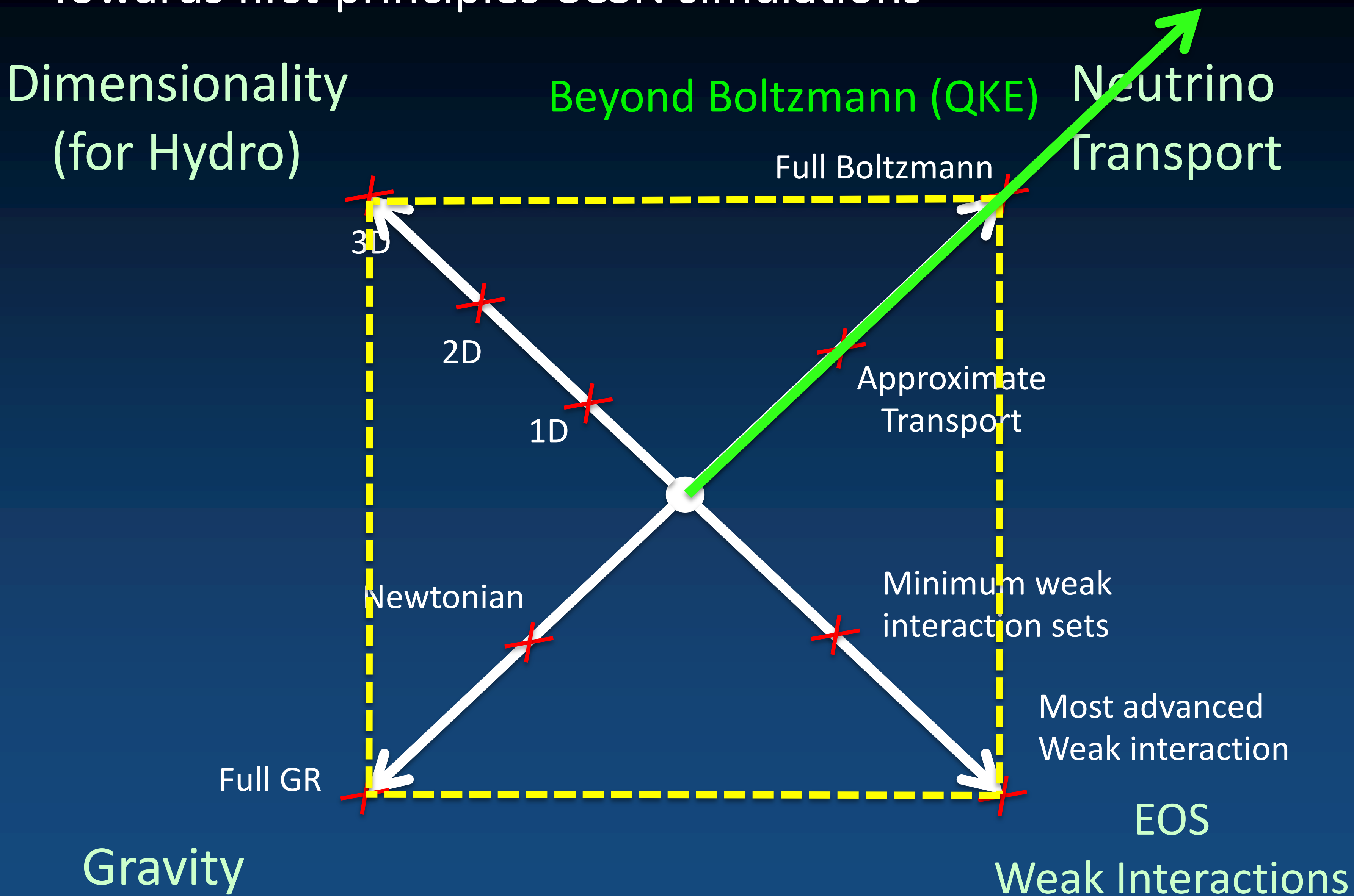
Akaho et al. 2023

Gravitational redshift in Black hole spacetime

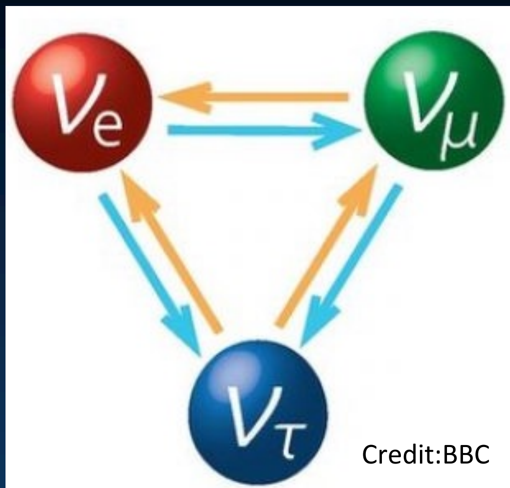


Akaho et al. 2020

- Towards first-principles CCSN simulations

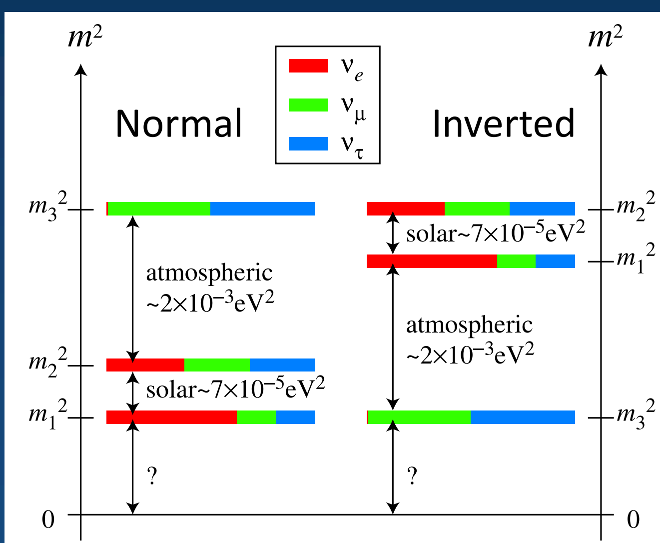


Neutrino oscillations



Credit:BBC

- There are many experimental evidences that neutrinos can go through flavor conversion.
- Neutrinos have at least three different masses.
- Flavor eigenstates are different from mass eigenstates.



Feruglio et al. 2003

$$|\nu_i\rangle = \sum_\alpha U^*_{\alpha i}\,|\nu_\alpha\rangle\,,$$
Mass state

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}\,|\nu_i\rangle\,,$$
Flavor state

U represents Pontecorvo–Maki–Nakagawa–Sakata matrix (PMNS matrix)

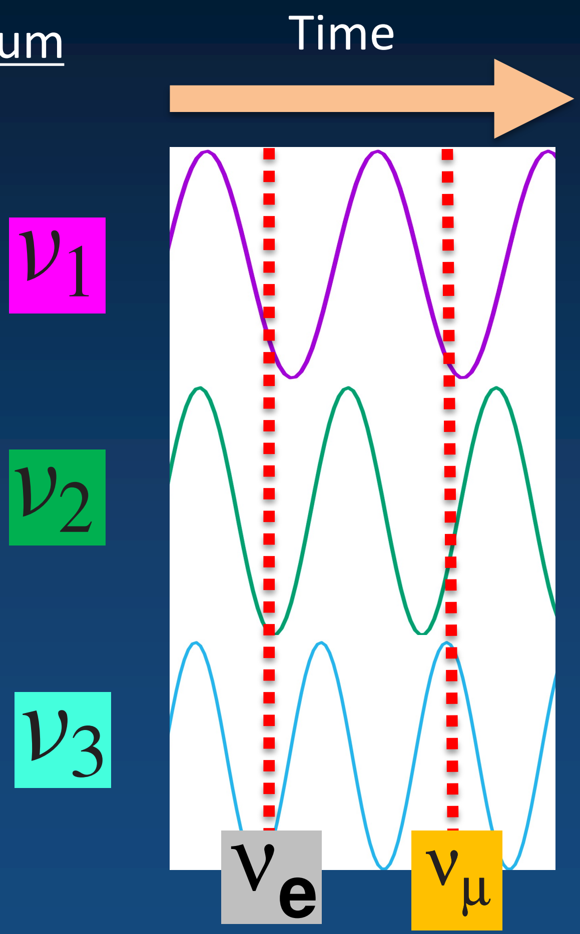
$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

