NJL模型を用いたテンサー型スピン偏極とカイラル対称性の研究

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§1 Introduction

Strong Magnetic Fields

- Early Universe:
  $B \sim 10^{16}$ T or $\sqrt{eB} \sim 1$ GeV
- Compact dense stars, such as magnetars:
  $B \sim 10^{10}$ T or $\sqrt{eB} \sim 1$ MeV
- Heavy-ion collisions in very strong laser pulses (experiments as PHELIX at GSI and ELI:)
  $B \sim 10^{7}$ T or $\sqrt{eB} \sim 0.01$ MeV
- Noncentral heavy-ion collisions:
  $B \sim 10^{15}$ T or $\sqrt{eB} \sim 10$ MeV $\ldots$ 300 MeV

Star Collaboration RHIC in BNL PRL 103, 251601 (09) , PRC81, 054908 (10)

spontaneous spin polarization above critical density

How to keep the Strong Magnetic Field

Magnetor : **Neutron-Star**
with Strong Magnetic Field ~ 10^{15} Gauss

Normal Neutron Star ~ 10^{12} Gauss

the mechanism of this strong magnetic field?

One Possible Candidate Spin-Polarization Matter

T. Tatsumi, Phys. Lett. B489 (2000) 280.

Quark Matter

Vector-Vector (One-Photon, One-Gluon Exchange)

Fock Term $\Rightarrow$ $-\,\sigma\cdot\sigma$ $\Rightarrow$ **spin-alignment**

History

T. Tastumi, PL B489, 280 (2000)
One-Gluon Exchange Force ⇒ AV-Type Spin-Spin interaction

T.M & T.Tatsumi, Nucl. Phys. A693, 710
Perturbative Calculation AV & Tensor in Nuclear Matter in RMF

E. Nakano, T.M., T. Tatsumi, Phys. Rev. D68, 105001 (2003).
Spin polarization and color superconductivity in quark matter

E.Nakano and T. Tatsumi, Phys. Rev. D71 (2005) 114006.
Dual Chiral Density Wave (DCDW)

T. Tatsumi and K. Sato, Phys. Lett. B663 (2008) 322.
Fermi Liquid Theory in Quark Matter

AV-Force in Quark Matter

T. M., E. Nakano, T. Tatsumi,
Horizons in World Physics. Vol.276, p209 ,((2011)
Review Tensor Type Spin Polarization

Y. Tsue, J. de. Providencia, C. Providencia and M. Yamamura,
PTP 128, p 507 (2012).
Tensor Spin-Polarization with Zero Mass Quark

§2 Spontaneous Spin Polarization in Relativistic Mean-Field Approach

Fermion Propagator

Relativistic HF Approx.

$$\varepsilon_{\rm int}=\int\frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\,\mathrm{Tr}\{iS(p)\}\mathscr{D}_S(p-k)\mathrm{Tr}\{iS(k)\}$$

$$+\int\frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\,\mathrm{Tr}\{iS(p)\gamma^\mu\}\mathscr{D}_V(p-k)\mathrm{Tr}\{iS(k)\gamma_\mu\}$$

$$+\int\frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\,\mathrm{Tr}\{iS(p)\gamma_5\}\mathscr{D}_P(p-k)\mathrm{Tr}\{iS(k)\gamma_5\}$$

AV channel

$$+\int\frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\,\mathrm{Tr}\{iS(p)\gamma_5\gamma^\mu\}\mathscr{D}_A(p-k)\mathrm{Tr}\{iS(k)\gamma_5\gamma_\mu\}$$

Tensor channel

$$+\int\frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\,\mathrm{Tr}\{iS(p)\sigma^{\mu\nu}\}\mathscr{D}_T(p-k)\mathrm{Tr}\{iS(k)\sigma_{\mu\nu}\}$$

Spin-Spin Int.

$$\vec{\sigma}\cdot\vec{\sigma}\ \text{(N.R.)}\ \Rightarrow\ \vec{\sigma}\cdot\vec{\sigma}\ \text{(AV)}\qquad \beta\vec{\sigma}\cdot\beta\vec{\sigma}\ \text{(Tensor)}$$

Relativistic Hartree-Fock ← σ、ω、η

$$\mathscr{D}_S(q) = -\frac{g_\sigma^2}{m_\sigma^2} + \frac{1}{8} g_\sigma^2 \Delta_\sigma(q) - \frac{1}{2} g_\omega^2 \Delta_\omega(q) + \frac{1}{8} g_\eta^2 \Delta_\eta(q)$$

$$\mathscr{D}_V(q) = \frac{g_\omega^2}{m_\omega^2} + \frac{1}{8} g_\sigma^2 \Delta_\sigma(q) + \frac{1}{4} g_\omega^2 \Delta_\omega(q) - \frac{1}{8} g_\eta^2 \Delta_\eta(q)$$

$$\mathscr{D}_P(q) = -\frac{g_\eta^2}{m_\eta^2} - \frac{1}{8} g_\sigma^2 \Delta_\sigma(q) - \frac{1}{2} g_\omega^2 \Delta_\omega(q) + \frac{1}{8} g_\eta^2 \Delta_\eta(q)$$

$$\mathscr{D}_A(q) = \frac{1}{8} g_\sigma^2 \Delta_\sigma(q) - \frac{1}{4} g_\omega^2 \Delta_\omega(q) + \frac{1}{8} g_\eta^2 \Delta_\eta(q)$$

$$\mathscr{D}_T(q) = \frac{1}{8} g_\sigma^2 \Delta_\sigma(q) - \frac{1}{8} g_\eta^2 \Delta_\eta(q)$$

$$\Delta_\alpha(q) = \frac{1}{m_\alpha^2 - q^2}$$

Two Type Spin-Forces

Two Body Forces : Zero-Range approx.

