強い場の物理と その天体物理への 応用の可能性

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2013年11月2日@千葉工大

plan

- Conceptual introduction to strong field physics and nonlinear QED
- A bit of technicalities
- Photons and hadrons in strong magnetic fields
- Possible application to astrophysics

Conceptual introduction to strong field physics and nonlinear QED effects

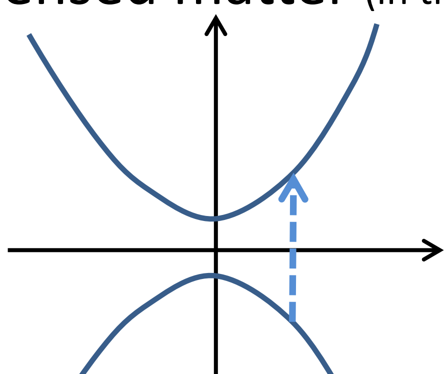
When is a field "strong"?

"strongness" depends on systems

- Consider an external field which couples to a system. Applying an external field drives the system into "excited" states.
- A field is called "strong" when its energy is much larger than typical excitation energy of the system (or "vacuum").
- "Critical field" is defined by the typical excitation energy. Thus we could define multiple critical fields.

ex) In QED vacuum: electron-positron excitation

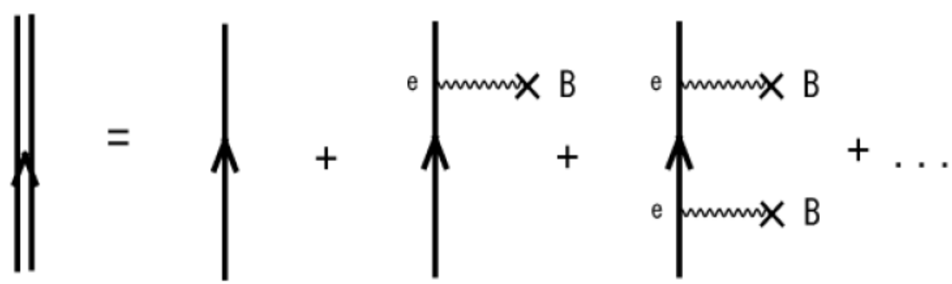
In condensed matter (in the presence of a gap) : electron-hole excitation



What is "strong field physics"?

- Characteristic phenomena that occur under **strong gauge fields** (EM fields and Yang-Mills fields)
- Typically, **weak-coupling** but **non-perturbative**

ex) electron propagator in a strong magnetic field



$$1 + O\left(\frac{eB}{m_e^2}\right) + O\left(\left(\frac{eB}{m_e^2}\right)^2\right)$$

must be resummed when $B >> B_c$

$$eB_c \equiv m_e^2$$

$$eE_c \sim m_e^2$$

Schwinger's critical field

→ **"Nonlinear QED"**

- **A new interdisciplinary field:** involving high-intensity LASER physics, hadron physics (heavy-ion physics), condensed matter physics (exciton), **astrophysics** (neutron stars, magnetars, early universe)

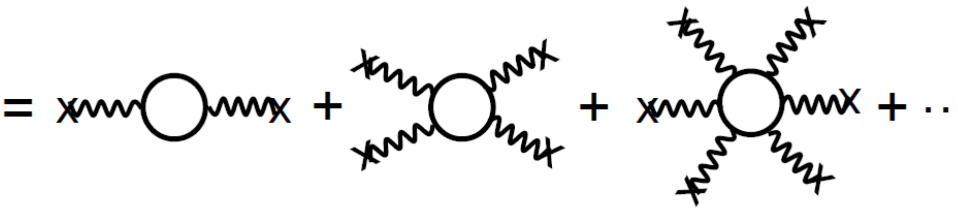
Nonlinear QED effects

- **Euler-Heisenberg action**

effective potential of constant EM fields


Z. Phys. 98, 714 (1936)
arXiv:physics/0605038

$$\mathfrak{L} = \frac{1}{2}(\mathfrak{E}^2 - \mathfrak{B}^2) + \frac{e^2}{\hbar c}\int_0^\infty e^{-\eta}\frac{d\eta}{\eta^3}\left\{ i\eta^2(\mathfrak{E}\mathfrak{B}) \cdot \frac{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2 - \mathfrak{B}^2 + 2i(\mathfrak{E}\mathfrak{B})}\right) + \text{konj}}{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2 - \mathfrak{B}^2 + 2i(\mathfrak{E}\mathfrak{B})}\right) - \text{konj}} + |\mathfrak{E}_k|^2 + \frac{\eta^2}{3}(\mathfrak{B}^2 - \mathfrak{E}^2)\right\}$$



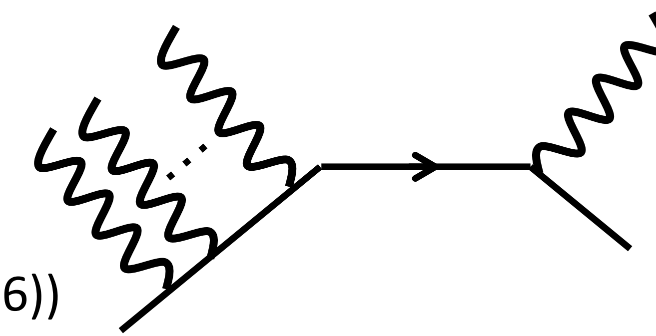
nonlinear w.r.t. *E* and *B*

- **Nonlinear Compton effects**

$e + n\gamma \rightarrow e' + m\gamma$

Multiple absorption of photons

(experimentally confirmed SLAC E144 (1996))



Nonlinear Compton scattering

VOLUME 76, NUMBER 17 PHYSICAL REVIEW LETTERS 22 APRIL 1996

E144 @ SLAC

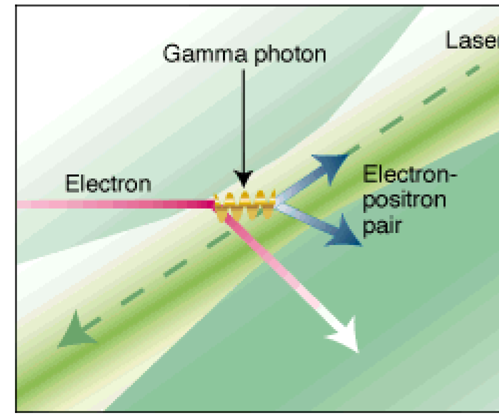
Observation of Nonlinear Effects in Compton Scattering

C. Bula, K. T. McDonald, and E. J. Prebys
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544

C. Bamber,* S. Boege, T. Kotseroglou, A. C. Melissinos, D. D. Meyerhofer,† and W. Ragg
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

D. L. Burke, R. C. Field, G. Horton-Smith, A. C. Odian, J. E. Spencer, and D. Walz
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

S. C. Berridge, W. M. Bugg, K. Shmakov, and A. W. Weidemann
Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennesee 37996
(Received 4 December 1995)



$$e + n\omega_0 \longrightarrow e' + \omega \; +m\omega_0$$

e-e+

Phys. Rev. Lett., 79,1626 (1997)

Electron energy 46.6 GeV
Laser Nd:glass 1054 and 527 nm
Peak intensity 10^{18} W/cm2
Measured up to n=4

これらはQCDでは常識的な反応。Initial state radiationとFinal state radiationとして常に考えられている

Why strong field physics?

- **Because there exist in Nature**

  Early Universe, Compact objects in universe, high-energy scattering, etc, etc.

- **Because we can learn something about "vacuum"**

  Vacuum has nontrivial structure in QFT whose information can be extracted by using strong fields.

- **Because it is a special tractable case of non-equilibrium physics** (can be formulated in weak-coupling theory)

  We can study a class of "non-equilibrium physics" which in general covers a broad range of phenomena

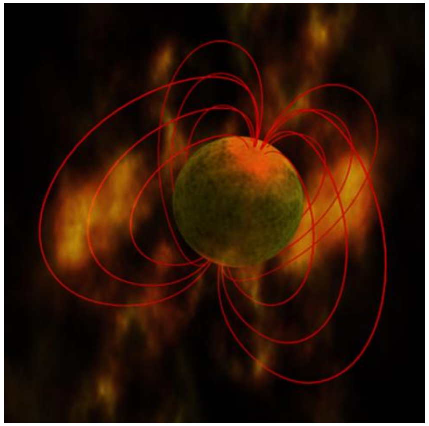
- **Because it is a kind of new <u>universal</u> picture of extreme states in Nature**

  It seems that extreme phenomena look very similar to each other even if they appear in completely different scales.

How strong?

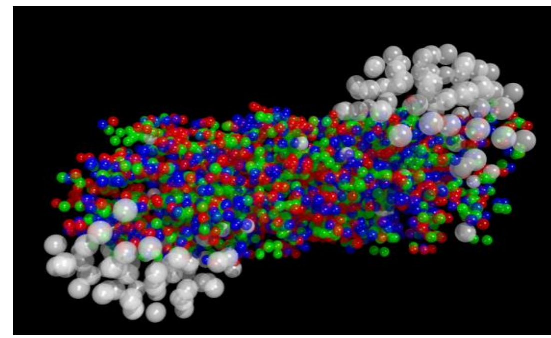
1 Tesla = 10^4 Gauss

10^{17}—10^{18} Gauss
$\sqrt{eB}$ ~ 1 – 10 m_π:
Noncentral heavy-ion coll. at RHIC and LHC
Also strong Yang-Mills fields $\sqrt{gB}$ ~ 1– a few GeV



10^{15}Gauss : Magnetars

4×10^{13} Gauss : "Critical" magnetic field of electrons
$\sqrt{eB_c} = m_e = 0.5$MeV



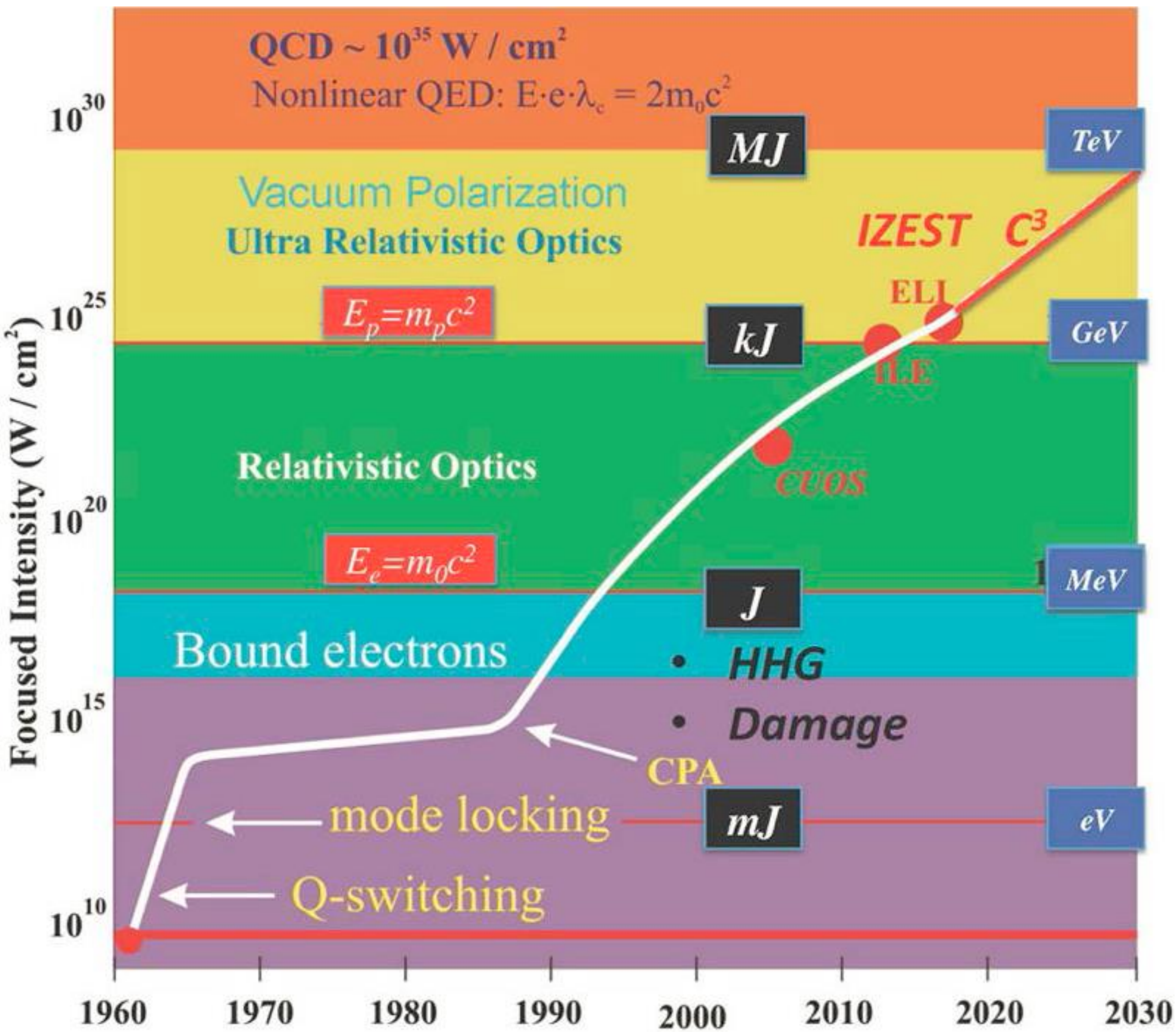
45 Tesla : strongest steady magnetic field
(High Mag. Field. Lab. In Florida)

10^8Tesla=10^{12}Gauss:
Typical neutron star surface

Super critical magnetic field may have existed in very early Universe. Maybe after EW phase transition? (cf: Vachaspati '91)

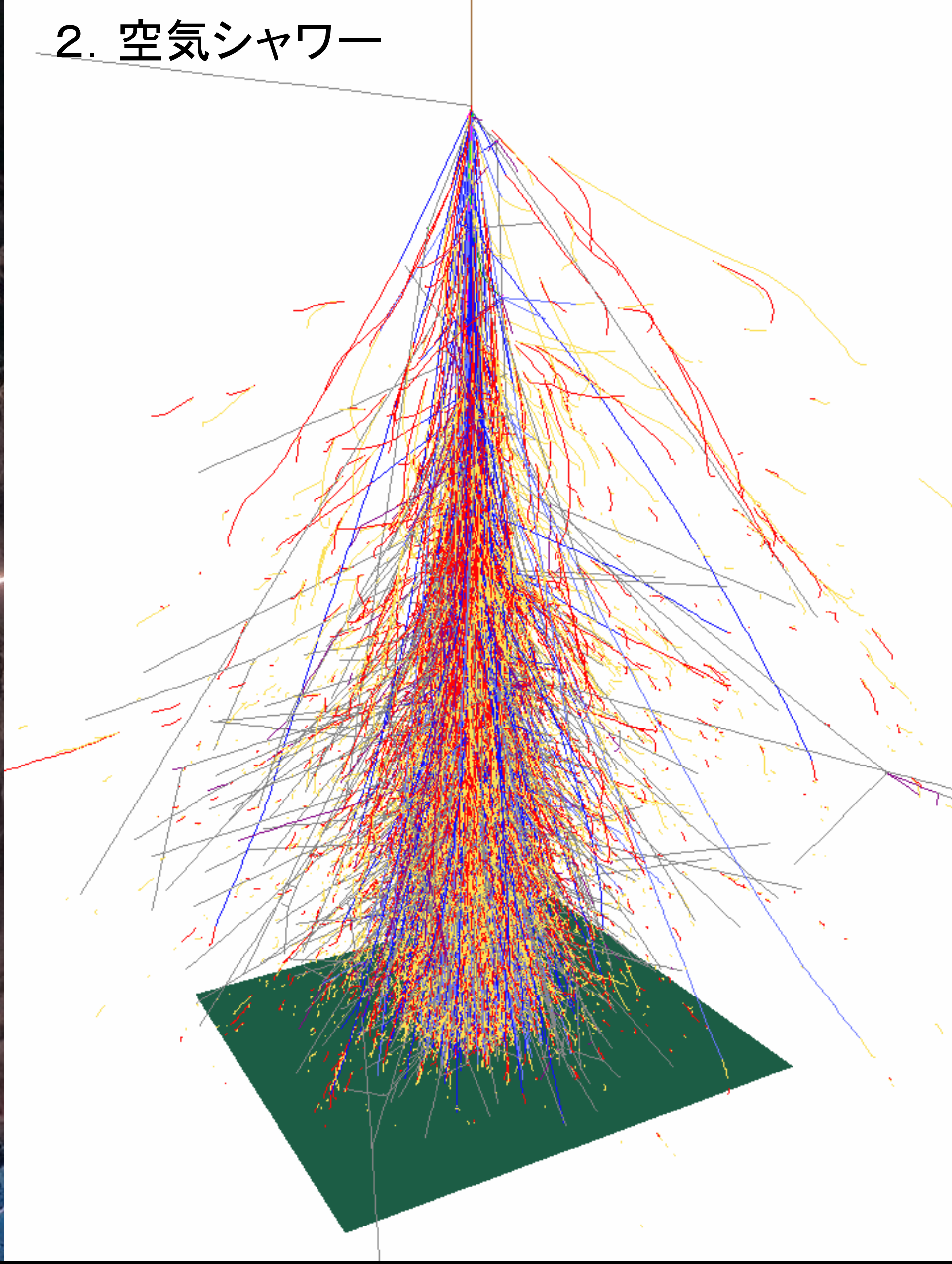
8.3 Tesla :
Superconducting magnets in LHC