Neutrino oscillation with a plane-wave picture

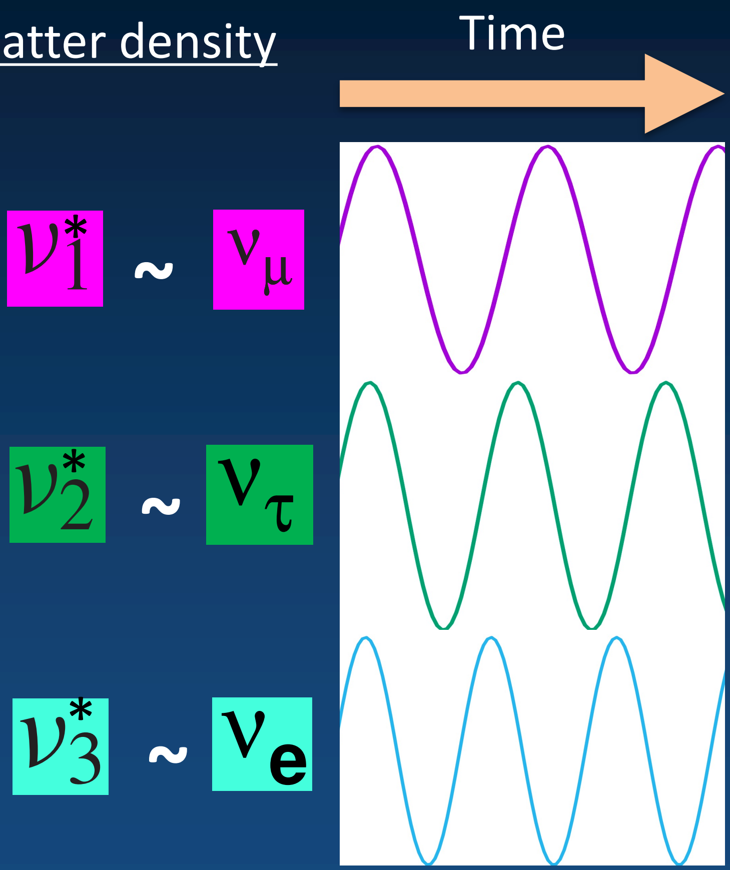
$$\psi_\ell(0,0) \equiv |\nu_\ell\rangle = \sum_{i=1}^{3} U^*_{\ell i}\,|\nu_i\rangle$$

$$\psi_\ell(t,x) = \sum_{i=1}^{3} U^*_{\ell i}\,|\nu_i\rangle\, e^{-i\phi_i}, \qquad \text{phase } \phi_i = E_i\,t - p_i\,x$$

Vacuum



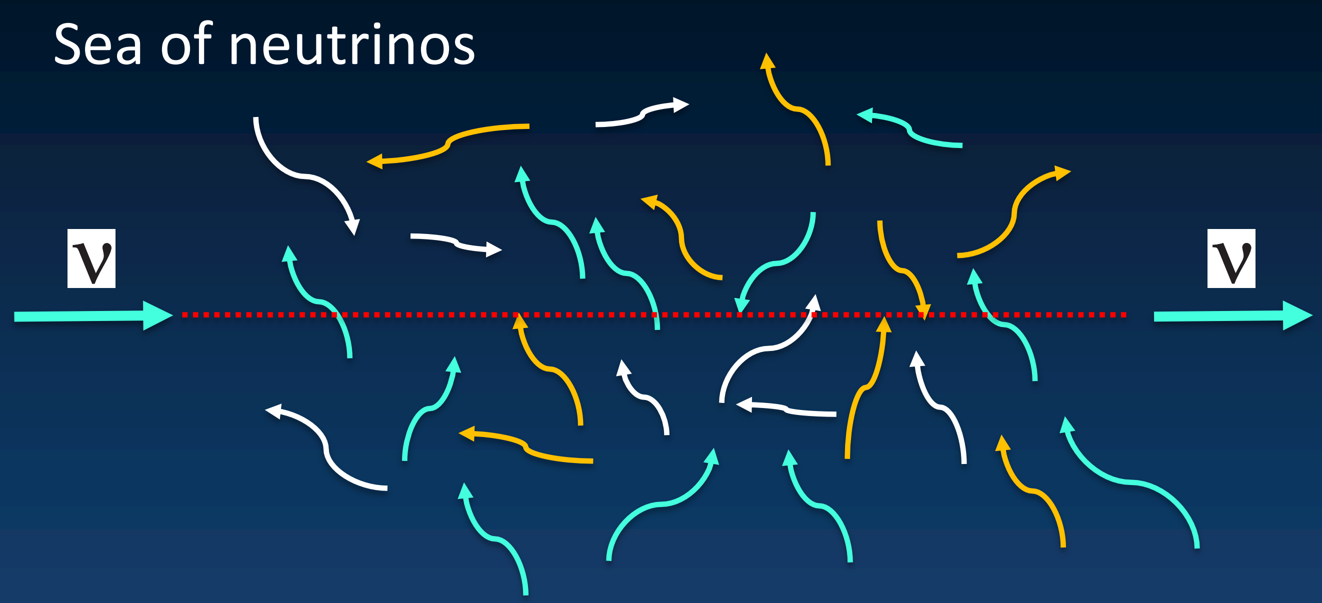
High matter density



Boltzmann transport becomes a reasonable approximation.

Neutrino oscillation induced by self-interactions

Pantalone 1992

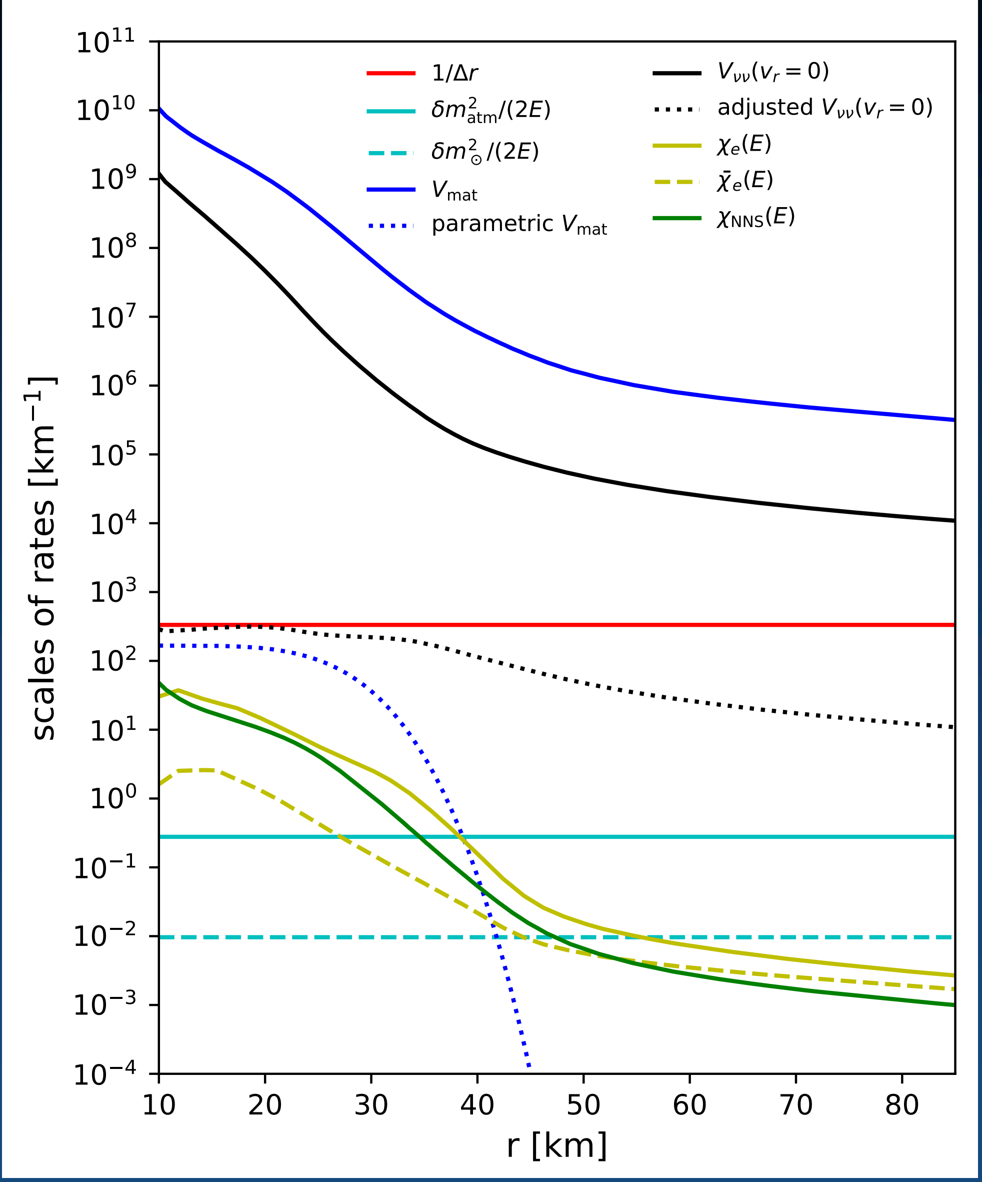


1. Refractions by self-interactions induce neutrino flavor conversions, which is analogy to matter effects (e.g., MSW resonance).