$$D_{\mu\nu}(q) \approx \frac{g_{\mu\nu}}{\Lambda^2}$$

$$\varepsilon_T = \varepsilon_K + \frac{G_A}{2}\rho_A^2 + \frac{G_T}{2}\rho_T^2$$

Axial Vector Density

$$\rho_A = \int \frac{d^4p}{(2\pi)^4} Tr\{S_D(p)\gamma_5\gamma^3\} = \rho_B < \Sigma_z >$$

Tensor Density

$$\rho_T = \int \frac{d^4p}{(2\pi)^4} Tr\{S_D(p)\sigma_{12}\} = \rho_B < \beta\Sigma_z >$$

Mean-Fields

$$U_A = G_A\rho_A \quad , \qquad U_T = G_T\rho_T .$$

Dirac Equation

$$\hat{h}_{sp}u(\boldsymbol{p},s) = [\boldsymbol{\alpha}\cdot\boldsymbol{p} + m + \Sigma_z U_A + \beta\Sigma_z U_T]\, u(\boldsymbol{p},s) = e(\boldsymbol{p},s)u(\boldsymbol{p},s),$$

Single Particle Energy

$$(p_0^2 - E_p^2)^2 - 2p_0^2(U_A^2 + U_T^2) - 8p_0 m U_A U_T - 2m^2(U_A^2 + U_T^2)$$
$$+(U_A^2 - U_T^2)^2 - 2(U_A^2 - U_T^2)(p_z^2 - \boldsymbol{p}_T^2) \;=\; 0.$$

AV density and T-density

$$\begin{aligned}
\varepsilon_K &= \sum_{s=\pm 1}\int \frac{d^3p}{(2\pi)^3} n(e(\boldsymbol{p},s))\left\{e(\boldsymbol{p},s) - U_A\rho_A - U_T\rho_T\right\},\\
\rho_A &= \sum_{s=\pm 1}\int \frac{d^3p}{(2\pi)^3} n(e(\boldsymbol{p},s))\frac{U_A(e^2+m^2-U_A^2+U_T^2+p_z^2-\boldsymbol{p}_T^2)+2mU_Te}{e(e^2-m^2-\boldsymbol{p}^2-U_A^2-U_T^2)-2mU_AU_T},\\
\rho_T &= \sum_{s=\pm 1}\int \frac{d^3p}{(2\pi)^3} n(e(\boldsymbol{p},s))\frac{U_T(e^2+m^2+U_A^2-U_T^2-p_z^2+\boldsymbol{p}_T^2)+2mU_Ae}{e(e^2-m^2-\boldsymbol{p}^2-U_A^2-U_T^2)-2mU_AU_T}.
\end{aligned}$$

§2-1 Perturbative Calculation

$$|U_A| \sim |U_T| \ll 1$$

Single Particle Energy

$$e(\boldsymbol{p},s) \approx E_p + s\frac{\tilde{U}_D}{E_p}$$

Dirac Spinor

$$u(\boldsymbol{p},s)\bar{u}(\boldsymbol{p},s) = \lim_{p_0 \to e(\mathbf{p},s)} (p_0 - e(\boldsymbol{p},s))S(p)$$

$$\approx \frac{(\not{p}+m)}{4E_p} + \frac{U_A \Delta S_A + U_T \Delta S_T}{8sE_p\tilde{U}_D} = \frac{(\not{p}+m)(1+s\gamma_5 \not{a}_N)}{4E_p}$$

$$a_N = \frac{U_A\sqrt{p_z^2+m^2}a_A + U_T\sqrt{\mathbf{p}_T^2+m^2}a_T}{\tilde{U}_D}$$

$$a_A \equiv \left(\frac{E_p p_z}{m\sqrt{p_z^2+m^2}}; \frac{m^2\hat{z}+p_z\boldsymbol{p}}{m\sqrt{p_z^2+m^2}}\right)$$

For Maximum < Σz >

$$a_T \equiv \left(\frac{p_z}{\sqrt{\mathbf{p}_T^2+m^2}}; \frac{E_p}{\sqrt{\mathbf{p}_T^2+m^2}}\hat{z}\right)$$

For Maximum < βΣz >

we rewrite that $U_A \to \lambda_A$ and $U_T \to \lambda_T$

AV & T-density

$$\begin{aligned}\rho_A &= \lambda_A \mathscr{A} + \lambda_T \mathscr{R},\\ \rho_T &= \lambda_A \mathscr{R} + \lambda_T \mathscr{T}\end{aligned}$$

$$\mathscr{A} = -\frac{N_d}{\pi^2} k_F E_F \left(1 - \frac{2}{3} v_F^2\right)$$

$$\mathscr{T} = -\frac{N_d}{\pi^2} k_F E_F \left(1 - \frac{1}{3} v_F^2\right)$$

$$v_F = k_F / E_F \text{ with } E_F = \sqrt{k_F^2 + m^2}$$

$$\mathscr{R} = -\frac{N_d}{\pi^2} m k_F$$

Energy Variation in NR limit $k_F << m$

$$\begin{aligned}\Delta\varepsilon = \Delta\varepsilon_K + \varepsilon_{SD} &\approx \frac{N_d}{\pi^2}\frac{m^2}{k_F}(\lambda_A + \lambda_T)^2 \left\{ m + \frac{G_A + G_T}{2}\frac{N_d}{\pi^2} p_F^3 \right\}\\ &\approx \frac{N_d}{\pi^2}\frac{m^2}{k_F}(\lambda_A + \lambda_T)^2 \{m + 3(G_A + G_T)\rho_B\}.\end{aligned}$$