Development of high-intensity laser



GEKKO-EXA (Japan)
XFEL, POLARIS, NIF, etc

Mourou Tajima



1. 空気（絶縁体）にかかった高電位差が、電子の雪崩的な生成に伴う雷で解消する。
2. 高エネルギーの粒子が大気中の原子核に衝突し、生成粒子がさらに粒子を放出。

現代物理学における「真空」

素粒子の標準模型は「場の量子論」で定義

場の量子論　＝　空間の各点に付随する「場」が振動し、
それが空間全体に伝播する力学を記述

「真空」　＝　その系の最低エネルギー状態であり、系の対称性を
反映して「選ばれる」もの。一般に非自明な「構造」を
持ち、常に「ゆらぎ」がある。

真空状態を理解することは世界を理解することの第一歩

物性物理でも同様。最低エネルギー状態とその励起の理解

例）「Higgs粒子」　は　Higgs場の対称性が破れた
真空（Higgs場が凝縮している）からの揺らぎ・励起

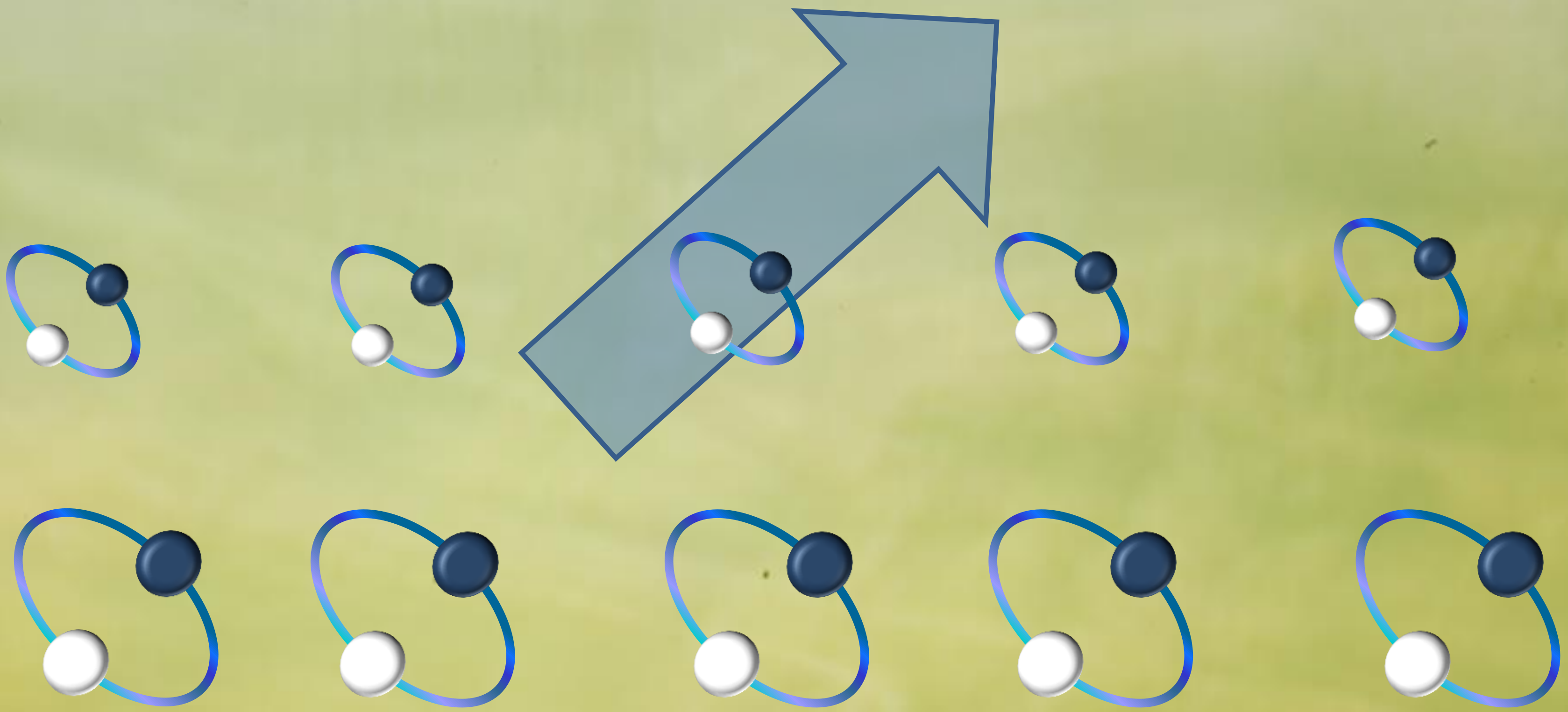
量子電磁力学 (Quantum ElectroDynamics, QED) ・・・ 光、電子、陽電子の世界