2. The oscillation timescale is much shorter than the global scale of CCSN/BNSM.

3. Collective neutrino oscillation induced by neutrino-self interactions commonly occurs in CCSNe and BNSM environments.

Huge disparity of scales between neutrino oscillations and other ingredients



Xiong et al. 2023

Quantum Kinetics neutrino transport:

Vlasenko et al. 2014, Volpe 2015, Blaschke et al. 2016, Richers et al. 2019

$$p^\mu \frac{\partial \overset{(-)}{f}}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial \overset{(-)}{f}}{\partial p^i} = -p^\mu u_\mu \overset{(-)}{S}_{\mathrm{col}} + i p^\mu n_\mu [\overset{(-)}{H}, \overset{(-)}{f}],$$

Advection terms (Same as Boltz eq.)

Collision term

Oscillation term

f is not a "distribution function"

Density matrix

$$\overset{(-)}{f} = \begin{bmatrix} \overset{(-)}{f}_{ee} & \overset{(-)}{f}_{e\mu} & \overset{(-)}{f}_{e\tau} \\ \overset{(-)}{f}_{\mu e} & \overset{(-)}{f}_{\mu\mu} & \overset{(-)}{f}_{\mu\tau} \\ \overset{(-)}{f}_{\tau e} & \overset{(-)}{f}_{\tau\mu} & \overset{(-)}{f}_{\tau\tau} \end{bmatrix}$$

Hamiltonian

$$\overset{(-)}{H} = \overset{(-)}{H}_{\mathrm{vac}} + \overset{(-)}{H}_{\mathrm{mat}} + \overset{(-)}{H}_{\nu\nu},$$

$$H_{\mathrm{vac}} = \frac{1}{2\nu} U \begin{bmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{bmatrix} U^\dagger,$$

$$H_{\mathrm{mat}} = D \begin{bmatrix} V_e & 0 & 0 \\ 0 & V_\mu & 0 \\ 0 & 0 & V_\tau + V_{\mu\tau} \end{bmatrix},$$

$$H_{\nu\nu} = \sqrt{2} G_F \int \frac{d^3 q'}{(2\pi)^3} \left(1 - \sum_{i=1}^{3} \ell'_{(i)} \ell_{(i)}\right) \left(f(q') - \bar{f}^*(q')\right),$$

Rich flavor-conversion phenomena driven by neutrino-neutrino self-interactions

Vacuum: $\omega = \frac{\Delta m^2}{2E_\nu},$
Matter: $\lambda = \sqrt{2} G_F n_e,$
Self-int: $\mu = \sqrt{2} G_F n_\nu,$

- Slow-mode (Duan et al. 2010)

- Energy-dependent flavor conversion occurs.
- The frequency of the flavor conversion is proportional to $\sqrt{\omega\mu}$

- Fast-mode (FFC) (Sawyer 2005)

- Collective neutrino oscillation in the limit of ω → 0.
- The frequency of the flavor conversion is proportional to μ
- Anisotropy of neutrino angular distributions drives the fast flavor-conversion.

- Collisional instability (Johns 2021)

- Asymmetries of matter interactions between neutrinos and anti-neutrinos drive flavor conversion.

$$\mathrm{Im}\ \Omega \cong \pm \frac{\Gamma - \bar{\Gamma}}{2} \frac{\mu S}{\sqrt{(\mu D)^2 + 4\omega\mu S}} - \frac{\Gamma + \bar{\Gamma}}{2}$$

Γ: Matter-interaction rate

- Matter-neutrino resonance (Malkus et al. 2012)

- The resonance potentially occur in BNSM/Collapsar environment (but not in CCSN).
- Essentially the same mechanism as MSW resonance.

$$|\lambda + \mu| \sim |\omega|$$

Linear stability analysis of flavor instabilities

Dispersion relation approach

Example: FFC (Izaguirre et al. 2017)

1. $\rho_v = \dfrac{f_{\nu_e} + f_{\nu_x}}{2} I + \dfrac{f_{\nu_e} - f_{\nu_x}}{2} \begin{pmatrix} s_v & S_v \\ S_v^* & -s_v \end{pmatrix}.$

2. $$i(\partial_t + \boldsymbol{v} \cdot \nabla_r) S_v = -v^\mu (\Lambda_\mu + \Phi_\mu) S_v + \int d\Gamma' v^\mu v'_\mu G_{v'} S_{v'},$$

Momentum-space integration

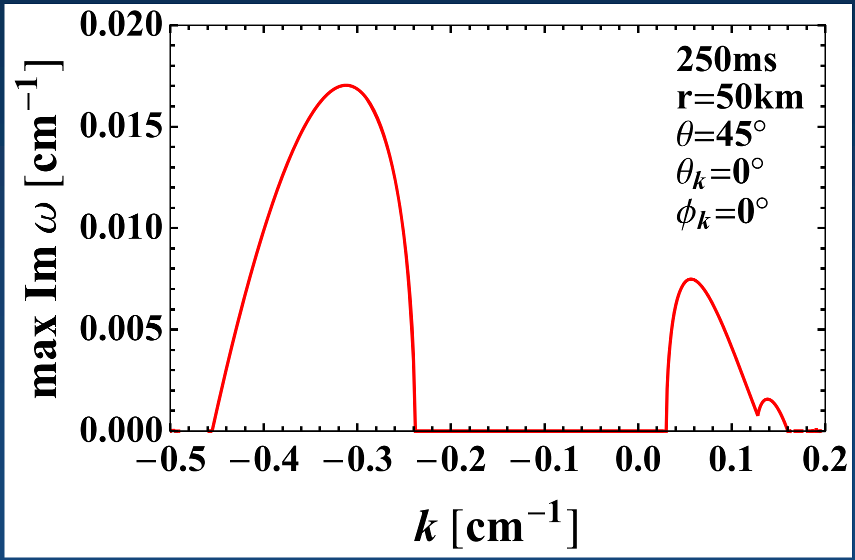
3. $S_v = Q_v \exp[-i(\Omega t - \boldsymbol{k} \cdot \boldsymbol{r})].$

4. $$\Pi^{\mu\nu} \equiv \eta^{\mu\nu} + \int d\Gamma G_v \frac{v^\mu v^\nu}{v^\gamma k_\gamma} = \eta^{\mu\nu} - \int d\Gamma G_v \frac{v^\mu v^\nu}{\omega - \boldsymbol{v} \cdot \boldsymbol{k}}.$$