Spontaneous Spin-Polarization occurs if $G_A + G_T < 0$

Energy Variation in Ultra Relativistic limit, $k_F >> m$

$$\Delta\varepsilon \approx \frac{N_d}{\pi^2}k_F^2\left[\lambda_A^2+\lambda_T^2(1+v_F^2)\right]+\frac{G_A}{18\pi^4}N_d^2k_F^4\lambda_A^2+\frac{2G_T}{9\pi^4}N_d^2k_F^4\lambda_T^2$$
$$\approx \frac{N_d}{\pi^2}k_F^2\left(1+\frac{G_A}{18\pi^2}N_dk_F^2\right)\lambda_A^2+\frac{2N_d}{\pi^2}k_F^2\left(1+\frac{G_T}{9\pi^2}N_dk_F^2\right)\lambda_T^2.$$

If $G_A < 0 < G_T$, AV-Type Spin Polarization

$\rho_T \to 0$, ρ_A :finite, **Weak Magnetic Field**

If $G_T < 0 < G_A$, Tensor-Type Spin Polarization

$\rho_A \to 0$, ρ_T :finite **Strong Magnetic Field**

$$u(\mathbf{p},s)\bar{u}(\mathbf{p},s)=\frac{(\not p+m)(1+s\gamma_5\not a)}{4E_p},$$