絶えず粒子・反粒子対生成と消滅が揺らぎとして起こっている

絶えず粒子・反粒子対生成と消滅が揺らぎとして起こっている

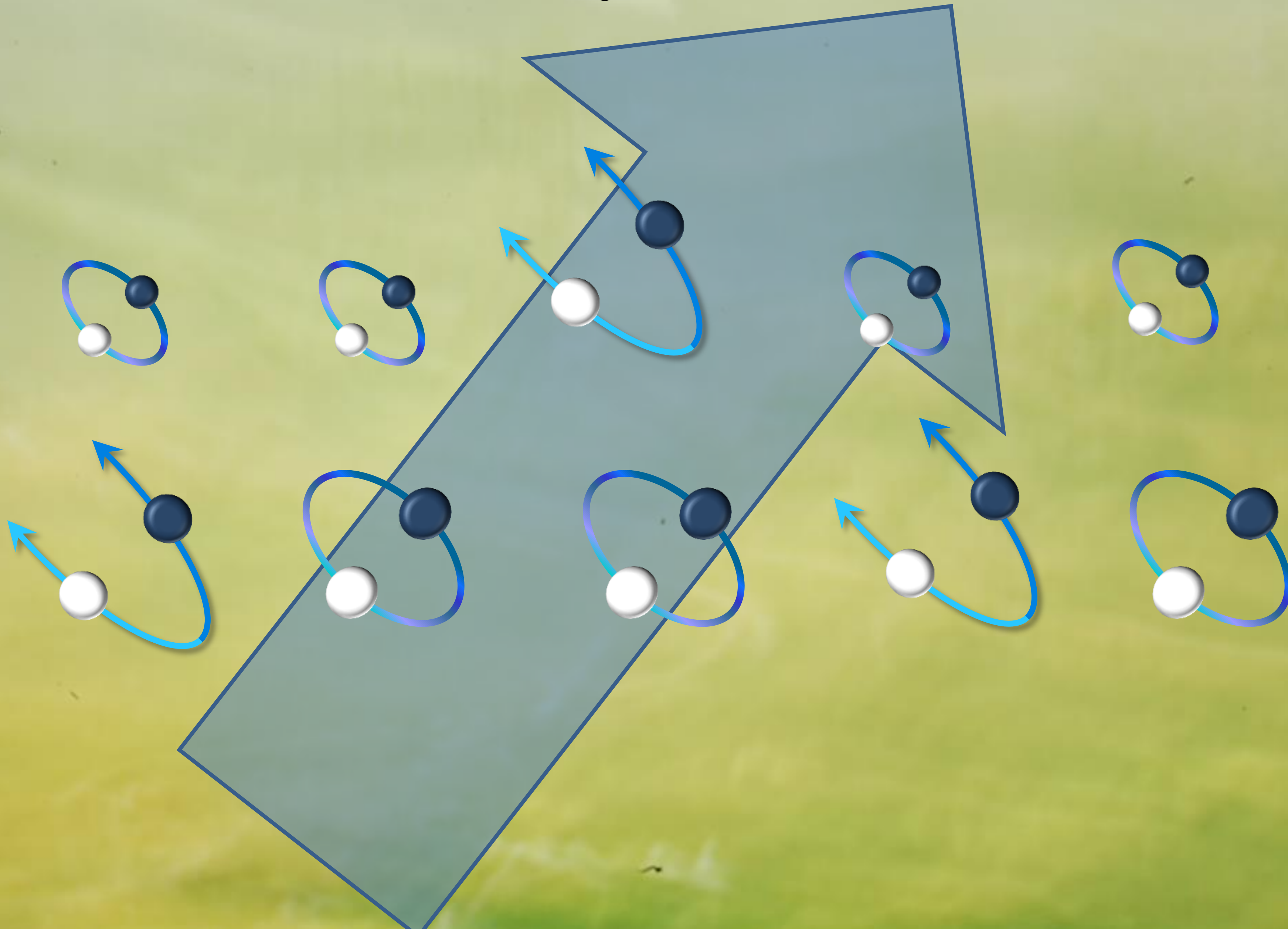
絶えず粒子･反粒子対生成と消滅が揺らぎとして起こっている

電場・磁場をかけると揺らぎが「そろう」 → 全体で「コヒーレント」な動作をする

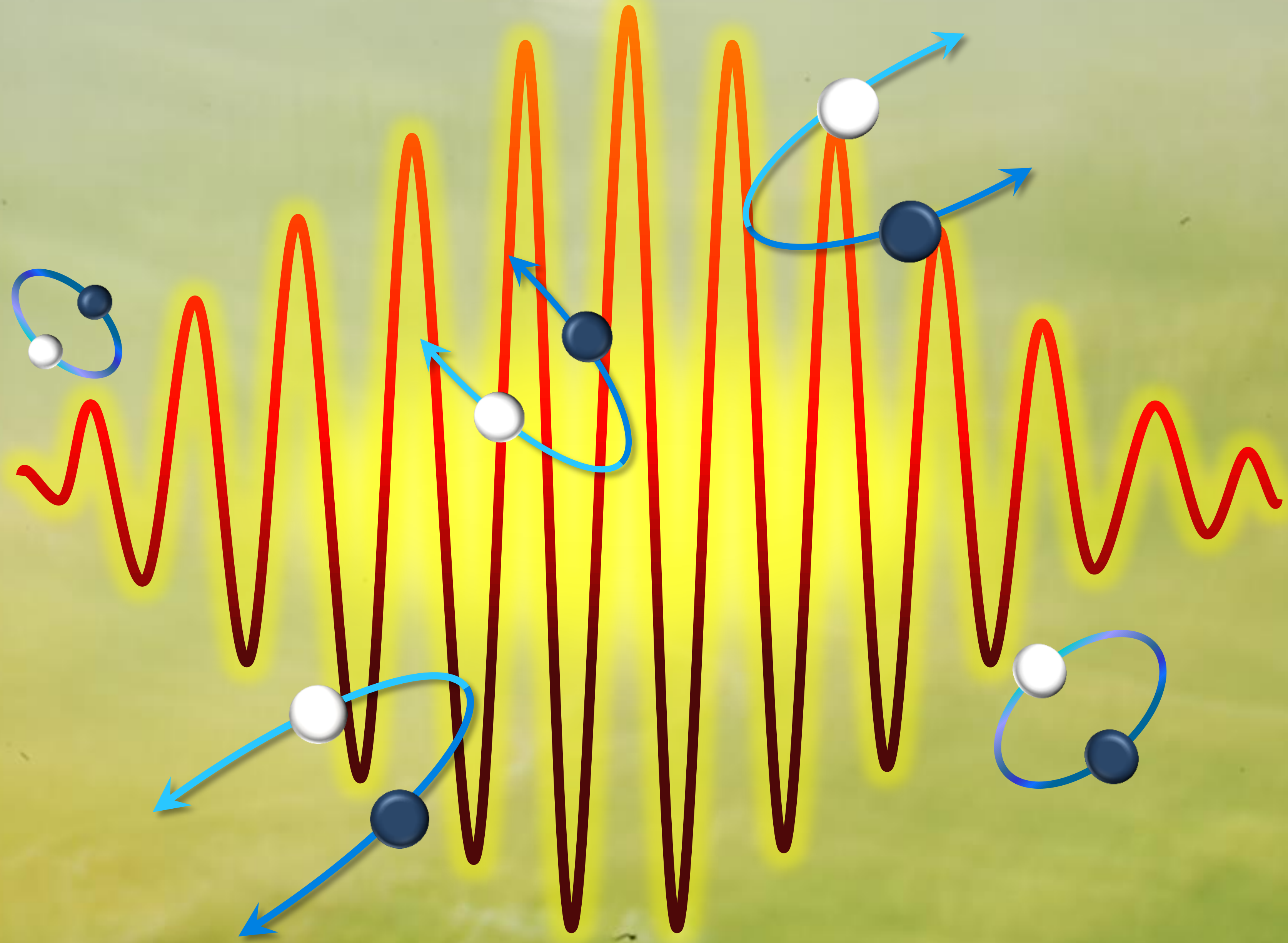


電場を強くすると、粒子・反粒子対が元に戻らず、粒子と反粒子が生まれる

→　真空の「崩壊」　Schwinger機構

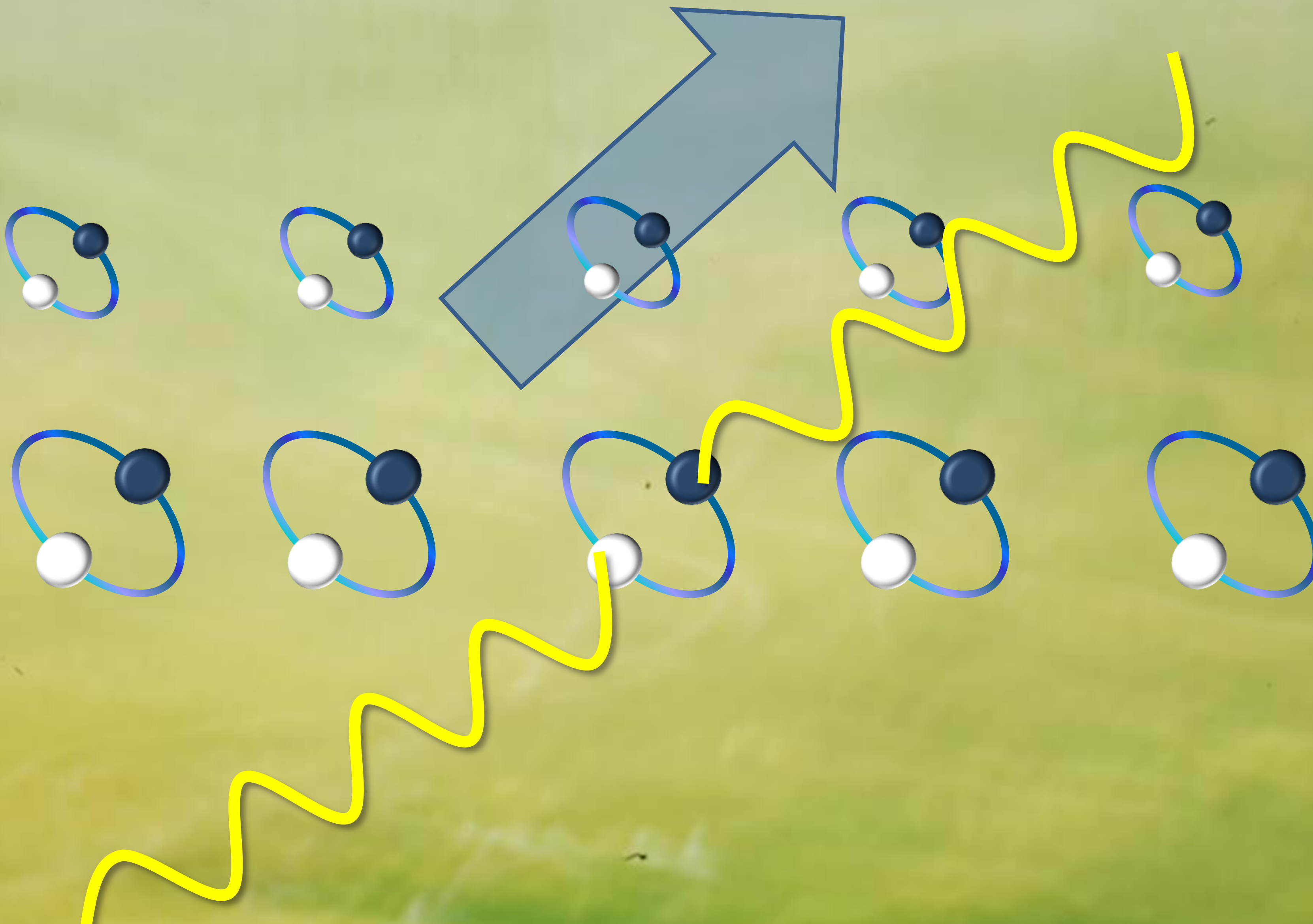


超強力なレーザーを用いて、Schwinger機構を実験的に検証しつつある



「コヒーレント」にそろったゆらぎに外から光が入射すると、光の性質が変化する

cf) exiton-polariton



高強度場を課して真空を探る

- 「真空」では絶え間ない粒子・反粒子対生成・消滅の繰り返し

- 電場や磁場などをかけると揺らぎが「揃い」、さらに電場を強くすることで、ゆらぎを構成する粒子対を実体化させることができる

  真空の崩壊　　　**Schwinger**機構

- 高強度レーザーを用いることで、実験的に検証が可能

- このような「構造が揃った真空」中を伝播する光子は
  性質を変える

  強磁場中では光の速度が「遅く」なり（屈折率の変化）、
  高いエネルギーの光は、電子・陽電子に崩壊してしまう

A bit of technicalities

Volkov solution, Furry picture, …

Furry Picture

$$\mathcal{L}^{\text{Int}}_{\text{QED}} = \bar\psi(i\partial\!\!\!/ - m)\psi - \tfrac{1}{4}(F_{\mu\nu})^2 - e\bar\psi(A\!\!\!/^{\text{ext}} + A\!\!\!/)\,\psi$$

$$\mathcal{L}^{\text{FP}}_{\text{QED}} = \bar\psi^{\text{FP}}(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} - \tfrac{1}{4}(F_{\mu\nu})^2 - e\bar\psi^{\text{FP}}A\!\!\!/\,\psi^{\text{FP}}$$

Equations of Motion

$$(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} = 0$$

Wavefunction

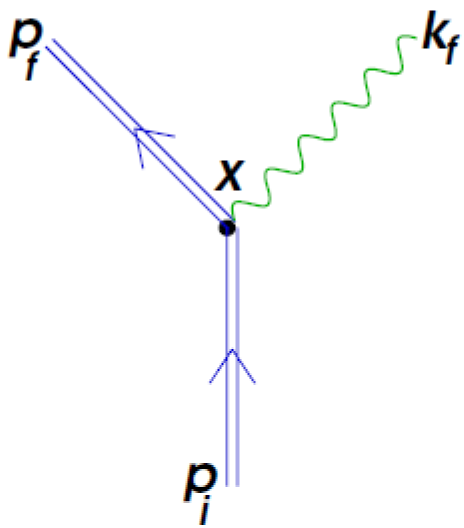
$$\psi^{\text{FP}} = E_p\, e^{-ipx}\, u_p, \quad E_p = \exp\left[-\frac{1}{2(k\cdot p)}\left(eA\!\!\!/^{\text{ext}}k\!\!\!/ + i2e(A^e p) - ie^2 A^{\text{ext}\,2}\right)\right]$$

Propagator

$$G^{\text{FP}} = \int \frac{d^4p}{(2\pi)^4} E_p(x)\frac{p\!\!\!/ + m}{p^2 - m^2}\bar E_p(x')\, e^{-ip\cdot x}\, u_p$$

Slides by A.Hartin @ PIF2013

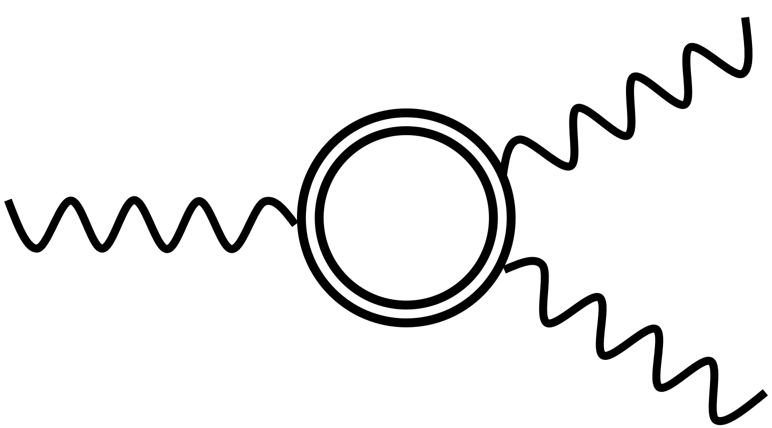
Feynman rules in Furry picture



- double fermion lines are Volkov-type solutions
- Volkov E_p functions can be grouped around the vertex
- Only need one new Feynman picture element - the dressed vertex

$$\gamma_\mu^{\mathrm{FP}}(p_f, p_i, kx) = \int d^4x\, \bar{E}_{p_f}(x)\gamma_\mu E_{p_i}(x)\, e^{i(p_f - p_i + k_f)\cdot x}$$

Note: Furry's theorem does not work in Furry picture



Fermion loops with odd number of external lines survive

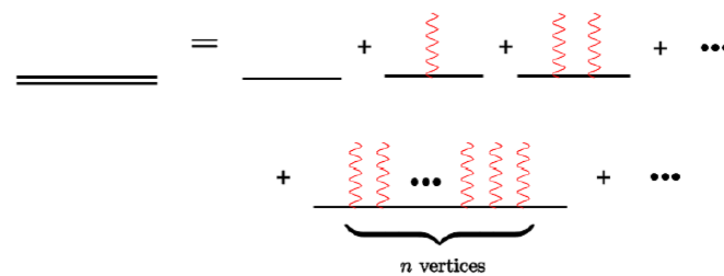
Proper-time method

J. S. Schwinger, Phys. Rev. **82** (1951) 664-679

Electron propagator in external EM field

$$G(p|A_{\rm cl}) = \frac{i}{\not{p} - e\not{A}_{\rm cl} - m}$$

$$= \frac{i}{\not{p} - m}\sum_{n=0}^{\infty}\left[\,(-ie\not{A}_{\rm cl})\frac{i}{\not{p} - m}\,\right]^n$$



can be equivalently rewritten as

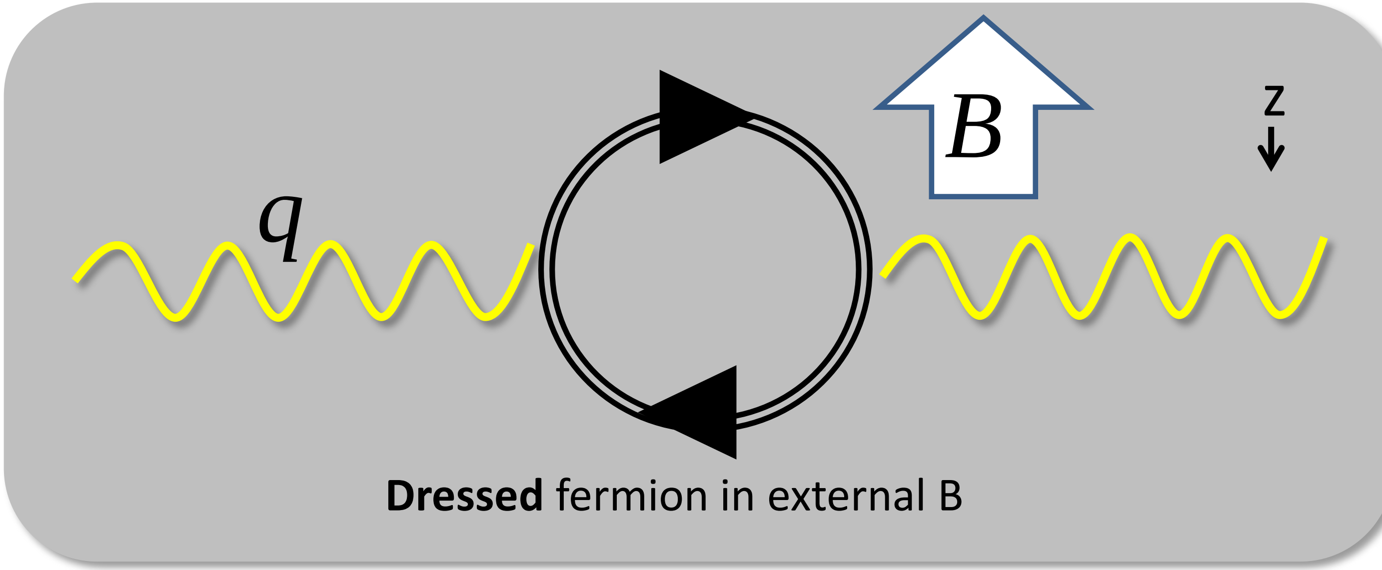
$$G(p|A_{\rm cl}) = i\left(\not{p} - e\not{A}_{\rm cl} + m\right) \times \frac{1}{i}\int_0^{\infty} d\hat{\tau}\ e^{i\hat{\tau}\left\{(\not{p}-e\not{A}_{\rm cl})^2-(m^2-i\varepsilon)\right\}}$$

τ: proper time

This form can incorporate <u>all order</u> contributions w.r.t. external field

Photons and hadrons in strong magnetic fields

Photons in strong magnetic fields



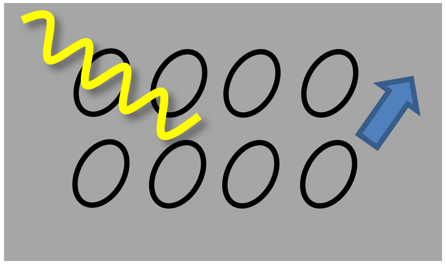
- **Properties of a photon propagating in a magnetic field**

  ← vacuum polarization tensor $\Pi^{\mu\nu}(q,B)$

- **Old but new problem** [Weisscopf 1936, Baier-Breitenlohner 1967, Narozhnyi 1968, Adler 1971]

  - Polarization tensor $\Pi^{\mu\nu}(q,B)$ has been known in *integral* form
  - Analytic representation obtained very recently [Hattori-Itakura 2013]

Magnetic vacuum as a media



Propagating photon in strong magnetic field

= probing magnetic vacuum "polarized" by external fields

~ photon couples to virtual excitation of vacuum (cf: exciton-polariton)

B dependent anisotropic response of a fermion (Landau levels)

- discretized transverse vs unchanged longitudinal motion
  - → Two different refractive indices : ***VACUUM BIREFRINGENCE***
- energy conservation gets modified
  - → Pol. Tensor can have imaginary part : ***PHOTON DECAY INTO e+e- PAIR***

(lots of astrophysical applications)

$$\Pi^{\mu\nu}_{\rm ex}(q) = \chi_0(q^2\eta^{\mu\nu} - q^\mu q^\nu) + \chi_1(q_\parallel^2\eta_\parallel^{\mu\nu} - q_\parallel^\mu q_\parallel^\nu) + \chi_2(q_\perp^2\eta_\perp^{\mu\nu} - q_\perp^\mu q_\perp^\nu)$$

present only in external fields

|| parallel to B

⊥ transverse to B

$$\eta_\parallel^{\mu\nu} = diag(1,0,0,-1)$$

$$\eta_\perp^{\mu\nu} = diag(0,-1,-1,0)$$

$$q^\mu = (q^0, q_\perp, 0, q^3)$$

$$q_\parallel^\mu = (q^0, 0, 0, q^3)$$

$$q_\perp^\mu = (0, q_\perp, 0, 0)$$

Vacuum birefringence

- **Maxwell eq. with the polarization tensor :**

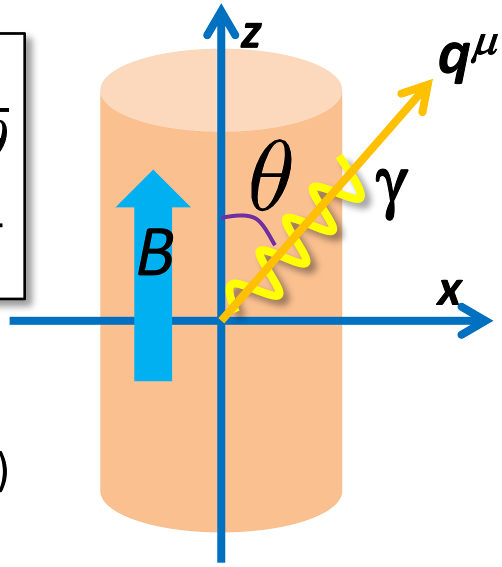
$$\left(q^2\eta^{\mu\nu} - q^\mu q^\nu + \hat{\Pi}^{\mu\nu}_{\rm ex}\right) A_\nu(q) = 0$$

$$\Pi^{\mu\nu}_{\rm ex}(q) = \chi_0(q^2\eta^{\mu\nu} - q^\mu q^\nu) + \chi_1(q^2_\parallel \eta^{\mu\nu}_\parallel - q^\mu_\parallel q^\nu_\parallel) + \chi_2(q^2_\perp \eta^{\mu\nu}_\perp - q^\mu_\perp q^\nu_\perp)$$

- **Dispersion relation of two physical modes gets modified**

→ **Two refractive indices : "Birefringence"**

$$n^2 \equiv \frac{|\boldsymbol{q}|^2}{\omega^2} \Rightarrow \begin{cases} n_1^2 = \dfrac{1+\chi_0+\chi_1}{1+\chi_0+\chi_1\cos^2\theta} \\ n_2^2 = \dfrac{1+\chi_0}{1+\chi_0+\chi_2\sin^2\theta} \end{cases}$$



1. Compute χ_0, χ_1, χ_2 analytically at the one-loop level

Hattori-Itakura Ann. Phys. 330 (2013)

2. Solve them self-consistently w.r.t *n in LLL approx.*

Hattori-Itakura Ann. Phys. 334 (2013)