$(G_v \equiv \sqrt{2} G_F f_{\nu_e}(\boldsymbol{v}),)$

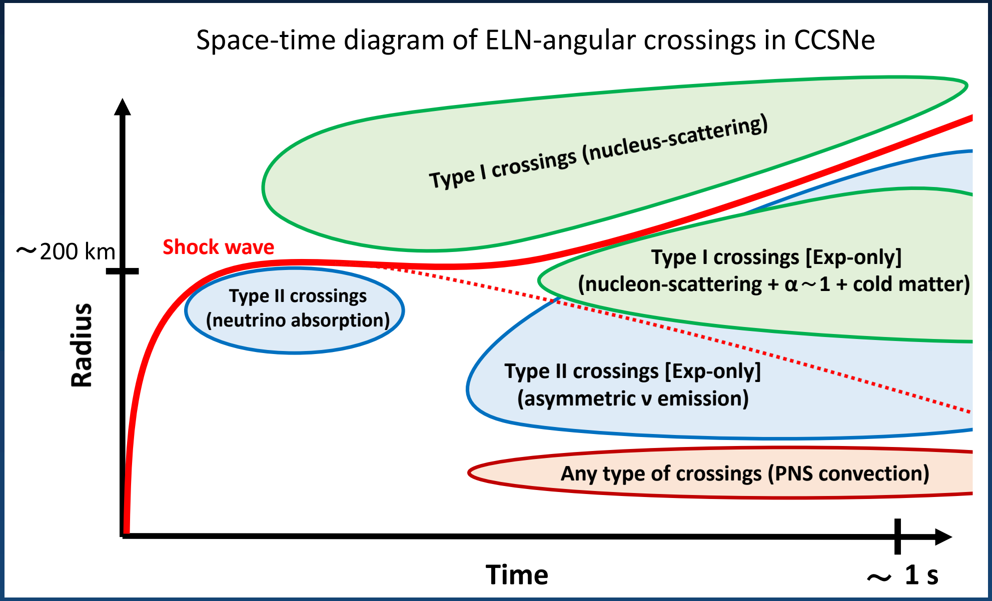
$$\det \Pi = 0,$$

1. Decomposing traceless part
2. Linearizing QKE equation
3. Plane-wave ansatz
4. Computing Dispersion relation



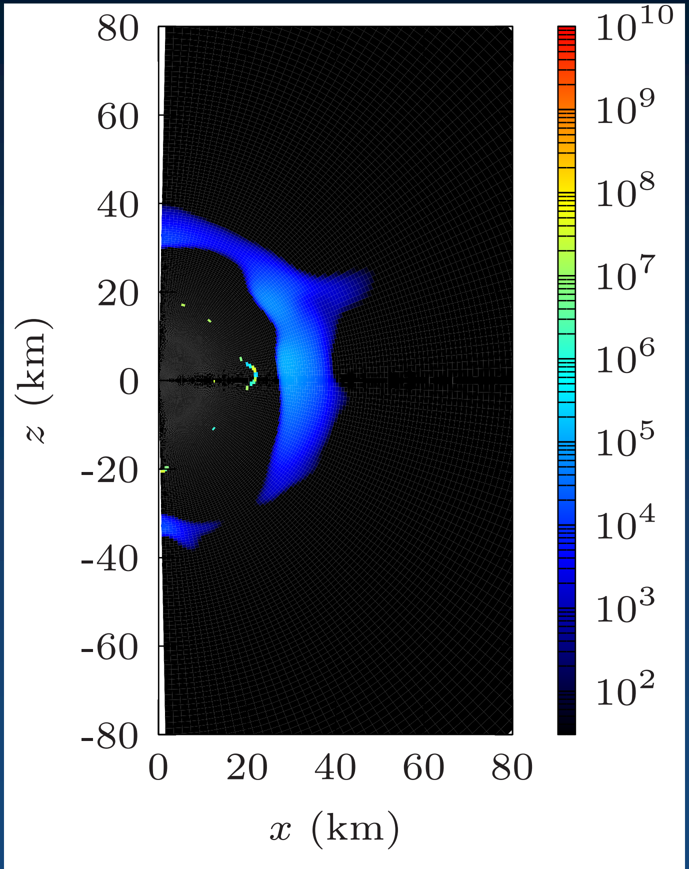
FFC and CFI can occur in CCSNe

Fast neutrino-flavor conversion (FFC)



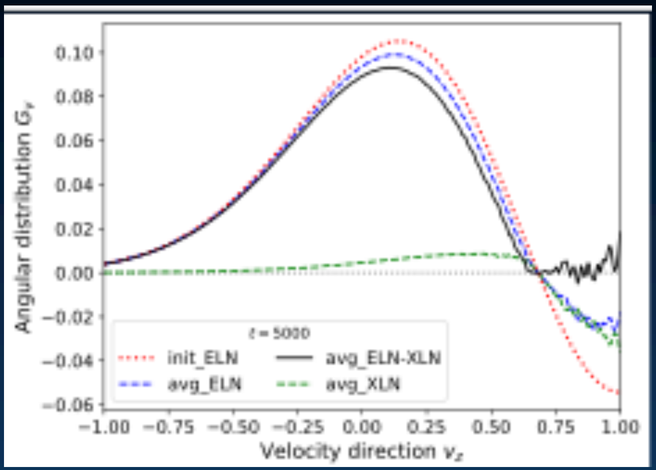
Nagakura et al. PRD 2021

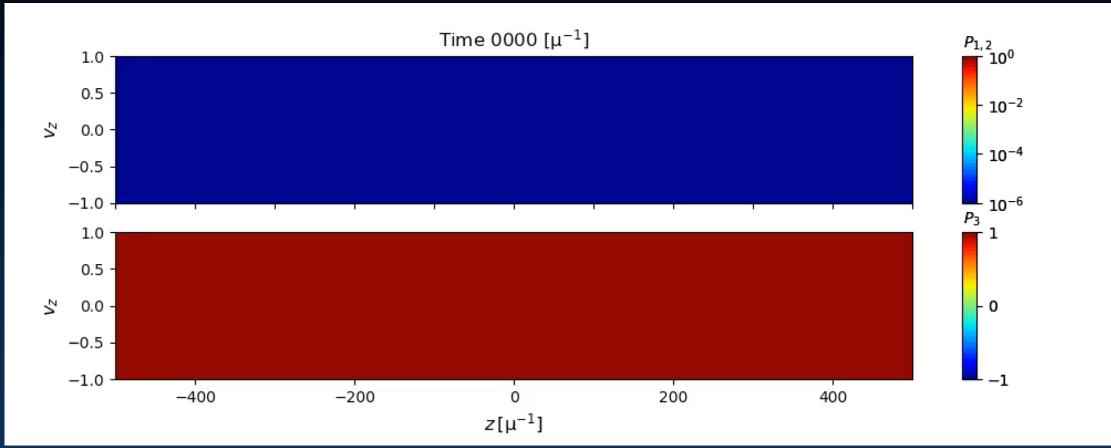
Collisional flavor instability (CFI)



Akaho et al. 2023

- Non-linear phase (local simulations of FFC)