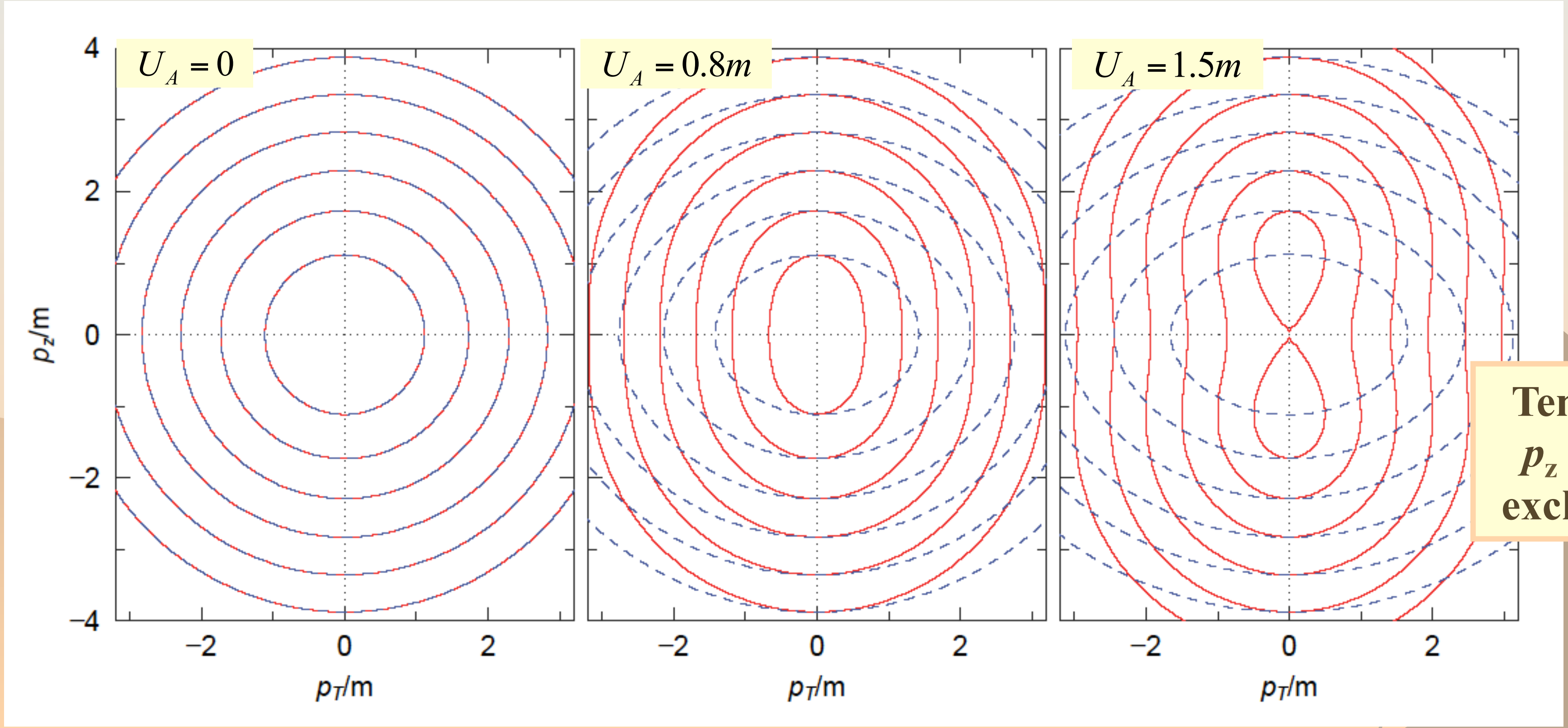
⇒ Same Conclusion in any λ

§2-2 AV-Type & T-Type Spin Polarization

Single Particle Energy

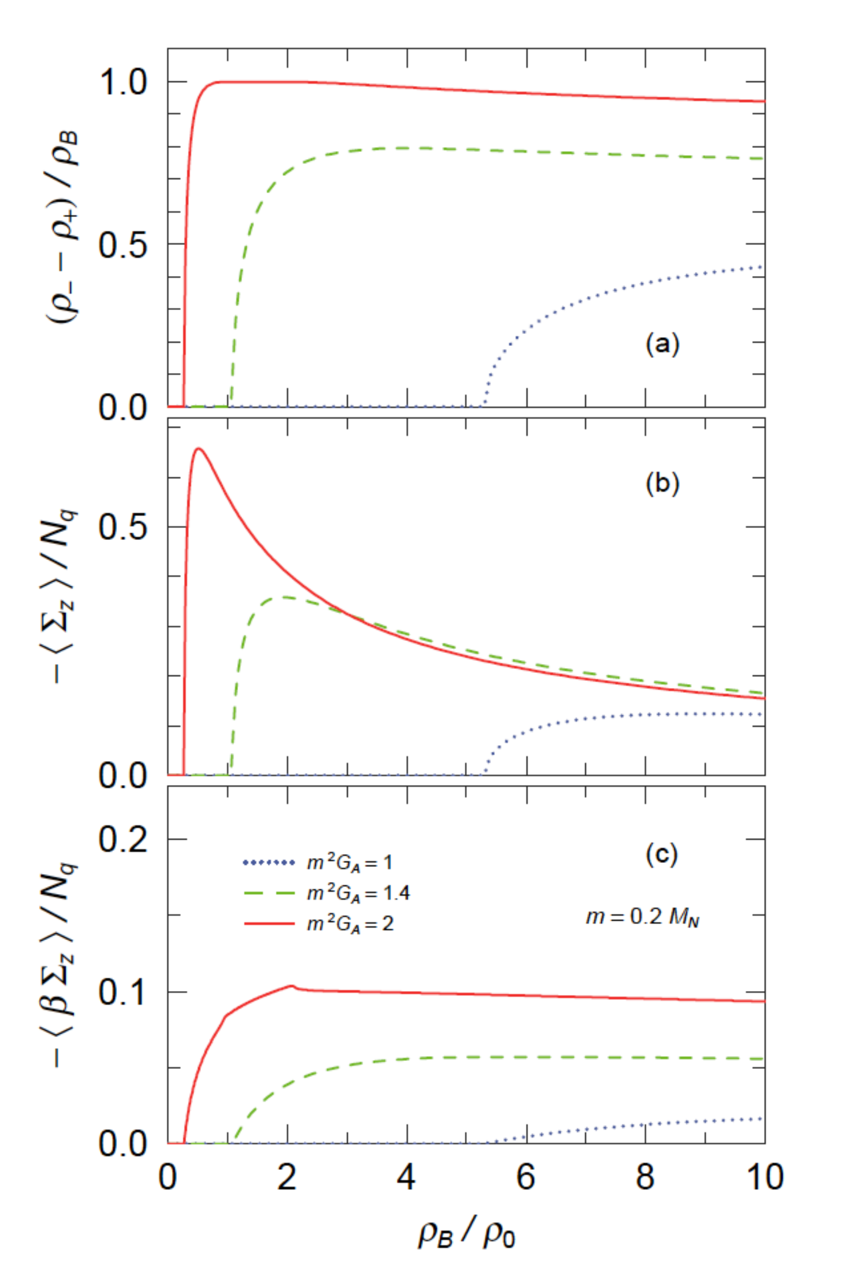
When $U_T = 0$, $e(\boldsymbol{p},s) = \sqrt{(\sqrt{m^2+p_z^2}+sU_A)^2+p_T^2}. = \sqrt{E_p^2+2sU_A\sqrt{m^2+p_z^2}+U_A^2}.$

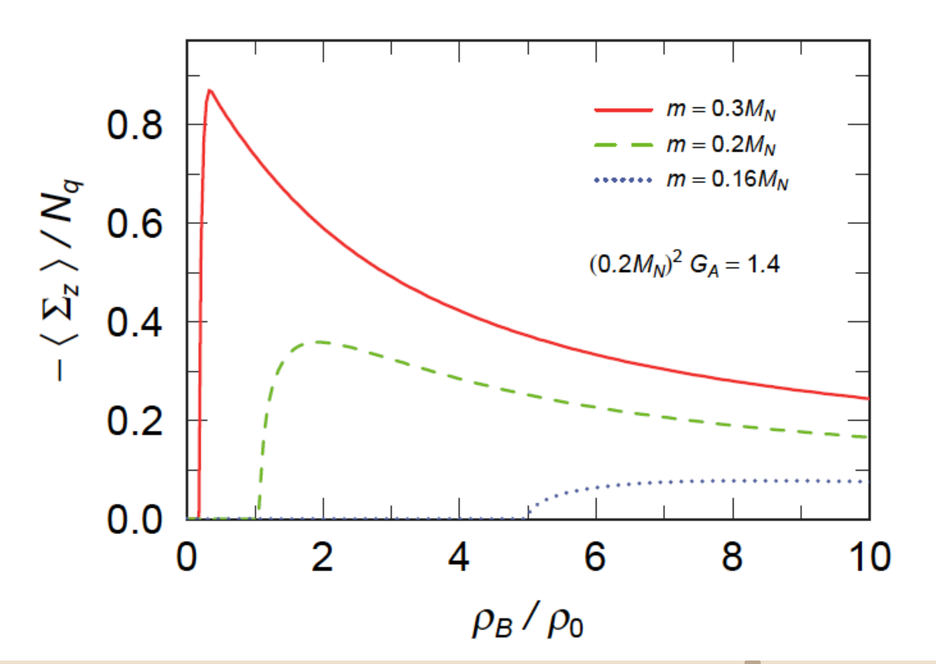
When $U_A = 0$, $e(\boldsymbol{p},s) = \sqrt{(\sqrt{m^2+\boldsymbol{p}_T^2}+sU_T)^2+p_z^2}. = \sqrt{E_p^2+2sU_T\sqrt{m^2+\boldsymbol{p}_T^2}+U_T^2}.$