Analytic representation of $\Pi^{\mu\nu}(q,B)$

Representation in double integral w.r.t. proper times

$$\chi_i(r_\parallel^2, r_\perp^2; B_\mathrm{r}) = \frac{\alpha}{4\pi}\int_{-1}^{1} d\beta \int_0^\infty d\tau \, \frac{\Gamma_i(\tau,\beta)}{\sin\tau} \, \mathrm{e}^{-iu\cos(\beta\tau)} \, \mathrm{e}^{i\eta\cot\tau} \, \mathrm{e}^{-i\phi_\parallel\tau} \, ,$$

$$B_\mathrm{r} = B/B_c, \; r_\parallel^2 = \frac{q_\parallel^2}{4m^2} \text{ and } r_\perp^2 = \frac{q_\perp^2}{4m^2} = -\frac{|q_\perp|^2}{4m^2}$$

$$\eta \equiv -2r_\perp^2/B_\mathrm{r} \text{ and } u \equiv \eta/\sin\tau$$

$$\phi_\parallel(r_\parallel^2, B_\mathrm{r}) = \frac{1}{B_\mathrm{r}}\left\{ 1 - (1-\beta^2)\, r_\parallel^2 \right\} ,$$

$$\Gamma_0(\tau,\beta) = \cos(\beta\tau) - \beta\sin(\beta\tau)\cot\tau \, ,$$

$$\Gamma_1(\tau,\beta) = (1-\beta^2)\cos\tau - \Gamma_0(\tau,\beta) \, ,$$

$$\Gamma_2(\tau,\beta) = 2\frac{\cos(\beta\tau) - \cos\tau}{\sin^2\tau} - \Gamma_0(\tau,\beta) \quad .$$

Analytic representation of $\Pi^{\mu\nu}(q,B)$

$$\chi_i = \frac{\alpha B_{\rm r}}{4\pi} \sum_{n=0}^{\infty} (2-\delta_{n0}) \left[\sum_{\ell=0}^{\infty} \Omega_{\ell i}^{n(0)} + \sum_{\ell=1}^{\infty} \Omega_{\ell i}^{n(1)} + \sum_{\ell=2}^{\infty} \Omega_{\ell i}^{n(2)} \right] ,$$

$$\begin{cases}
\Omega_{\ell 0}^{n(0)} = (1-\delta_{n0})\, C_\ell^{n-1}(\eta)\, F_\ell^n(\xi, B_{\rm r}) - n\eta^{-1}\, C_\ell^n(\eta)\, G_\ell^n(\xi, B_{\rm r}) , \\
\Omega_{\ell 0}^{n(1)} = (1+\delta_{n0})\, C_{\ell-1}^{n+1}(\eta)\, F_\ell^n(\xi, B_{\rm r}) - n\eta^{-1}\, C_{\ell-1}^n(\eta)\, G_\ell^n(\xi, B_{\rm r}) , \\
\Omega_{\ell 0}^{n(2)} = 0 . \\
\Omega_{\ell 1}^{n(0)} = C_\ell^n(\eta) \{ F_\ell^n(\xi, B_{\rm r}) - H_\ell^n(\xi, B_{\rm r}) \} - \Omega_{\ell 0}^{n(0)} , \\
\Omega_{\ell 1}^{n(1)} = C_{\ell-1}^n(\eta) \{ F_\ell^n(\xi, B_{\rm r}) - H_\ell^n(\xi, B_{\rm r}) \} - \Omega_{\ell 0}^{n(1)} , \\
\Omega_{\ell 1}^{n(2)} = 0 , \\
\Omega_{\ell 2}^{n(0)} = -\Omega_{\ell 0}^{n(0)} , \\
\Omega_{\ell 2}^{n(1)} = D_\ell^{n(1)}(\eta)\, F_\ell^n(\xi, B_{\rm r}) - \Omega_{\ell 0}^{n(1)} , \\
\Omega_{\ell 2}^{n(2)} = D_\ell^{n(2)}(\eta)\, F_\ell^n(\xi, B_{\rm r}) .
\end{cases}$$

$$D_\ell^{n(1)}(\eta) = -8 \sum_{\lambda=0}^{\ell-1} (\ell-\lambda) \{ (1-\delta_{n0})\, C_\lambda^{n-1}(\eta) - C_\lambda^n(\eta) \} ,$$

$$D_\ell^{n(2)}(\eta) = -8 \sum_{\lambda=0}^{\ell-2} (\ell-\lambda-1) \{ (1+\delta_{n0})\, C_\lambda^{n+1}(\eta) - C_\lambda^n(\eta) \} .$$

$$C_\ell^n(\eta) \equiv {\rm e}^{-\eta} \frac{\ell!}{(\ell+n)!} \eta^n \left[L_\ell^n(\eta) \right]^2 .$$

$$F_\ell^n(r_\parallel^2, B_{\rm r}) = \int_{-1}^{1} \frac{d\beta}{r_\parallel^2 \beta^2 - nB_{\rm r}\beta + (1-r_\parallel^2) + (2\ell+n)B_{\rm r}} \equiv I_{\ell\Delta}^n(r_\parallel^2)$$

$$G_\ell^n(r_\parallel^2, B_{\rm r}) = \frac{1}{2r_\parallel^2} \left[\Xi_\ell^n(B_{\rm r}) + nB_{\rm r}\, I_{\ell\Delta}^n(r_\parallel^2) \right] ,$$

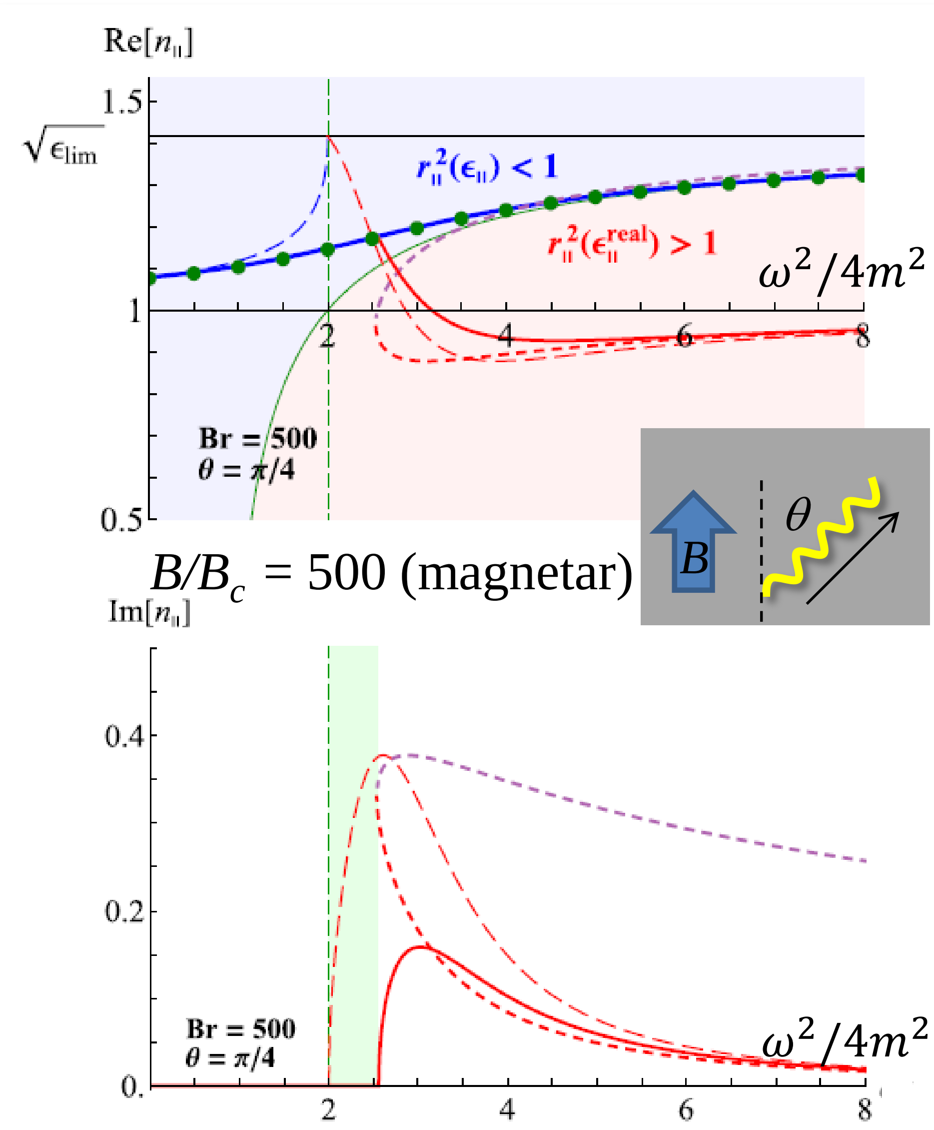
$$H_\ell^n(r_\parallel^2, B_{\rm r}) = \frac{1}{r_\parallel^2} \left[2 + \frac{nB_{\rm r}}{2r_\parallel^2}\, \Xi_\ell^n(B_{\rm r}) + \frac{1}{4r_\parallel^2} \left\{ (b^2-4ac) + (nB_{\rm r})^2 \right\} I_{\ell\Delta}^n(r_\parallel^2) \right] ,$$

$$\Xi_\ell^n(B_{\rm r}) \equiv \ln \left| \frac{1+2\ell B_{\rm r}}{1+2(\ell+n)B_{\rm r}} \right| = \ln \left| \frac{m^2+2\ell eB}{m^2+2(\ell+n)eB} \right|$$

$$I_{\ell\Delta}^n(r_\parallel^2) = \begin{cases}
\frac{1}{\sqrt{(r_\parallel^2-s_-^{\ell n})(r_\parallel^2-s_+^{\ell n})}} \cdot \frac{1}{2} \ln \left| \frac{a-c-\sqrt{b^2-4ac}}{a-c+\sqrt{b^2-4ac}} \right| & (r_\parallel^2 < s_-^{\ell n}) \\
\frac{1}{\sqrt{|(r_\parallel^2-s_-^{\ell n})(r_\parallel^2-s_+^{\ell n})|}} \left[\arctan\left(\frac{b+2a}{\sqrt{4ac-b^2}} \right) - \arctan\left(\frac{b-2a}{\sqrt{4ac-b^2}} \right) \right] & (s_-^{\ell n} < r_\parallel^2 < s_+^{\ell n}) \\
\frac{1}{\sqrt{(r_\parallel^2-s_-^{\ell n})(r_\parallel^2-s_+^{\ell n})}} \cdot \frac{1}{2} \left[\ln \left| \frac{a-c-\sqrt{b^2-4ac}}{a-c+\sqrt{b^2-4ac}} \right| + 2\pi i \right] & (s_+^{\ell n} < r_\parallel^2) .
\end{cases}$$

- Infinite summation w.r.t. n and l = summation over two Landau levels
- Numerically confirmed by Ishikawa, et al. arXiv:1304.3655 [hep-ph]
- couldn't find the same results starting from propagators with Landau level decomposition

Refractive index



- **Need to self-consistently solve the equation** (effects of back-reaction)
- **Use LLL solution for simplicity**
  → $\chi_0 = \chi_2 = 0, \chi_1 \neq 0$

$$n_{\parallel}^2 = \frac{1+\chi_1}{1+\chi_1 \cos^2\theta}, \quad \chi_1 = \chi_1(q_{\parallel}^2, q_{\perp}^2, B)$$

$$n_{\perp}^2 = 1$$

$$\begin{cases} q_{\parallel}^2 = \omega^2 - q_z^2 = \omega^2(1 - n_{\parallel}^2 \cos^2\theta) \\ q_{\perp}^2 = -|q_{\perp}|^2 = -\omega^2 n_{\parallel}^2 \sin^2\theta \end{cases}$$

- **Refractive index $n_{||}$ deviates from 1 and increases with increasing ω**

  cf: air n = 1.0003, water n = 1.333

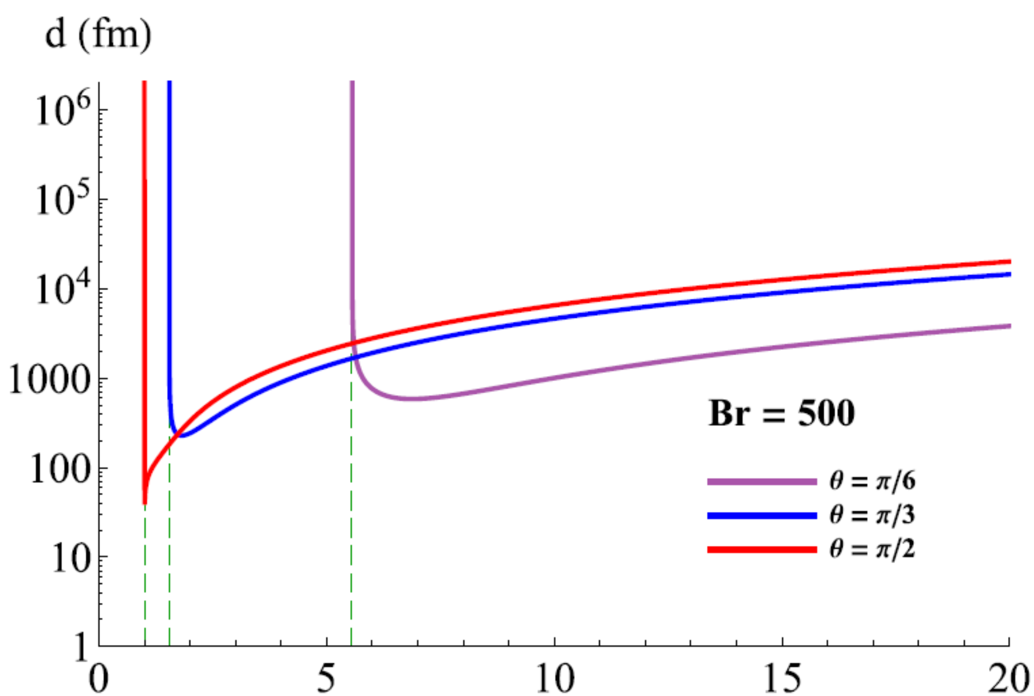
- **New branch at high energy is accompanied by an imaginary part**
  → decay into an e+e- pair

Decay length

Amplitude of an incident photon decays exponentially characterized by the decay length

$$d \equiv \frac{1}{2\omega\kappa} = \frac{1}{2\omega n_{\rm imag}}.$$

Surviving length ~ life time



$\omega^2/4m^2$

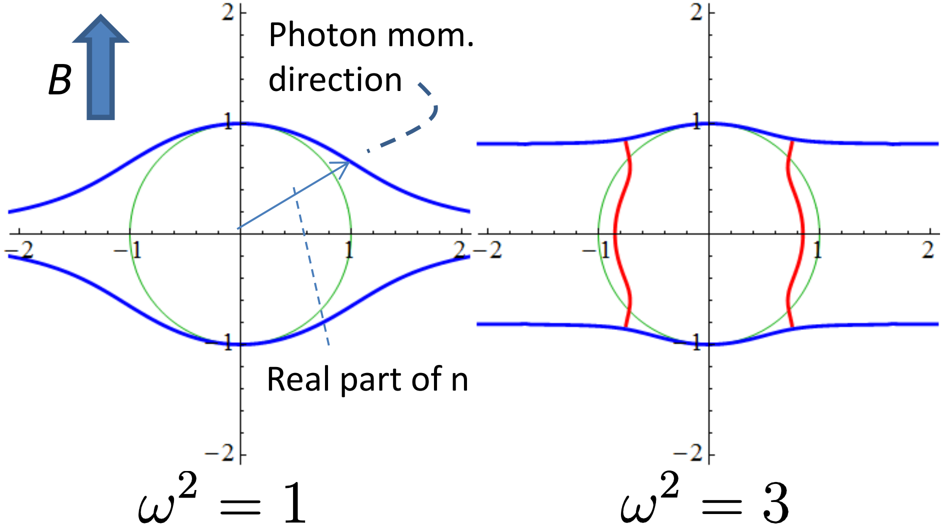
Very short length
→ relevant for magnetars

Even shorter in HIC
→ relevant for very soft photons generating anisotropic distribution