Zaizen and Nagakura 2022

Asymptotic state FFC can be estimated <u>"analytically"</u>

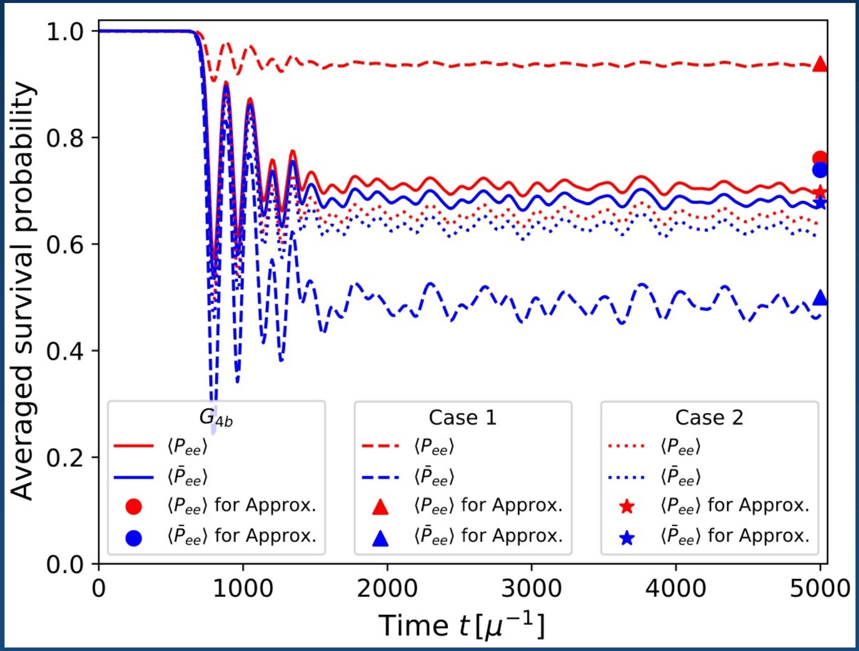
Conservation law of neutrinos

+

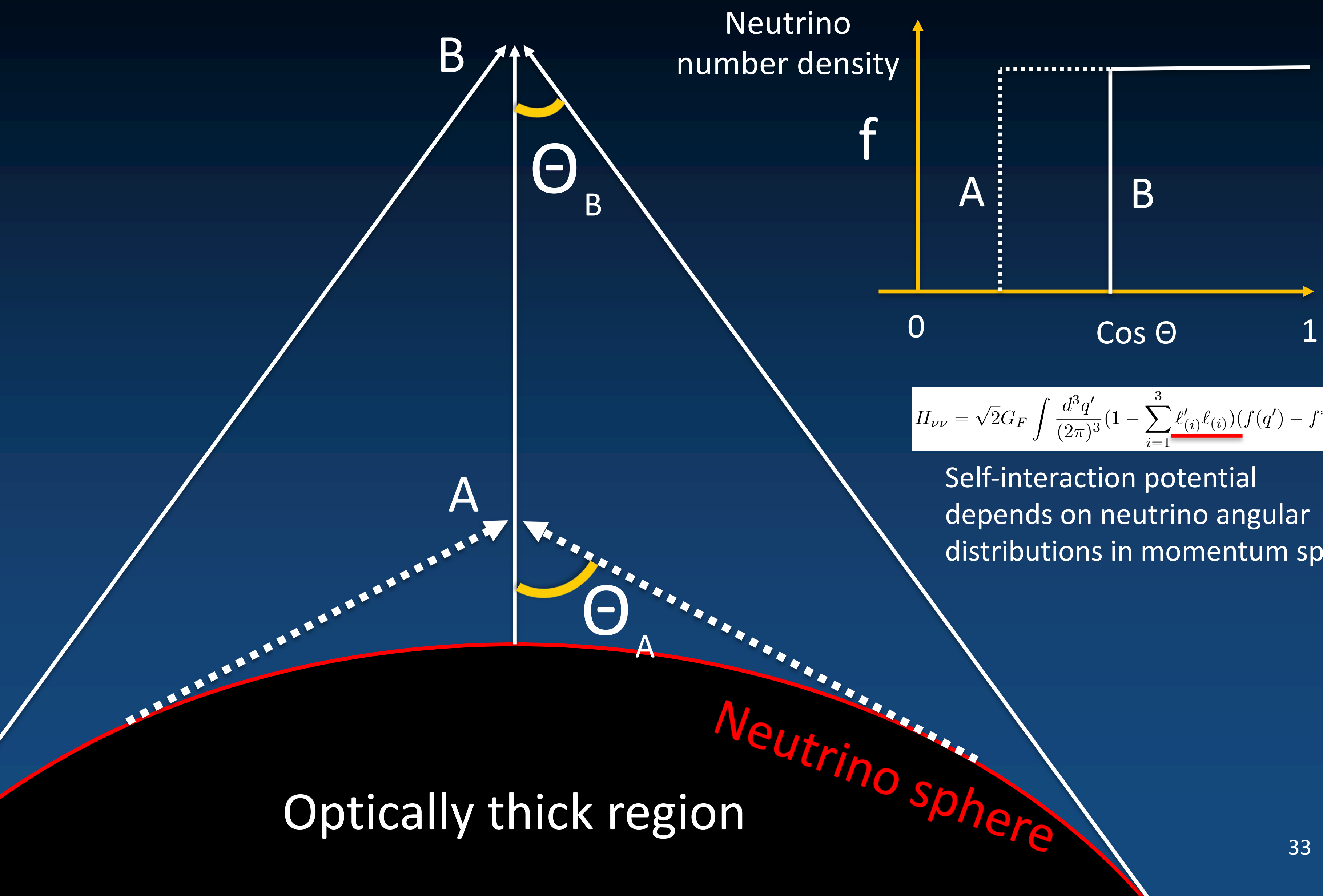
Stability condition

(disappearance of ELN-XLN crossings)

Zaizen and Nagakura 2022



- Need of global simulations in the study of flavor conversions in CCSN/BNSM



$$H_{\nu\nu} = \sqrt{2}G_F \int \frac{d^3q'}{(2\pi)^3}\left(1-\sum_{i=1}^{3} \ell'_{(i)}\ell_{(i)}\right)\left(f(q')-\bar{f}^*(q')\right),$$

Self-interaction potential depends on neutrino angular distributions in momentum space

- Global simulations:

General-relativistic quantum-kinetic neutrino transport (GRQKNT)

Nagakura 2022

$$p^{\mu}\frac{\partial \overset{(-)}{f}}{\partial x^{\mu}} + \frac{dp^{i}}{d\tau}\frac{\partial \overset{(-)}{f}}{\partial p^{i}} = -p^{\mu}u_{\mu}\overset{(-)}{S}_{\rm col} + ip^{\mu}n_{\mu}[\overset{(-)}{H}, \overset{(-)}{f}],$$

- Fully general relativistic (3+1 formalism) neutrino transport
- Multi-Dimension (6-dimensional phase space)
- Neutrino matter interactions (emission, absorption, and scatterings)
- Neutrino Hamiltonian potential of vacuum, matter, and self-interaction
- 3 flavors + their anti-neutrinos
- Solving the equation with Sn method (explicit evolution: WENO-5th order)
- Hybrid OpenMP/MPI parallelization

Time-dependent global simulations of FFC

Nagakura and Zaizen PRL 2022, PRD 2023

- Issue:

$$\ell_{\mathrm{n}_\nu} \equiv c\,\mathrm{T}_{\mathrm{n}_\nu} = 0.235\ \mathrm{cm} \left(\frac{L_\nu}{4 \times 10^{52} \mathrm{erg/s}} \right)^{-1} \left(\frac{E_{\mathrm{ave}}}{12\mathrm{MeV}} \right) \left(\frac{R}{50\mathrm{km}} \right)^{2} \left(\frac{\kappa}{1/3} \right)$$

Oscillation wavelength is an order of sub-centimeter.

Too short !!!!

How can we make FFC simulations tractable???

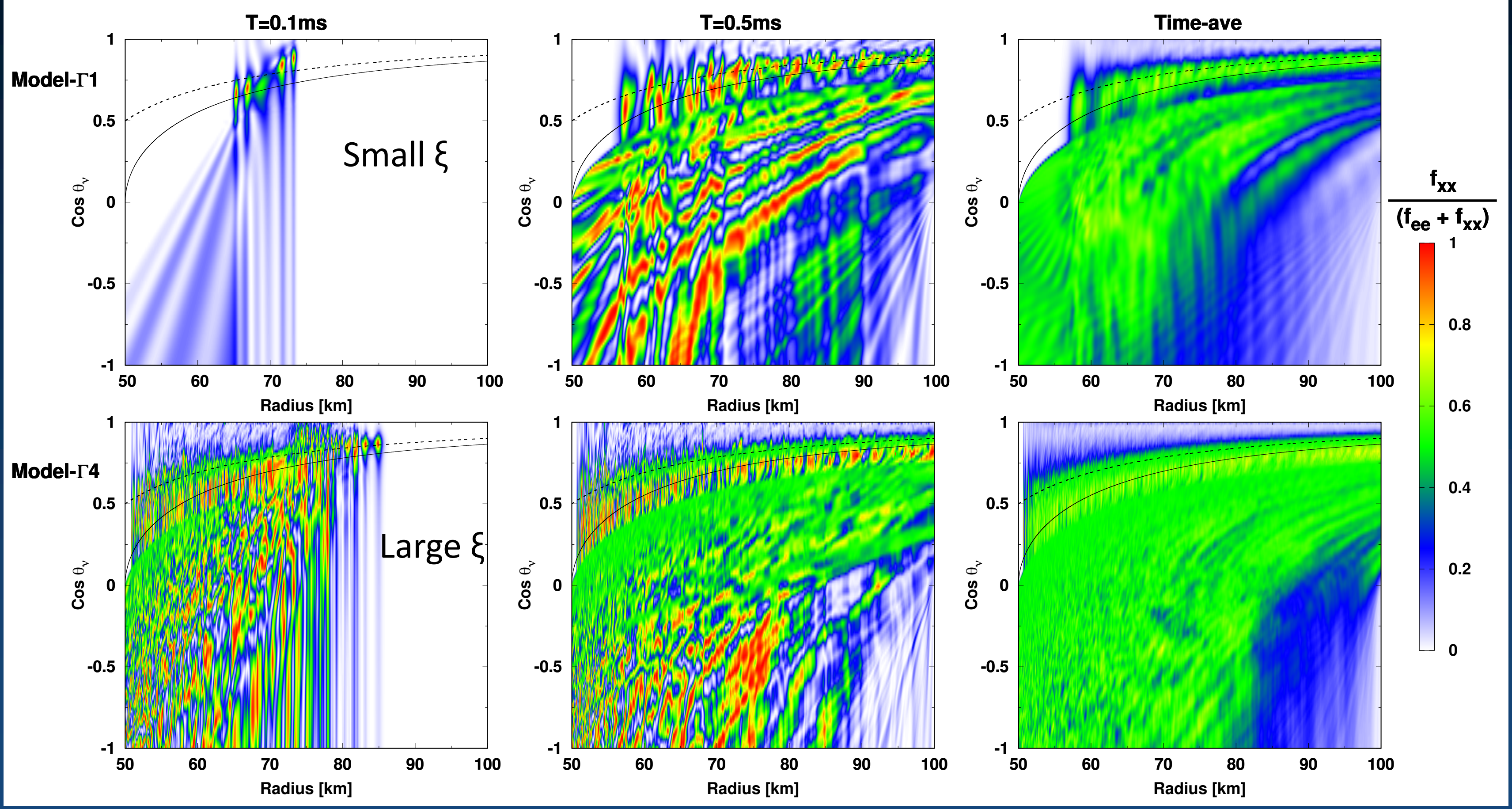
- Strategy:

$$\frac{\partial \overset{(-)}{f}}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \cos\theta_\nu \overset{(-)}{f}) - \frac{1}{r\sin\theta_\nu}\frac{\partial}{\partial \theta_\nu}(\sin^2\theta_\nu \overset{(-)}{f}) = -i\,\xi\,[\overset{(-)}{H}, \overset{(-)}{f}],$$