Tensor :
$p_z \leftrightarrow p_T$
exchange

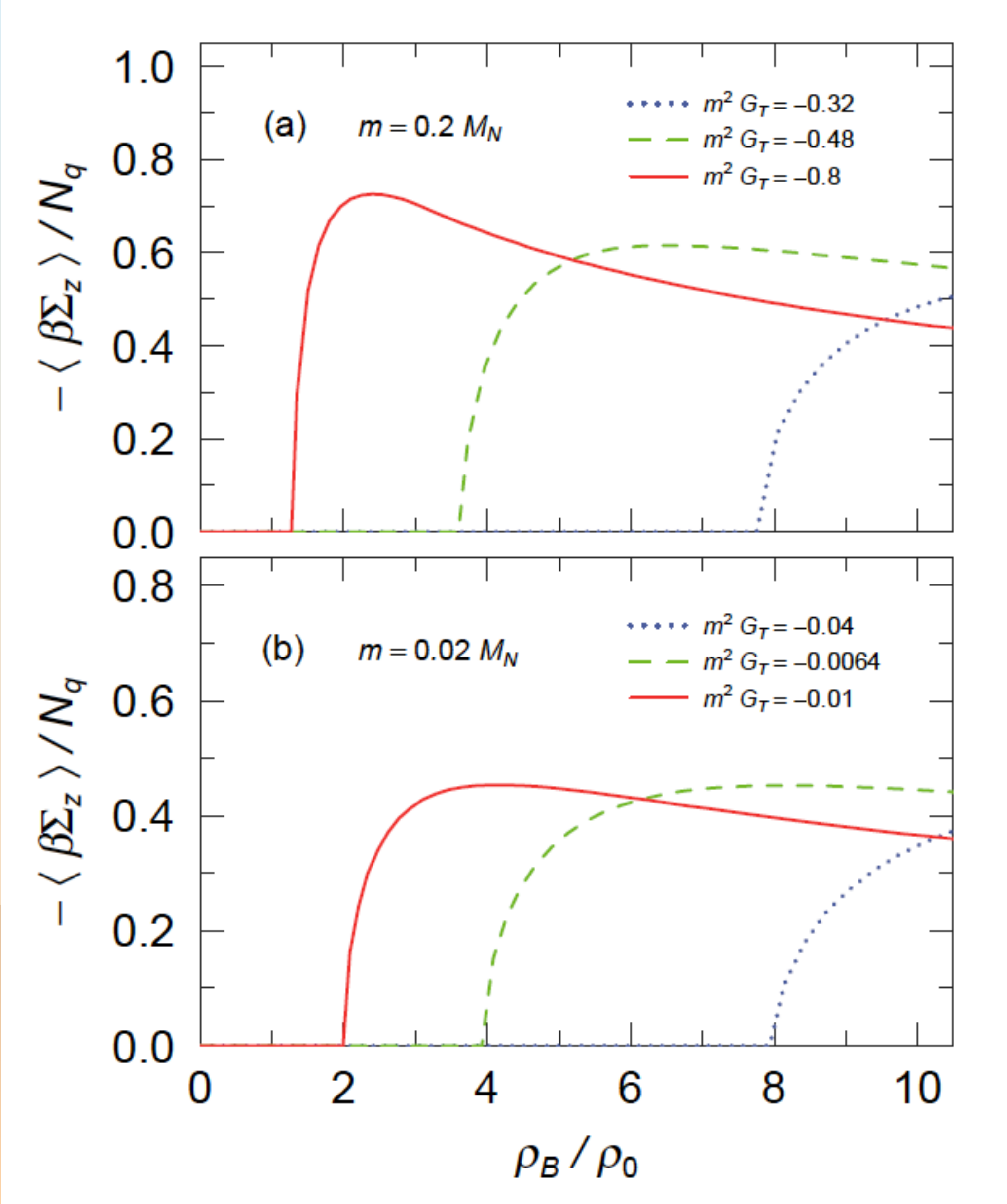
AV-Type Spin-Polarization





When the quark mass approaches to zero, Spin-Polarization rapidly disappears.

T-Type Spin Polarization



Even when the quark mass approaches to zero, Spin-Polarization does **not rapidly** disappear.

$|U_A| \ll 1$
$|U_T| \ll 1$

$$\rho_A \approx 2N_d U_A \int \frac{d^3p}{(2\pi)^3} \left\{ \boxed{\frac{\partial n(E_p)}{\partial E_p} \frac{p_z^2+m^2}{E_p^4}} + \boxed{n(E_p) \frac{E_p^2-(p_z^2+m^2)}{E_p^3}} \right\}$$

$$\approx \frac{N_d}{\pi^2} U_A \left\{ \boxed{-\frac{p_F(E_F^2+2m^2)}{3E_F}} + \boxed{\left[\frac{p_F(E_F^2+2m^2)}{3E_F} - m^2 \ln\left(\frac{p_F+E_F}{m}\right) \right]} \right\} \approx \rho_A(S) + \rho_A(C)$$

$$\rho_T \approx 2U_T N_d \int \frac{d^3p}{(2\pi)^3} \left\{ \boxed{\frac{\partial n(E_p)}{\partial E_p} \frac{p_T^2+m^2}{E_p^4}} + \boxed{n(E_p) \frac{p_z^2}{E_p^3}} \right\}$$

$$\approx \frac{U_T}{\pi^2} N_d \left\{ \boxed{-\frac{p_F(2E_F^2+m^2)}{3E_F}} + \boxed{\left[\frac{p_F(E_F^2+2m^2)}{6E_F} - \frac{m^2}{2} \ln\left(\frac{p_F+E_F}{m}\right) \right]} \right\} \approx \rho_T(S) + \rho_T(C)$$