Angle dependence at various photon energies

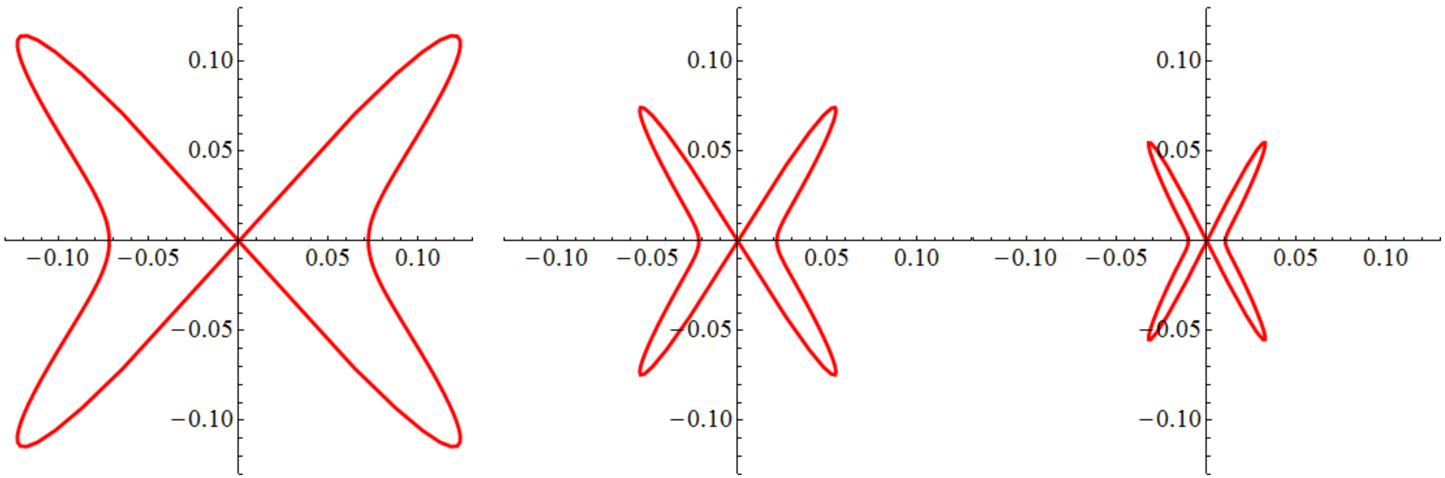
For magnetars → $B_\mathrm{r} = 500$

Real part



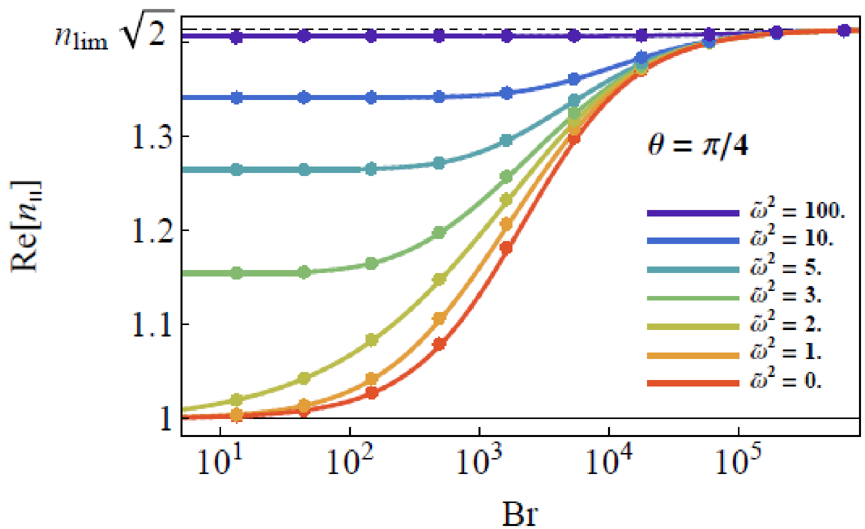
Imaginary part

No imaginary part

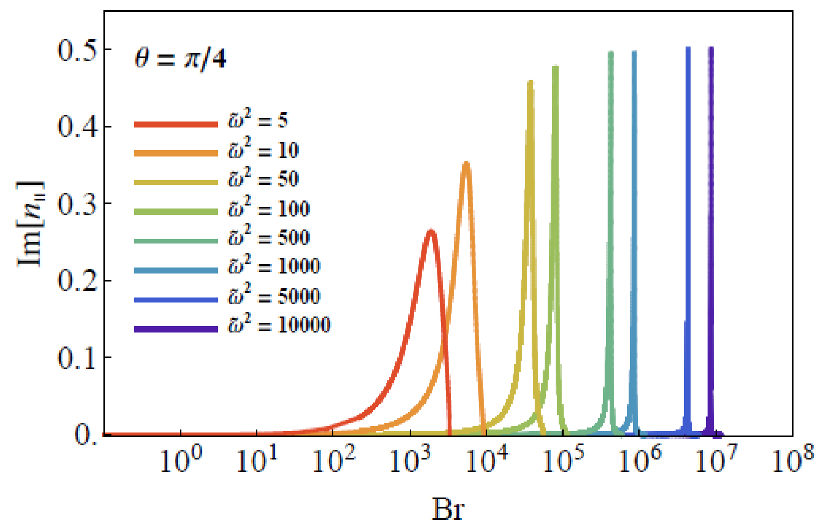


Effects are stronger with stronger magnetic fields or higher energy photons

Refractive index



Photon decay



Br=*B*/*B*c =
O(10^2) at magnetars
O(10^5) at RHIC

$\tilde{\omega}^2$=10000 ←→ ω = 200MeV

$$n_{\lim} = \sqrt{\epsilon_{\lim}} = |\cos\theta|^{-1}$$

Hadrons in strong B

- 磁場中でのdispersion relationの変化

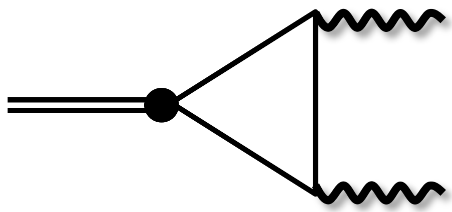
  最低エネルギー「有効質量」の変化

  崩壊モードの変化

- 全く新しい崩壊モードの出現

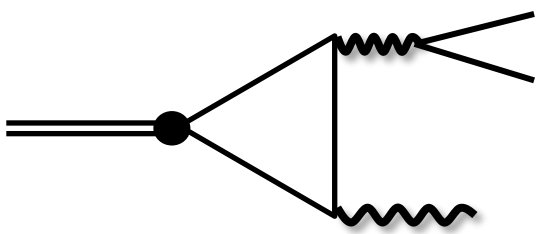
Neutral pion decay

- **Chiral anomaly** induces π^0 decay through triangle diagram



$\pi^0 \to 2\gamma : \mathcal{O}\left(e^2\right)$

Dominant (98.798 % in vacuum)



$\pi^0 \to \gamma + e^+e^- : \mathcal{O}\left(e^3\right)$

Dalitz decay (1.198 % in vacuum)
NLO contribution

99.996 %

- **Adler-Bardeen's theorem**

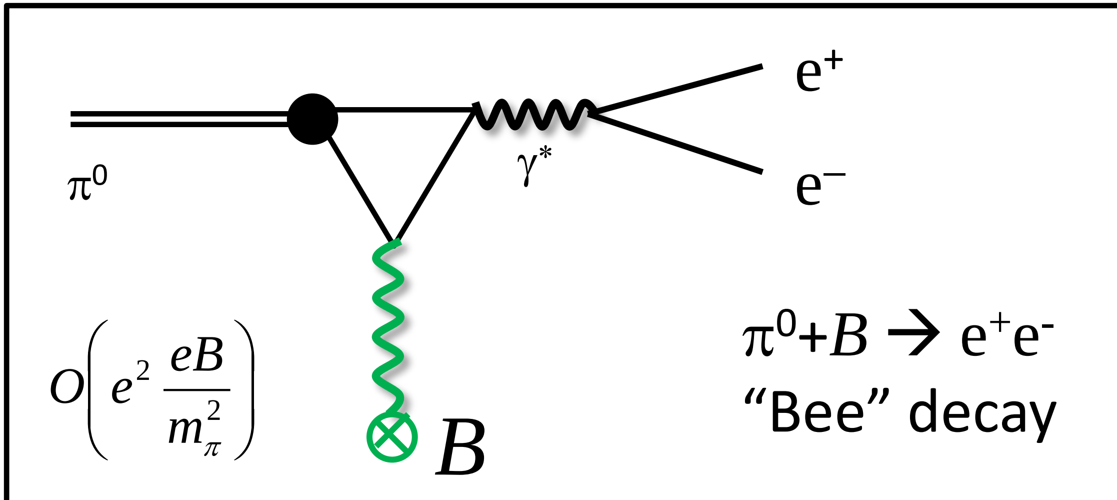
There is no radiative correction to the triangle diagram
Triangle diagram gives the exact result in all-order perturbation theory

→ only two photons can couple to π^0

Neutral pions in strong B

Hattori , KI, Ozaki, arXiv:1305.7224[hep-ph]

- There is only one diagram for a constant external field to be attached



cf: axion
(very light, but small coupling)

- Also implies

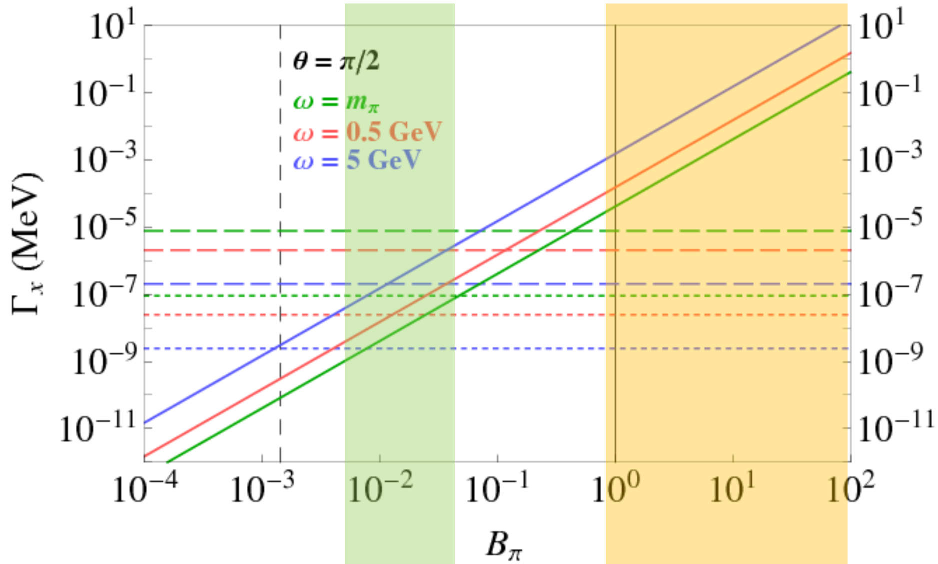
  -- conversion into γ with space-time varying B

  -- Primakoff process* ($\gamma^* + B \rightarrow \pi^0$): important in HIC

  -- mixing of π^0 and γ

* observed in nuclear Coulomb field

Decay rates of three modes

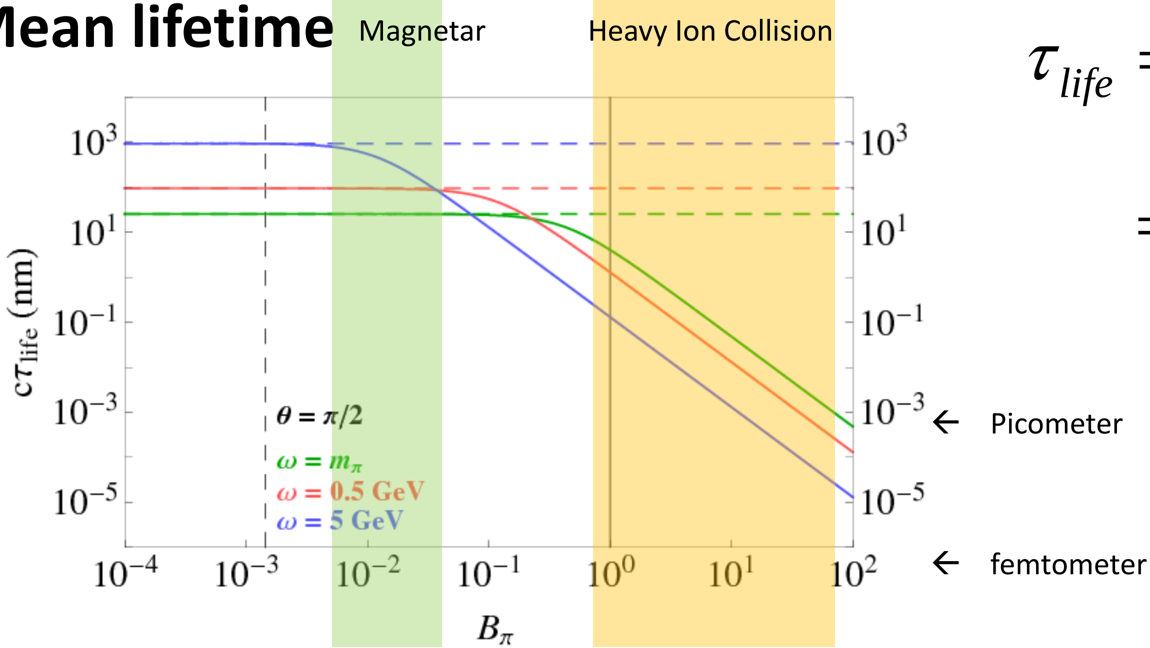


Solid : “Bee” decay
Dashed: 2γ decay
Dotted : Dalitz decay

$$\Gamma_{Be^+e^-} = \frac{q^2 q_\parallel^2}{12\pi\omega_\pi}\left(\lambda\frac{eB}{q^2}\right)^2\left(1+\frac{2m^2}{q^2}\right)\sqrt{1-\frac{4m^2}{q^2}}$$

$$B_\pi = B/m_\pi^{\ 2}$$

Mean lifetime



$$\tau_{life} = \Gamma_{total}^{-1}$$

$$= \frac{1}{\Gamma_{2\gamma} + \Gamma_{Dalitz} + \Gamma_{Bee}}$$

Energetic pions created in cosmic ray reactions will be affected

Response of hadrons to magnetic fields

- Naïve argument: spin *s*, magnetic moment *g,* charge *e*

$$E_n^2(p_z, s_z) = m^2 + p_z^2 + (2n+1)eB - gs_z eB$$

Landau levels spin-magnetic effect

- "Effective" mass in *B*

$$E_{n=0}^2(p_z = 0, s_z) = m^2 + (1 - gs_z)eB$$

- Spin 0 mesons : $m^2 + eB$ (pions) "heavier"
- Spin 1/2 , *g*=2 : m^2 (electron)
- Spin 1, *g*=2 : $m^2 - eB$ (rho meson) "lighter"

Decay of rho mesons

Chernodub, PRD82 (2010) 085011

- Dominant decay mode (>99%) : $\rho^{+/-} \rightarrow \pi^{+/-}\,\pi^0$
- Due to mass variation in B, this decay mode becomes impossible at high value of B

$$m_{\rho^\pm}(B_{\rho^\pm}) = m_{\pi^\pm}(B_{\rho^\pm}) + m_{\pi^0}$$

No change

This is realized when

$$B_{\rho^\pm} = \frac{1}{2e}\left[m_\rho^2 - m_\pi^2 - m_\pi(m_\pi^2 + 2m_\rho^2)^{\frac{1}{2}}\right]$$

$$\cong 0.36\frac{m_\rho^2}{e} \cong 11\frac{m_\pi^2}{e}$$

Beyond this magnetic field, charged rho mesons become <u>long lived</u>.