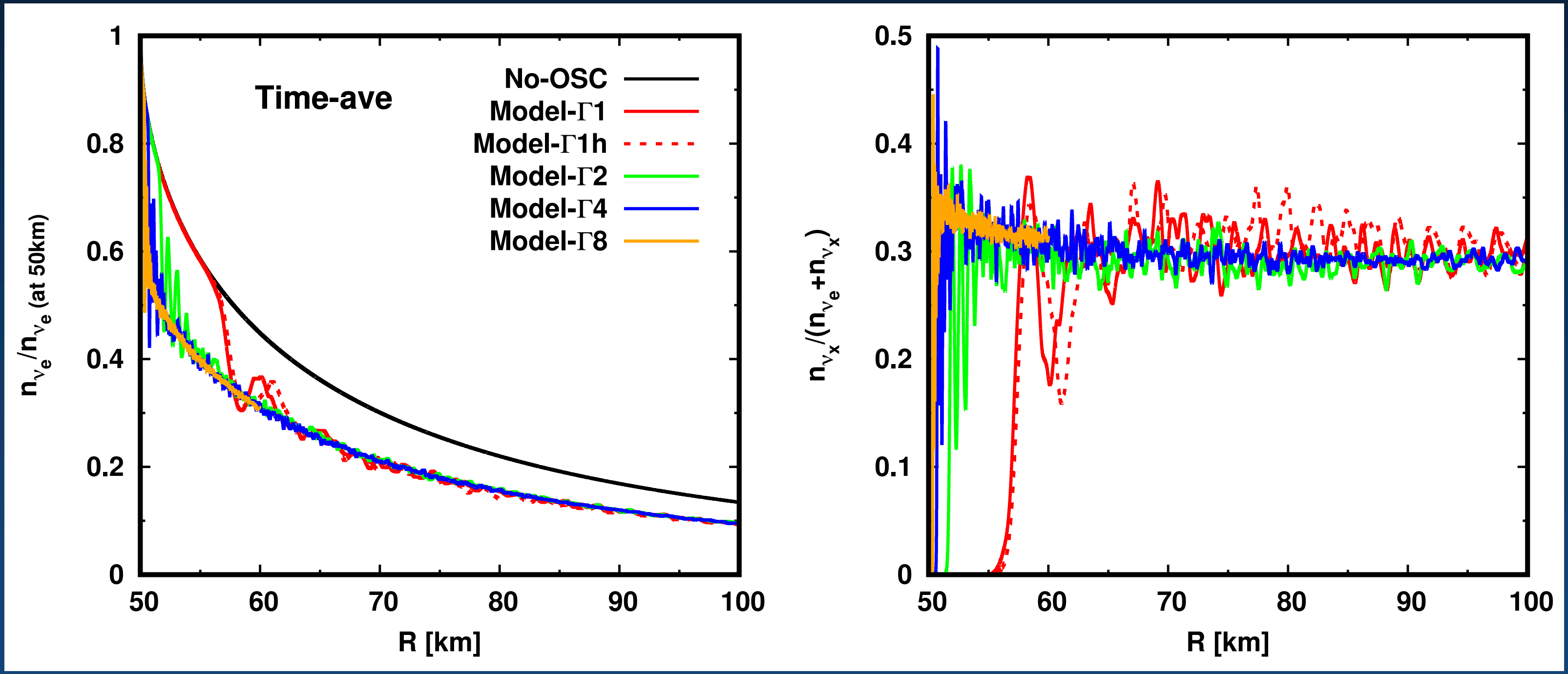
Attenuation parameter ($0 \leqq \xi \leqq 1$)

- Attenuating Hamiltonian makes global QKE simulations tractable.
- Realistic features can be extracted by a convergence study of ξ ($\rightarrow$ 1).

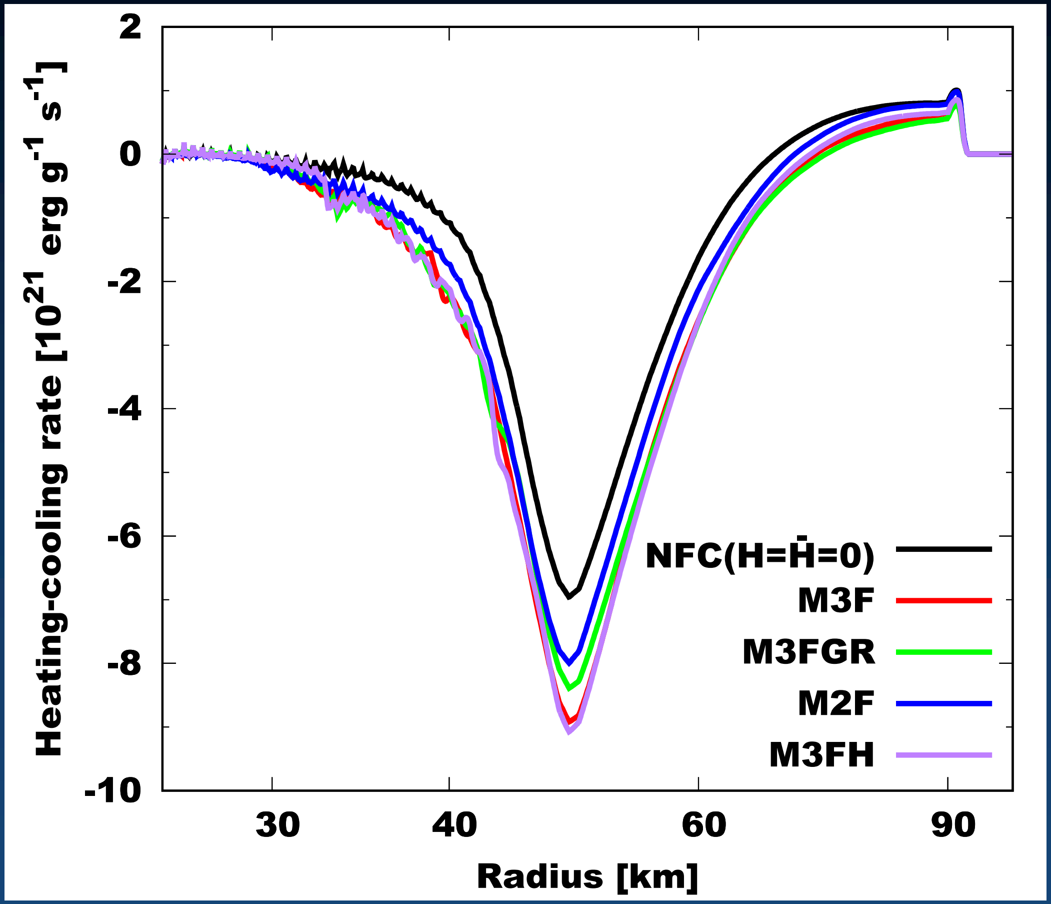
Temporal and quasi-steady features of FFC in global scale
(1D in space + 1D angle in momentum space)



Attenuating Hamiltonian potential does not change the degree of flavor conversion in asymptotic states.



- Global Simulations of FFC (in CCSN) Nagakura PRL 2023



<u>Numerical setup:</u>

Collision terms are switched on.

Fluid-profiles are taken from a CCSN simulation.

General relativistic effects are taken into account.

A wide spatial region is covered.