$\rho_{A,T}$(S) : contr. from Fermi Surface **Variation of Fermi Level**

$\rho_{A,T}$(C) : contr. from Core of Fermi Distr. **Change of Dirac Spinor**

non.rela limit, $\rho_A / U_A \approx \rho_T / U_T \approx \rho_A(S)/U_A \approx \rho_T(S)/U_T \approx N_d m p_F / \pi^2$

Ultra-Relativistic limit

$p_F \gg m$

$$\rho_A \approx -\frac{U_A}{\pi^2} N_d m^2 \ln\left(\frac{p_F+E_F}{m}\right) \approx 0, \quad \rho_T \approx -\frac{U_T}{\pi^2} N_d p_F^2 \neq 0$$

AV- Type **ρ_A(S) and ρ_A(C) are cancelled each other**

Mass Zero Limit

AV density

$$\rho_A = N_d \sum_{s=\pm1} \int \frac{d^3p}{(2\pi)^3} n\left(\sqrt{(|p_z|+sU_A)^2+p_T^2}\right) \frac{s|p_z|+U_A}{\sqrt{(|p_z|+sU_A)^2+p_T^2}}$$

$$= N_d \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{n(\sqrt{(p_z+U_A)^2+p_T^2})(p_z+U_A)}{\sqrt{(p_z+U_A)^2+p_T^2}} + \frac{n(\sqrt{(p_z-U_A)^2+p_T^2})(-p_z+U_A)}{\sqrt{(p_z-U_A)^2+p_T^2}} \right\}$$

$$= N_d \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{n(\sqrt{p_z^2+p_T^2})p_z}{\sqrt{p_z^2+p_T^2}} + \frac{n(\sqrt{p_z^2+p_T^2})(-p_z)}{\sqrt{p_z^2+p_T^2}} \right\} = 0.$$

No Spin-Polarization

T density

$$\rho_T \approx \sum_{s=\pm1} \int \frac{d^3p}{(2\pi)^3} \left\{ n(E_p) + \frac{\partial n(E_p)}{\partial E_p} \frac{s\sqrt{p_T^2+m^2}U_T}{E_p} \right\} \times \frac{s\sqrt{p_T^2+m^2}+U_T}{E_p} \left\{ 1 - \frac{s\sqrt{p_T^2+m^2}U_T}{E_p^2} \right\}$$

$$\approx 2U_T \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{\partial n(E_p)}{\partial E_p} \frac{p_T^2+m^2}{E_p^2} + n(E_p)\frac{p_z^2}{E_p^3} \right\}$$

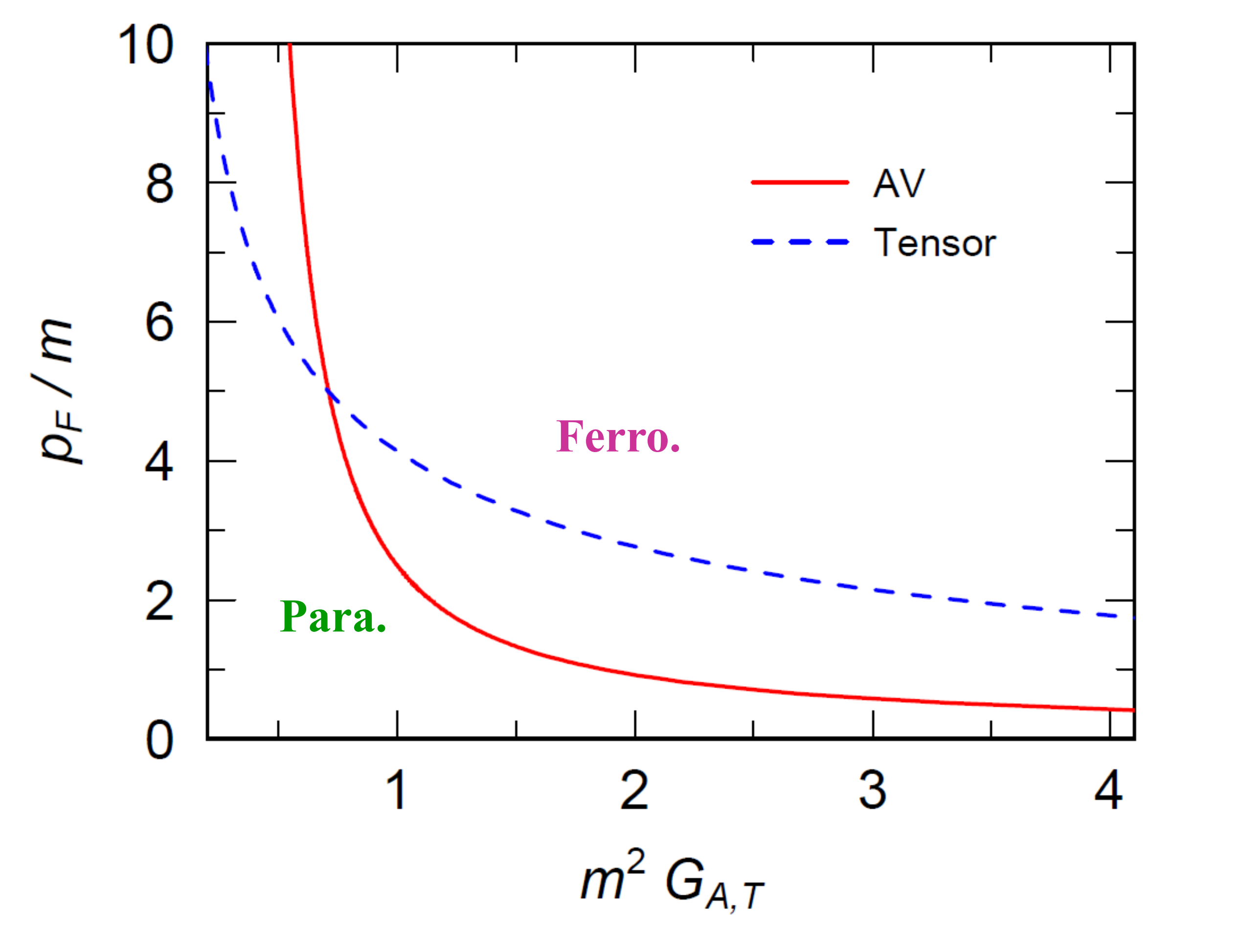
$$\approx \frac{1}{\pi^2} U_T \left\{ -\frac{k_F(2E_F^2+m^2)}{3E_F} + \left[\frac{k_F(E_F^2+2m^2)}{6E_F} - \frac{m^2}{2} \ln\left(\frac{k_F+E_F}{m}\right) \right] \right\}.$$