Also, neutral rho meson cannot decay into pi+ pi- when masses of charged pions become Large. This happens when $B=(m_\rho^2-4m_\pi^2)/4e \sim 6.5\ m_\pi^2/e$

Instability??

$$\varepsilon^2_{n,s_z}(p_z) = p_z^2 + (2n - 2s_z + 1)eB_{\rm ext} + m^2$$

- For vector mesons (s=1), the LLL with p_z=0

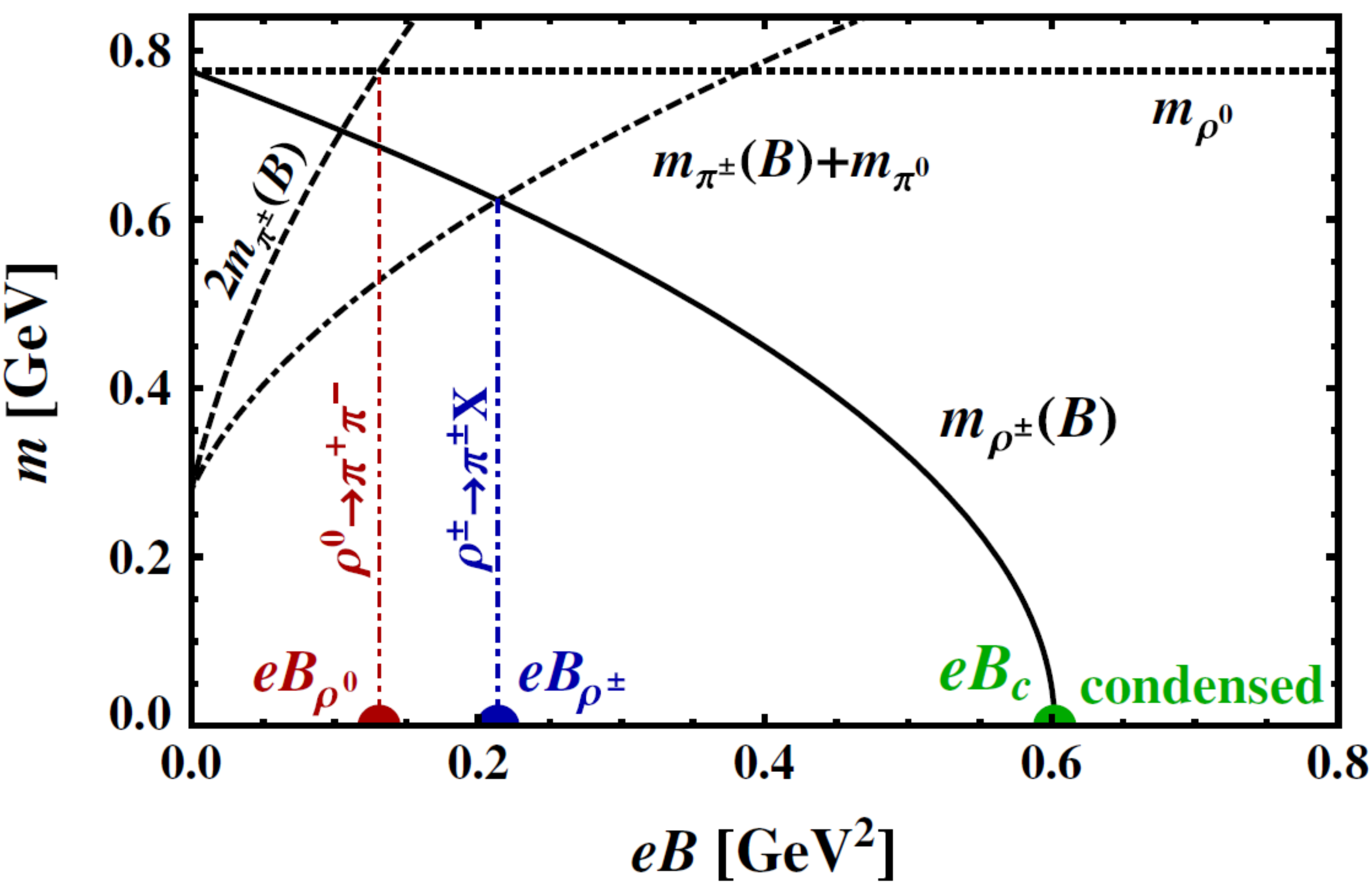
$$\varepsilon^2_{0,1}(p_z = 0) = m_\rho^2 - eB_{ext}$$

can be NEGATIVE when

$$B_{ext} > B_c \equiv \frac{m_\rho^2}{e} \sim 30\frac{m_\pi^2}{e}$$

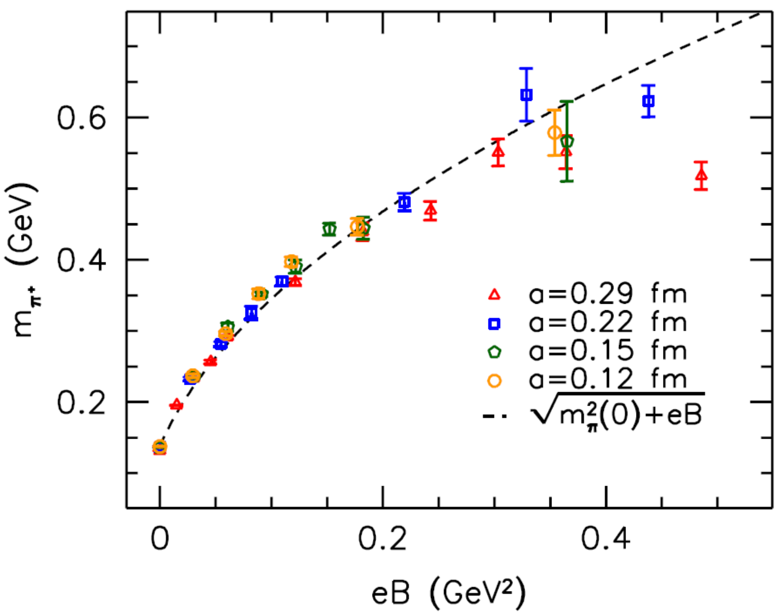
→ Charged rho mesons are unstable in very high magnetic field

Summary of Chernodub



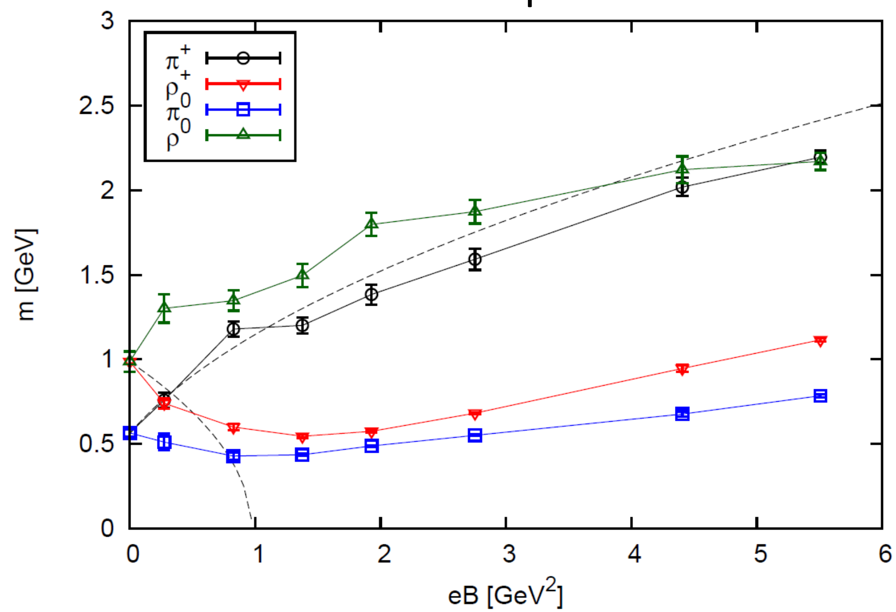
Lattice does not support instability scenario

Charged pion



Bali, Bruckmann, Endroedi, Fodor, Katz, Krieg, Schaefera and Szaboeb (2011)

Rhos and pions



Hidaka, Yamamoto (2012)

崩壊モードの抑制はありそう

Lessons from electron

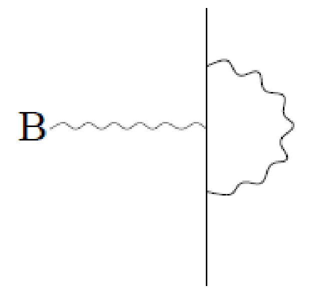
- Naïve picture : spin ½, *g*=2

  → electron mass does not change in magnetic field

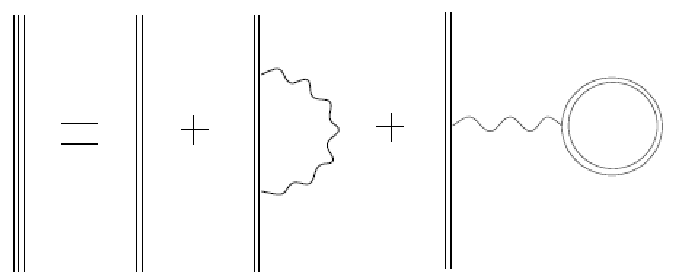
BUT THIS IS NOT TRUE IN TWO FOLDS.

(1) *g*-factor deviates from 2 due to radiative corrections

→ "anomalous" magnetic moment

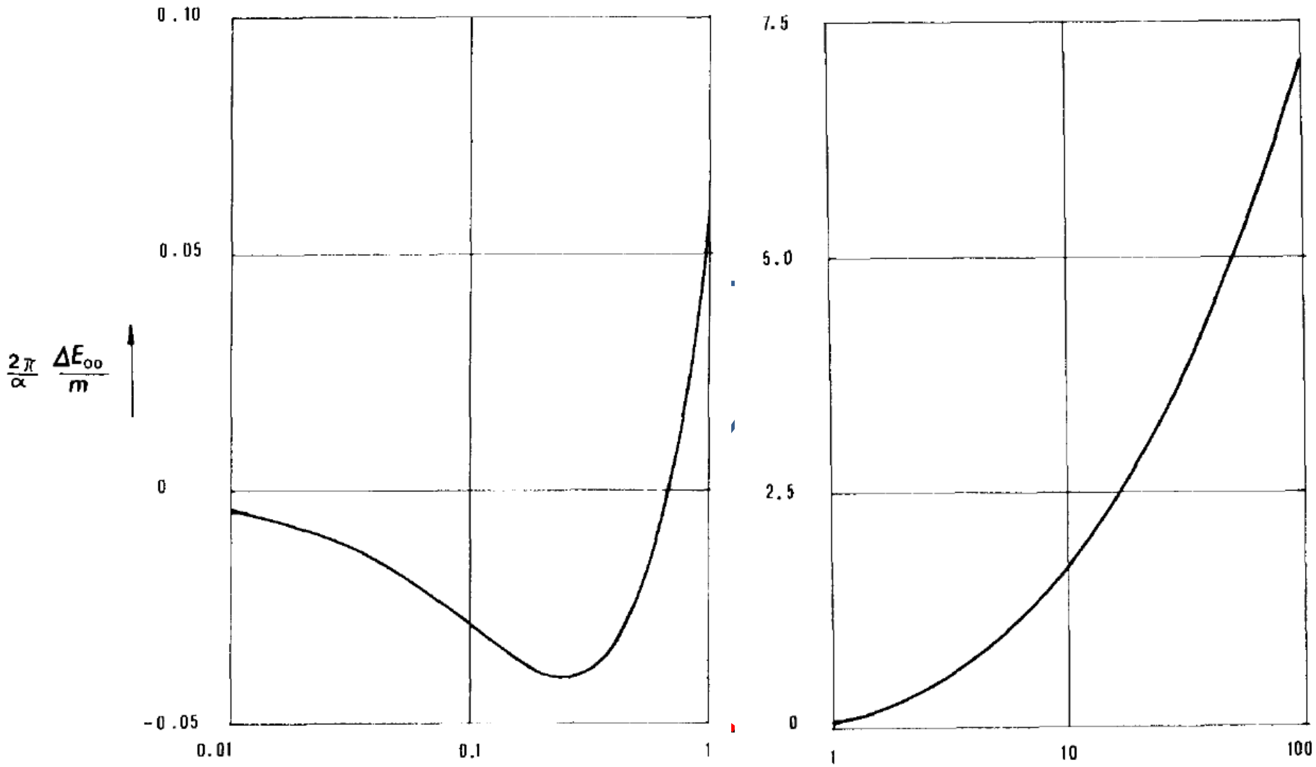


(2) When *B* is strong enough, we have to resum all the diagram with external field insertion. (resum all orders wrt eB)



Double line: dressed electron

Electron's lowest energy in magnetic fields



$L = B/B_c$

$$\frac{E_0}{m_e} \sim 1 + \frac{\alpha}{4\pi}\left(\left(\ln\frac{2B}{B_c} - \gamma - \frac{3}{2}\right)^2 + 3.9\right)$$

Jancovici, 1969
Constantinescu 1972

$E_0 = m_e\left[1 - (\alpha/4\pi)\, B/B_c\right]$

J.Schwinger, PR73(1948)416

Possible application to astrophysics (or to-do list for future research)

Strong fields in astrophysics

- **Early universe**

  QCD phase transition?

  QGP in laboratory is really QGP in early universe?

- **Compact stars (neutron stars, magnetars)**

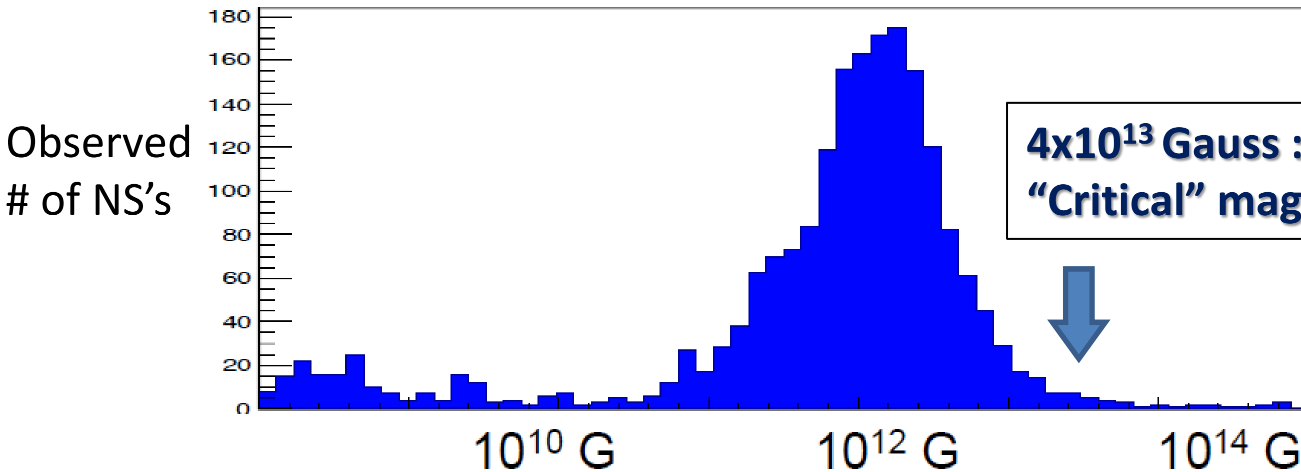
  inner region   EOS?

  outer region   mechanism of radiation?

- **Black Holes, Gamma-ray bursts**

  jet production?