Three-flavor framework

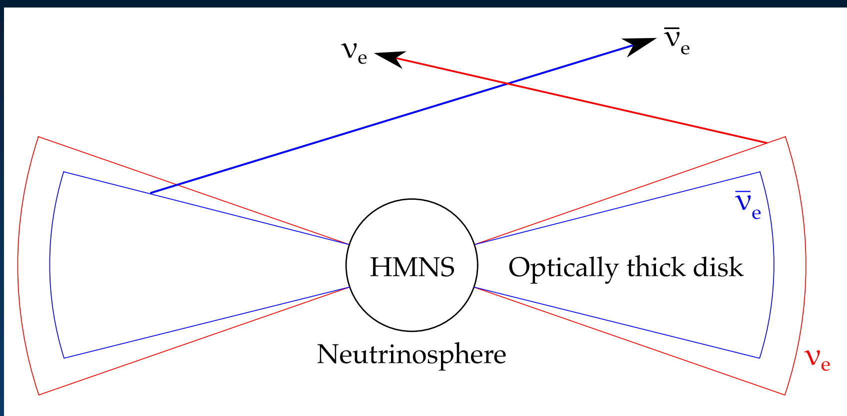
Neutrino-cooling is enhanced by FFCs
Neutrino-heating is suppressed by FFCs

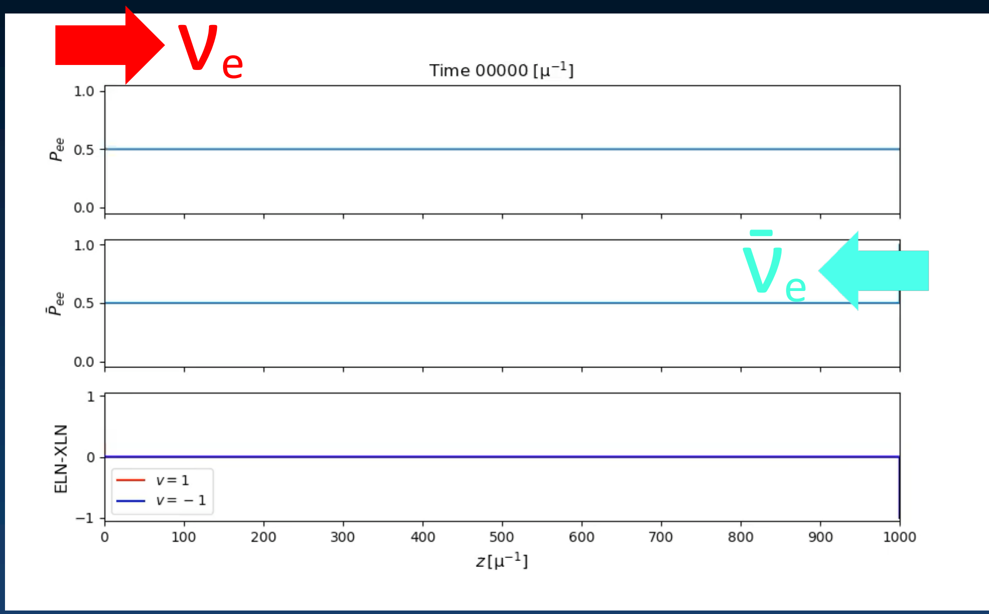
Impacts on the explodability of CCSN

Fast flavor swap would be ubiquitous in BNSM

Zaizen and Nagakura 2024

- Colliding-beam model





$$\partial_{t'}^2 P_3 \sim -4\mu^2 \left(1 - (P_3)^2\right)$$

$P_3 = 1$: electron-type → Unstable

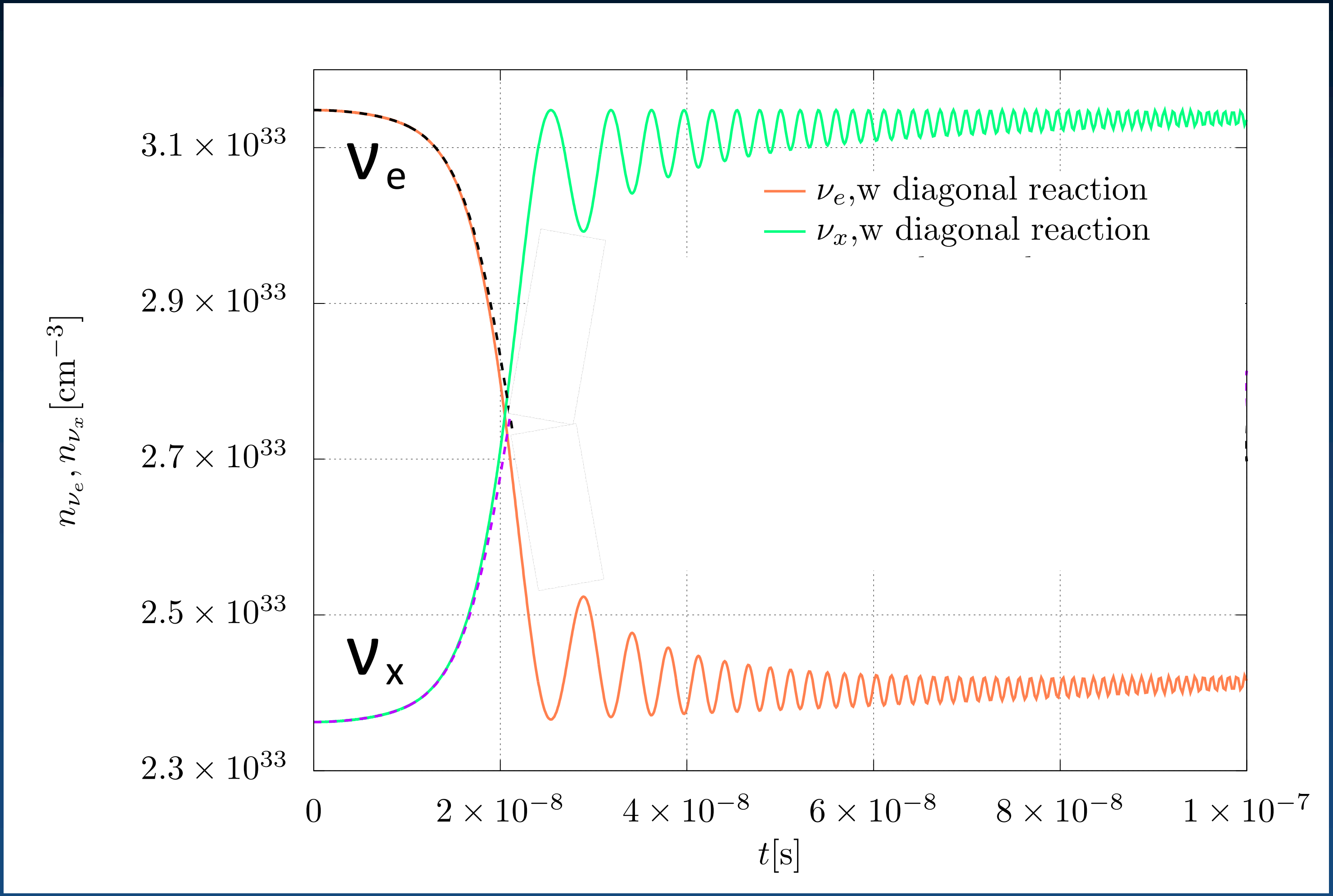
$P_3 = 0$: equipartition → Non-steady

$P_3 = -1$: heavy-lepton type → Stable

Neutrinos undergo flavor swaps in asymptotic states.