Phase-Diagram between Paramag. and Ferromag.

$$F_b(U_b) = \frac{G\rho_b}{U_b} - 1 \quad , \; b = A, T$$

$$F_b(U_b) \to -1, \quad U_b \to \infty$$

$F_b(U_b = 0) > 0$: a condition of Spontaneous Spin-Polarization



Large Mass

$AV > T$

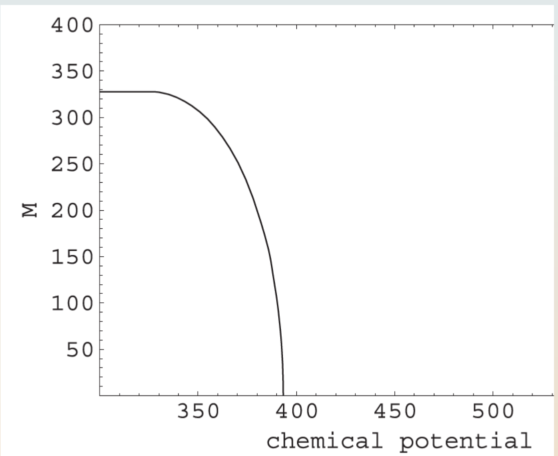
Small Mass

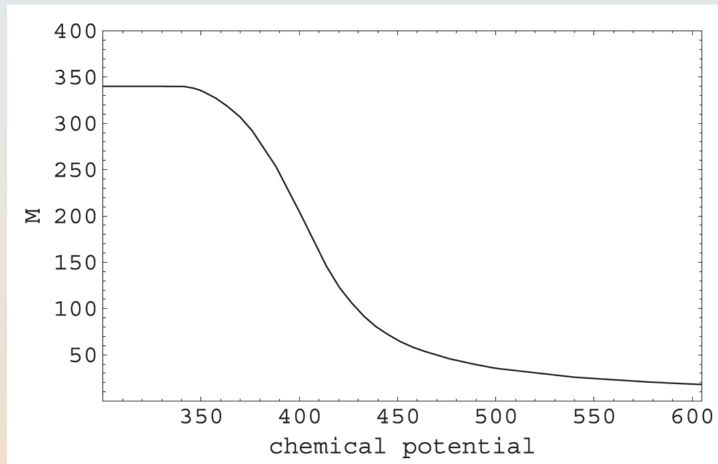
$T > AV$

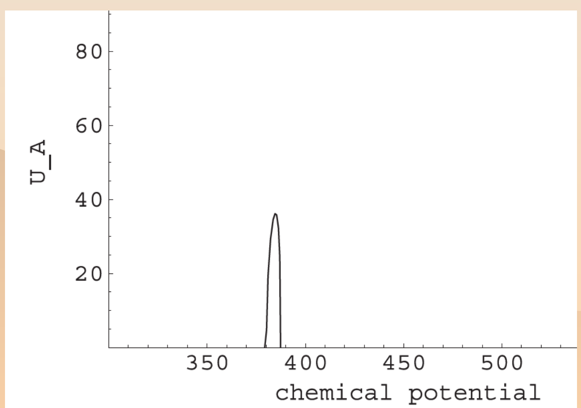
§2-3 Spin-Polarization in NJL model

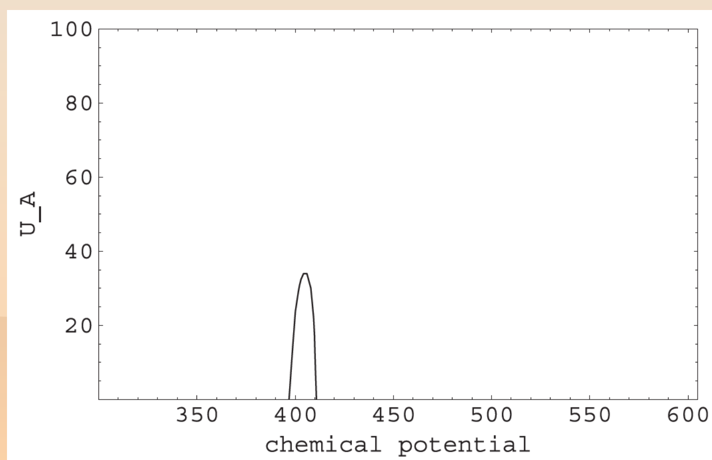
Spin-Polarization Phase below Chiral Restoration Density