Magnetic fields of neutron stars



4x10^{13} Gauss : $\sqrt{eB_c} = m_e = 0.5\text{MeV}$
"Critical" magnetic field of electrons

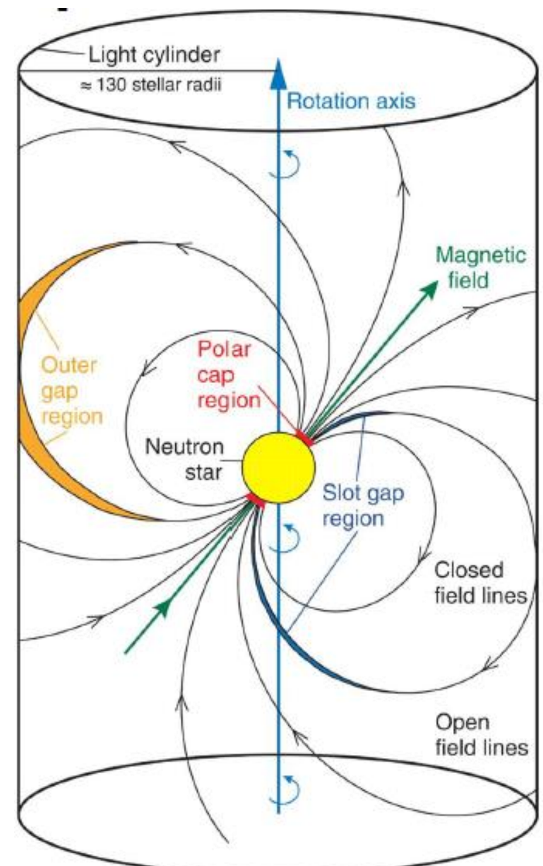
There is no ***static*** electric field because it is immediately screened by a plasma.

However,

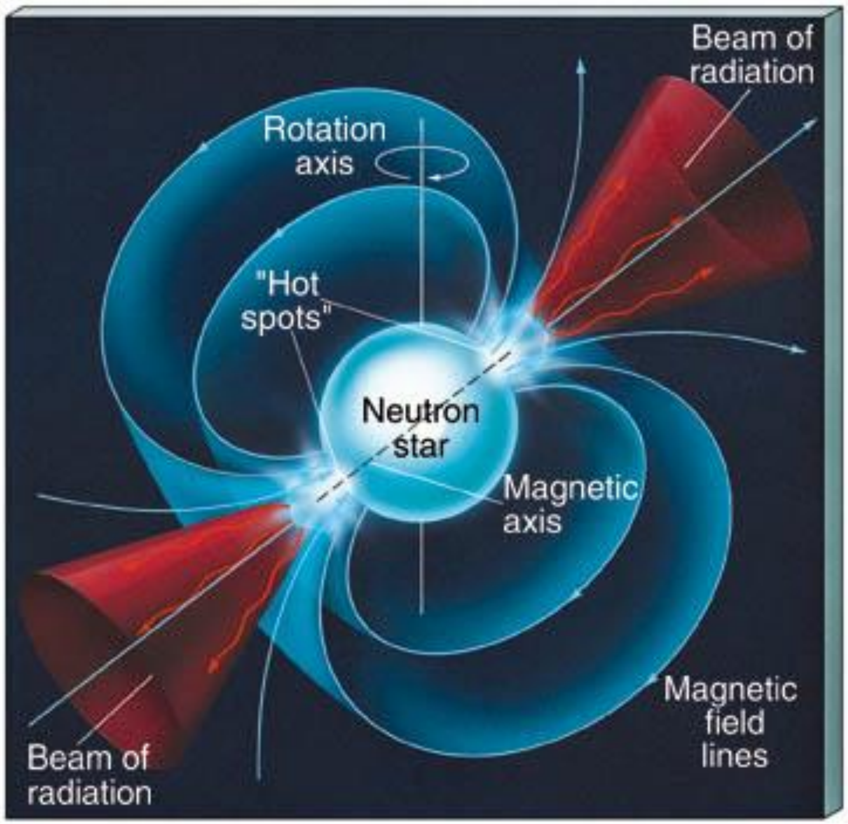
Pulsar

→ rapid rotation of magnetic field

→ Electric field is induced and strong too



Strong field physics in NS/magnetar



- **OUTSIDE of the star**

Both electric and magnetic fields are strong enough around the polar regions.

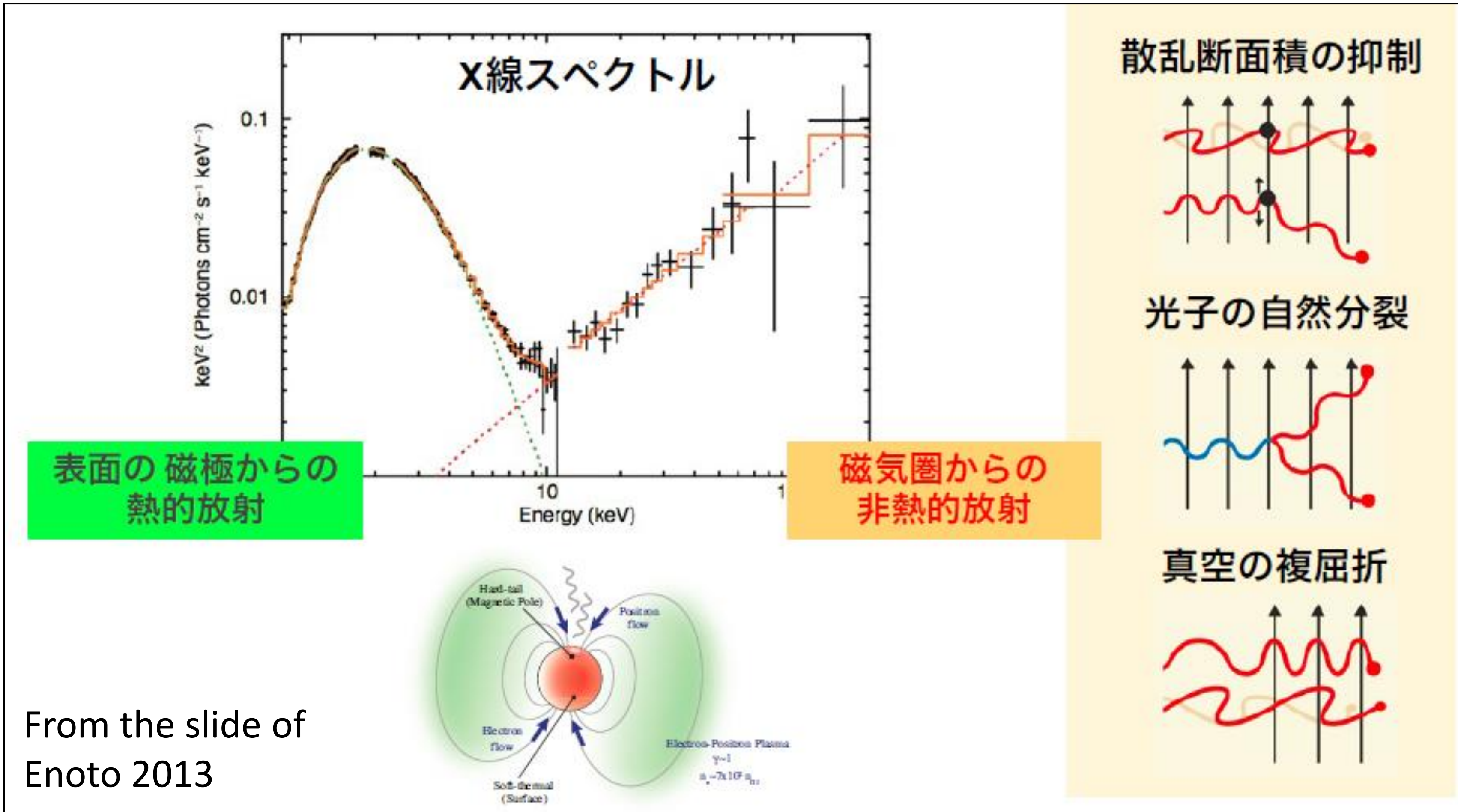
→ anomalous photon emission due to photon splitting and Schwinger mechanism?

→ origin of intense radiation ?

- **INSIDE of the star**

If the magnetic field is present in the stars, there must be a big effect on the equation of state of nuclear matter.

Unique X-ray spectrum in magnetars



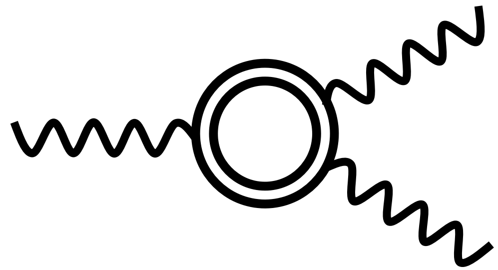
From the slide of Enoto 2013

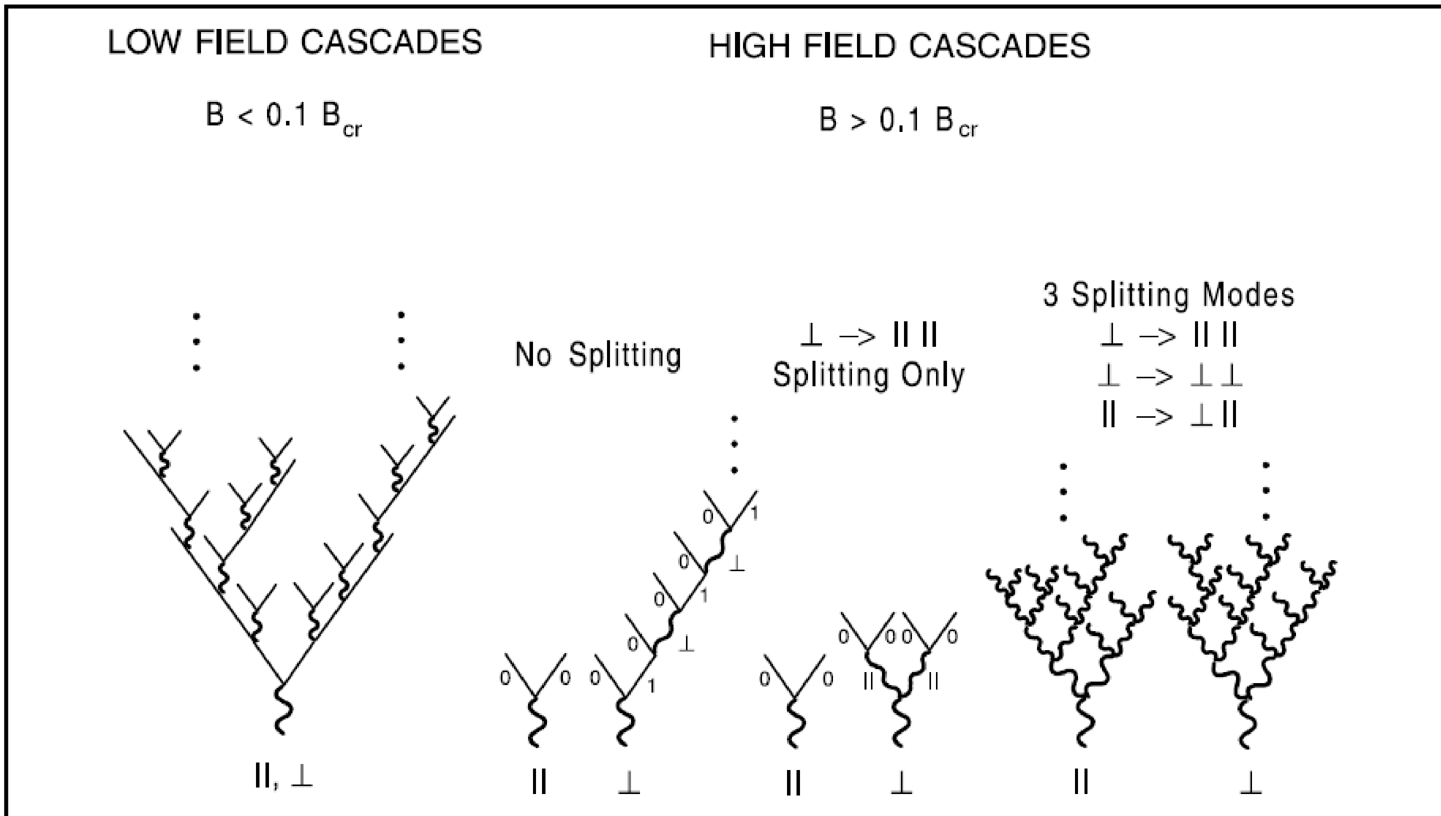
磁場の存在　→　光子の応答の異方性　→　偏極の効果
高エネルギー光子（E>500keV）が、分裂して低エネルギーに？

現在、複屈折を取り込んだ光子分裂の効果を解析中　（服部、郡、板倉）

Photon splitting

真空中では不可能





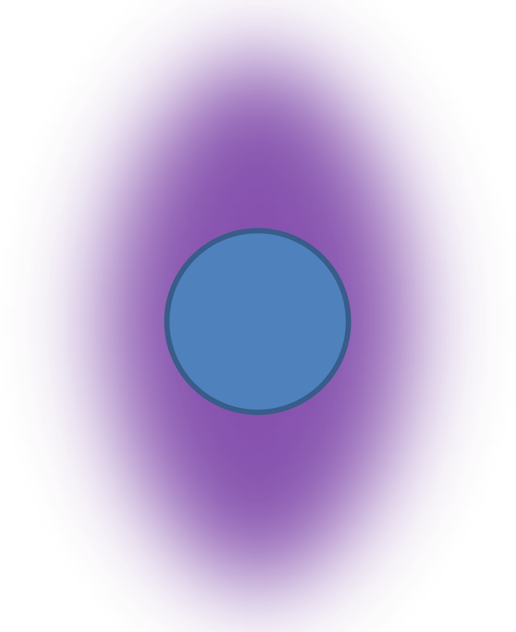
場の強さによって
カスケードの性質が
変化する

Baring, Harding
ApJ 547 (2001)

Magnetar磁極近傍で
ジェットを生成？

強磁場によるレンズ効果

- 磁場中で光子の屈折率は変化する。その変化は空気中よりも大きく、水よりは小さい。偏光依存性あり。



中性子星の像の歪み
＋背後の像の歪み

コップの水によって、
スプーンの像が歪み
背後の像も歪む

双極子型の磁場の配位が回転した場合に
どのような像の変化が得られるか？ （電場の効果も必要）

Effects of magnetic fields on EoS

- **Three possible effects to be considered**

1. Landau quantization for electrons and protons

→ anisotropy of chemical potential (beta equilibrium)

2. Mass shift of protons (due to large anomalous magnetic moment)

→ new balance of beta equilibrium (more protons?)

3. Mass shift of pions

→ anisotropic nuclear force? Charge asymmetry?

- **Earlier attempt**

Broderick, Prakash, and Lattimer, Astrophys. J 537 (2000) 351

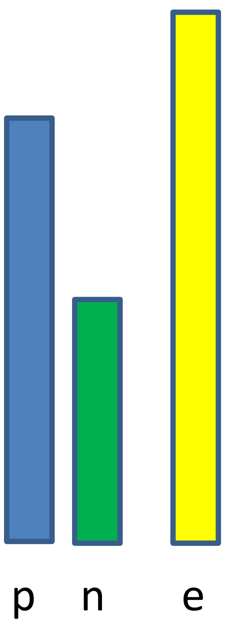
- reduction of electron μ → increase of proton fraction
- softening of EOS due to Landau quantization
- stiffening due to anomalous magnetic moment of nucleons

Lowest proton mass in strong *B*

$$\varepsilon_{n,s_z}^2(p_z) = p_z^2 + (2n - \cancel{2}s_z + 1)eB_{\text{ext}} + m^2$$

$g_{\text{p}}=5.58$

磁場中で異常磁気能率のために
陽子の「有効質量」は大きく減少する（電子よりも効果大）
中性子は電荷を持たないので変化せず



Mn > Mp 差が広がる → B=0 の時よりも 陽子を作りやすい
（中性子を作ることがそれほど得でなくなる）
→ 中性子から陽子への崩壊が促進される
→ 中性子星の中で陽子の比率が増える

陽子がNS中で増えると、磁場を支えやすくなるのではないか？
（超流動と結合して超伝導流の生成？渦糸内部に強力な磁場を保持？）

磁気能率に対する非線形磁場効果の吟味も必要（cf 電子）

Chiral symmetry breaking at finite B

- **Relatively understood at μ=0 thanks to lattice**

  application to QGP formation at

  colliders and early universe

  → $T_c(B)$

- **Not understood at large μ relevant for compact stars**

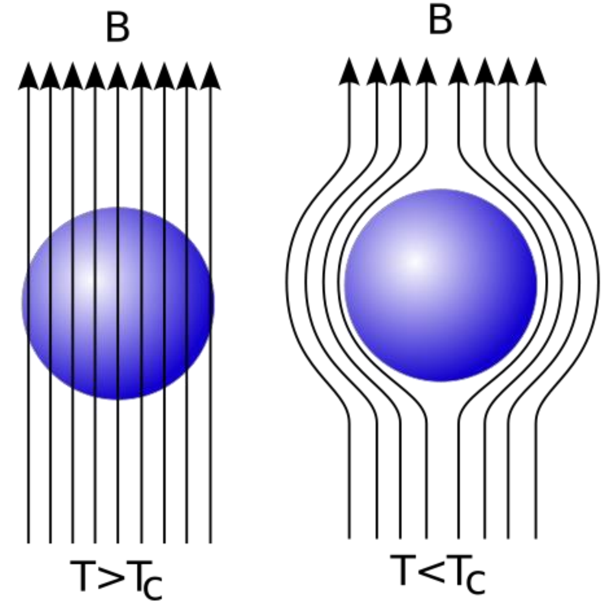
  need to understand B-dependence of

  critical chemical potential $\mu_c(B)$

***Tc* decreases** in superconductors in *B*

- Magnetic field is expelled from the superconductors (Meissner effect). Stronger magnetic field creates vortices (if possible), and let the material back to a normal state.