Collisional flavor swap
(associated with collisional instability)

Kato, Nagakura, and Johns 2024

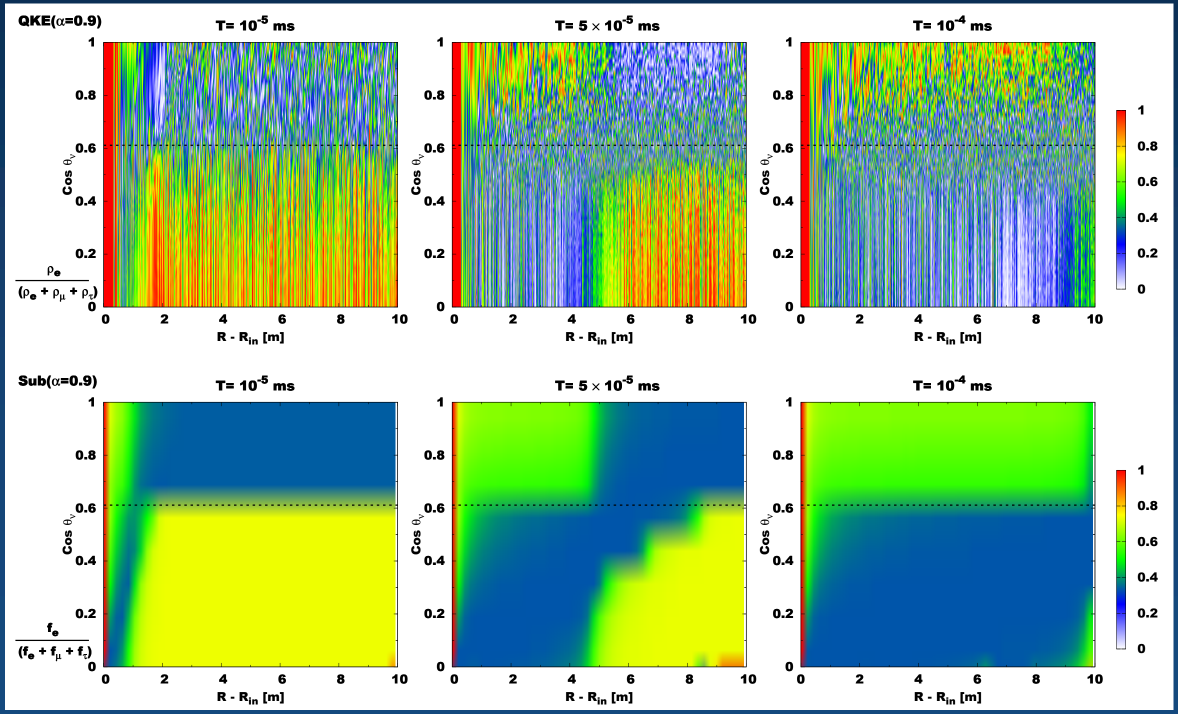


- BGK Subgrid model Nagakura et al. 2024

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + i p^\mu n_\mu [H, f]$$: Full QKE

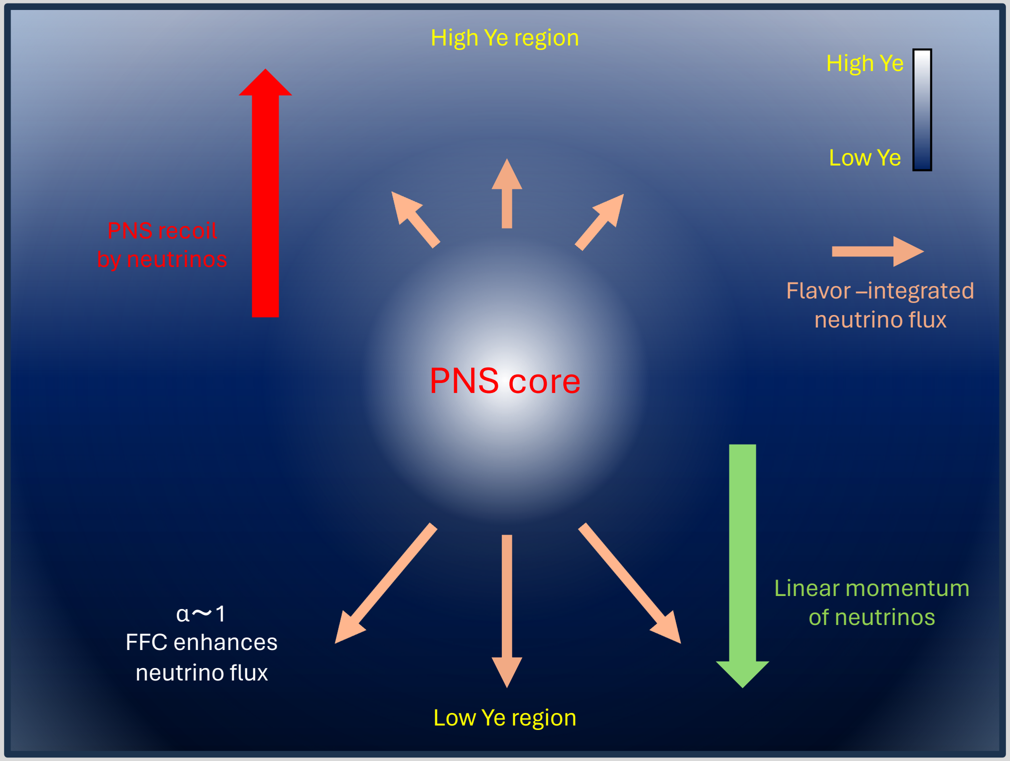
$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + p^\mu n_\mu \frac{1}{\tau_a}(f - f^a)$$: Relaxation-time approximation

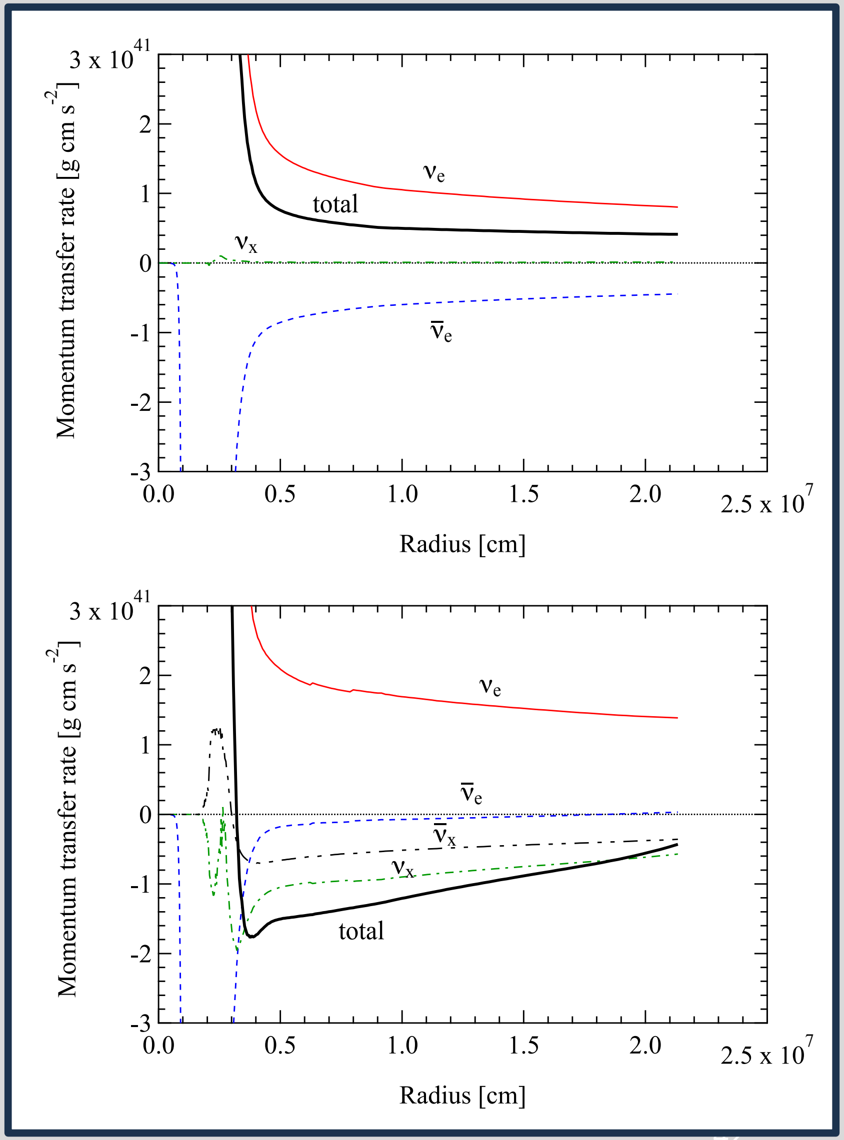
Radial-angular distributions for survival probability of electron-type neutrinos



- Neutron star kick powered by neutrino flavor conversions

Nagakura and Sumiyoshi 2024





Summary

- Radiation-hydrodynamic simulations under classical treatments of neutrino kinetics have been matured in CCSN theory.
- Collective neutrino oscillations, one of the quantum kinetics features of neutrinos, ubiquitously occur in CCSN and BNSM environments.
- Fast neutrino-flavor conversion (FFC) and collisional flavor instabilities potentially gives a huge impact on fluid-dynamics, nucleosynthesis, and neutrino signal.
- We developed a new GRQKNT code for time-dependent global simulations of neutrino quantum kinetics (QKE).
- Global simulations are currently available, that show qualitatively different features of flavor conversions from those found in local simulations.
- More discussions will be made for astrophysical consequences of flavor conversions.