- Cooper pairs are formed as spin 0 states (up and down). Magnetic field lets the spin aligned to help to break the pairs.



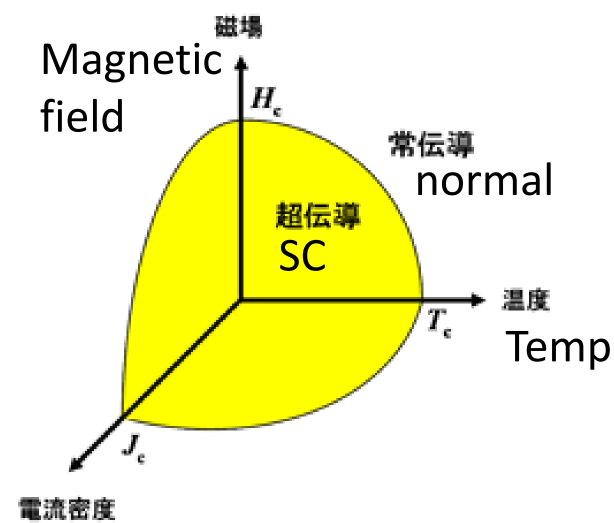
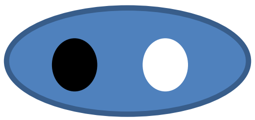


Fig.5 超伝導領域

What about in QCD?

- Chiral condensate:

  spin 0 quark-antiquark pair



  Spin up quark and spin down antiquark feel the same Lorentz force

  → decrease or increase Tc ?

  cf) enhancement of symm. br.

  (magnetic catalysis)

  flavored meson will be broken

- Confinement (Polyakov loop) :

  couples to B only through quark loops

Tc decrease?

Chiral and deconfinement transitions split?

Explicit breaking due to B

- Finite B introduces **<u>explicit breaking of chiral symmetry</u>** even in the chiral limit.

  $| L_{int} | = Q \mu eB$ $Q_u = +2/3$, $Q_d = -1/3$

  explicit breaking is large for large B

  However, flavored meson will be destroyed by B

- **Different GOR relation should be realized**

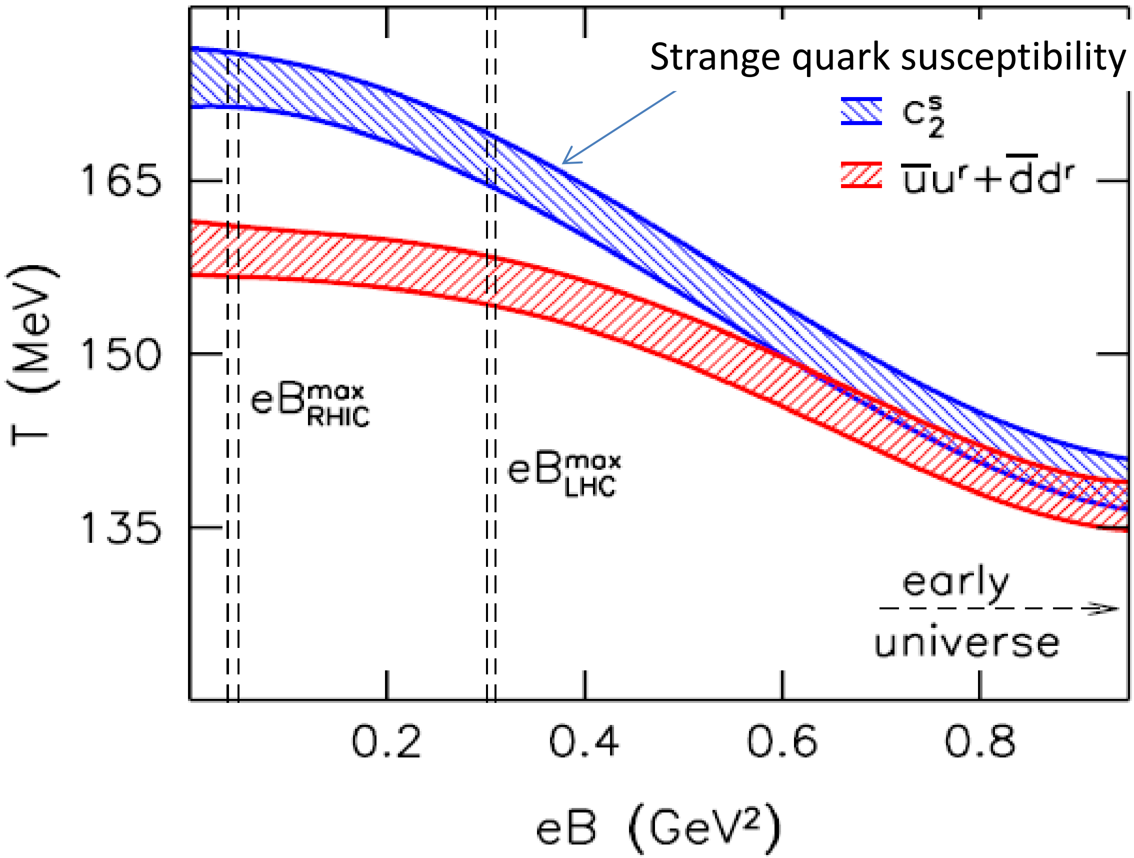
  pions are not massless

  pion mass will be related to B

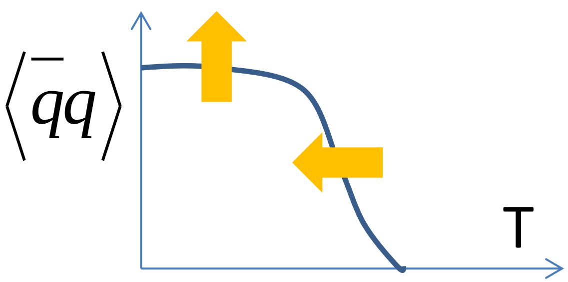
  such relation could be anisotropic wrt direction of B

  [now under consideration with Hattori]

Lattice results



G.S.Bali, F.Bruckmann, G.Endrodi, Z.Fodor, S.D.Katz, S.Krieg, A.Schafer and K.K.Szabo, JHEP 02 (2012) 044



-- Tc significantly decreases with increasing B

-- Chiral condensates enhanced

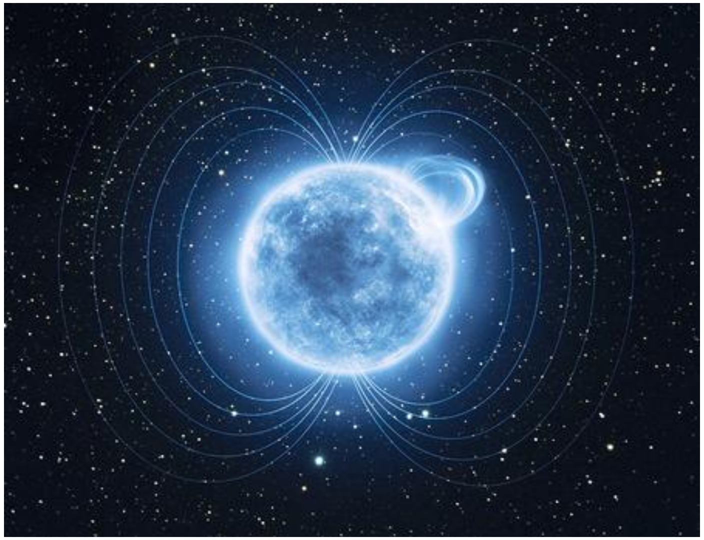
-- No splitting of chiral sym. br. and deconfinement

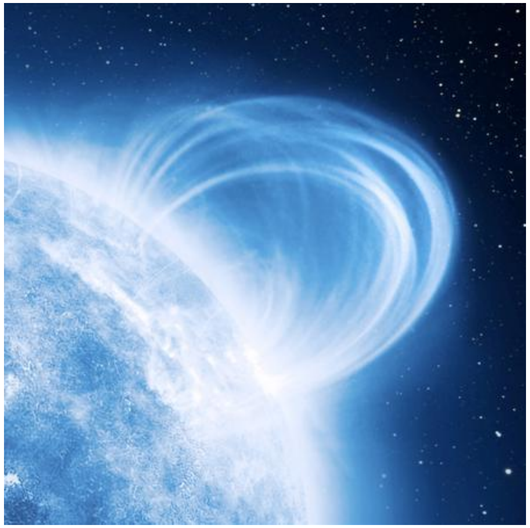
-- transition seems to be still crossover

New observation of SGR 0418+5729

Nature 500 (2013) 312 15 August 2013

- Magnetic dipole model suggests moderate value of magnetic fields 6×10^{12} Gauss. But this object shows bursts, a property typically seen in magnetars.
- "Phase dependent" X-ray measurement suggested "***proton synchrotron resonance***" and spots with very high magnetic fields > 2×10^{14} Gauss.





Summary of the Nature paper

- Even if the global magnetic field is moderate, it could be **much stronger INSIDE the star**, exceeding the critical magnetic field strength. Strong magnetic loops sometimes appear on the surface and induce energetic bursts.

- **Very similar to the sun** where the global magnetic field is 10 Gauss, but there are local spots having very strong magnetic fields 1000 Gauss. This ``sunspots" are related to bursts and flare events (jets and explosions).

- This measurement opens up a new possibility to **extract the information inside NS's and magnetars**!! Physics inside and outside the stars could be related to each other!!

- **Questions to be asked:**

  - Are there enough protons so that they can absorb X-rays?

  - Is magnetic reconnection possible at such a strong B field?

  - Is the flare event related to glitch?

QGP in Early Universe

- 生成過程が本質的に異なる:

  超高温の熱的な背景のもとで、あるいは再加熱過程でクォークやグルオンが生成

  EW plasma + QGP → QGP

- 超巨大なQGPの生成:

  QGPからEM的自由度が逃げていかない

  QGPとQED plasmaの共存

  incoherentな強い光子場の存在

  quarkは電荷を持つ

  (EWPTの前は、EW plasma + QGP + QED plasma)

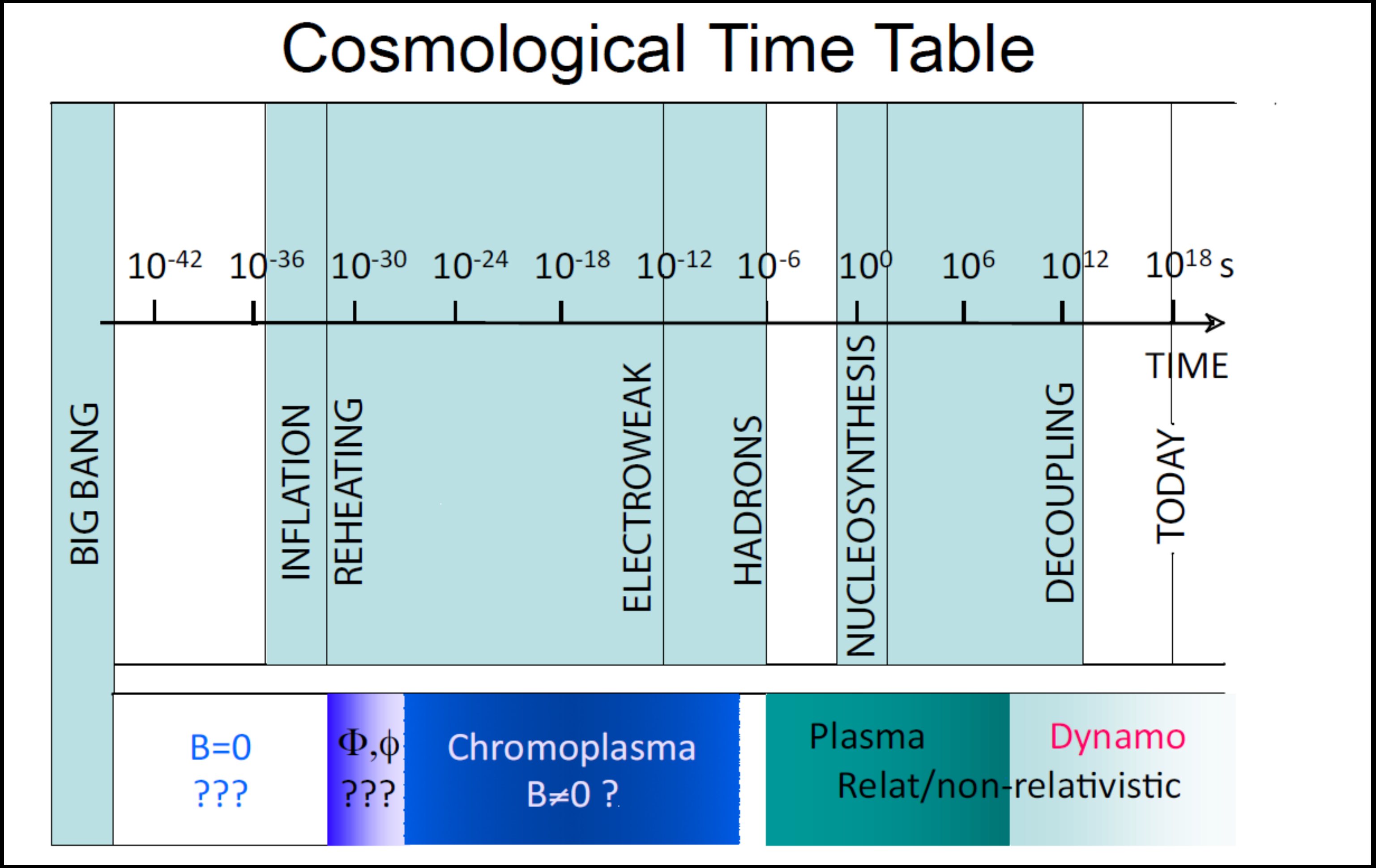
- そもそも最初から強い電磁場が存在した可能性

  ← 電弱相転移で超強磁場生成？ cf) Vachaspati (1991)

少なくとも、光子場と強く結合したQGPを扱う必要あり

Magnetic fields in Early Universe

Widrow, et al. “The First magnetic Fields” (arXiv:1109.4052)



はっきりしたことはわかっていないようだ、、、

Summary

- **When an external field is much larger than typical excitation energy of a system, one can find extraordinary non-perturbative phenomena called "strong field physics".**

- **Strong field physics reveals novel properties of ordinary particles such as photons and hadrons in strong external fields.**

- **Such extreme situations are seen in Nature, in particular, in the universe, and also realized in experiments with high-intensity laser or heavy-ion collisions.**

- **We need to incorporate strong field physics to understand the properties of compact stars like neutron stars and magnetars.**