NJL with AV-Int.   E. Nakano D-Thesis, S.Maedan PTP118, 729 (2008)









Spin-Polar. appears only in very small density region

まとめ

ゼロ・レンジ擬ベクトル(AV)相互作用、テンソル(T)相互作用による自発的スピン偏極の機構の比較を行った。

両者は非相対論極限では一致する

相対論では二つの寄与

(1) Spin-upとSpin-down のフェルミ面のズレ （数の差）

(2) AV場、T場による Dirac Spinor の変化　フェルミ分布全体から寄与

大きな質量：AVが現れやすい

小さい質量：Tが現れやすい

超相対論極限(質量ゼロ極限)

AV-Type では (1) と (2) が相殺　AV場はゼロ

T-Type では　相殺されない

§3 NJL model with T-int.

Lagrangian

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_s + \mathcal{L}_T$$

SU(2) Chiral Symmetry

$$\begin{aligned}
\mathcal{L}_K &= \bar{\psi} i \gamma \cdot \partial \psi, \\
\mathcal{L}_S &= -\frac{G_S}{2}\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5 \tau \psi)^2\right], \\
\mathcal{L}_T &= -\frac{G_T}{2}\left[(\bar{\psi}\sigma_{\mu\nu}\psi)(\bar{\psi}\sigma^{\mu\nu}\psi) + (\bar{\psi} i \tau_a \gamma_5 \sigma_{\mu\nu}\psi)(\bar{\psi} i \tau_a \sigma^{\mu\nu}\gamma_5 \psi)\right],
\end{aligned}$$

SU(2) chiral Symmetry ***UA(1) symmetry*** **is impossible**

$G_T < 0$: Iso-Scalar Spin-Polarization

same spin-direction of u- and d- quarks

$G_T > 0$: Iso-Vector Spin-Polarization

opposite spin-direction of u- and d- quarks

This Time

Tensor Type Spin Polarization (SP) Phase

Dirac Eq.

$$[\not{p} - M_q - U_T \Sigma_z]\, u(\boldsymbol{p}, s) = 0$$

Mean Field

$$\begin{aligned} M_q &= G_S \rho_s = G_S < \bar{\psi}\psi >, \\ U_T &= G_T \rho_T = G_T < \bar{\psi}\Sigma_z \psi > - G_T < \bar{\psi}\Sigma_z \tau_a \psi > \tau_a . \end{aligned}$$

Hartree Eq.

$$1 - \frac{G_s \rho_s}{M_q} = 0, \qquad 1 - \frac{G_T \rho_T}{U_T} = 0,$$

$$e(\boldsymbol{p}, s) = \sqrt{(\sqrt{m^2 + \boldsymbol{p}_T^2} + sU_T)^2 + p_z^2}. = \sqrt{E_p^2 + 2sU_T\sqrt{m^2 + \boldsymbol{p}_T^2} + U_T^2}.$$

Fermi Distribution is Oblately Deformed (AV -> Prolate)

T $\Rightarrow$ $<\beta\sigma_z>$ **: maximum** **AV** $\Rightarrow$ $<\sigma_z>$ **: maximum**

T-SP increases Magnetic Field

Quark Propagator

$$[\not{p} - M_q - U_T \Sigma_z]\, S(p) = 1.$$

$$S(p) = S_F(p) + S_D(p)$$

$$S_F(p) = \frac{[\gamma_\mu p^\mu + M_q + \Sigma_z U_T]\left\{p^2 - M_q^2 + U_T^2 + 2U_T(p_z\gamma_5\gamma^0 - p_0\gamma_5\gamma^3)\right\}}{[p_0^2 - e^2(\boldsymbol{p},1) + i\delta]\,[p_0^2 - e^2(\boldsymbol{p},-1) + i\delta]},$$

$$S_D(p) = \sum_{s=\pm1}[\gamma_0 e - \boldsymbol{\gamma}\cdot\boldsymbol{p} + M_q + \Sigma_z U_T]\left\{1 + \frac{s(p_z\gamma_5\gamma^0 - p_0\gamma_5\gamma^3) + sU_T}{\sqrt{\boldsymbol{p}_T^2 + M_q^2}}\right\} \times \frac{i\pi}{2e(\boldsymbol{p},s)} n(\boldsymbol{p},s)\delta[p_0 - e(\boldsymbol{p},s)],$$

Density

$$\rho_q = N_d \sum_{s=\pm1}\int \frac{d^3p}{(2\pi)^3}\theta[e_F - e(\boldsymbol{p},s)],$$

$$\rho_s = N_d \int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\,[iS(p)]\,,$$

$$\rho_T = N_d \int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\,[i\Sigma_z S(p)]\,.$$

Vacuum Polarization

Proper Time Regularization

Energy Denominator

$$D(p) = D_+(p)D_-(p) = [p_0^2 - e^2(\boldsymbol{p}, +1)][p_0^2 - e^2(\boldsymbol{p}, -1)]$$

$$= \left(p_0^2 - \boldsymbol{p}^2 - M_q^2 - U_T^2 - 2U_T\sqrt{\boldsymbol{p}_T^2 + M_q^2}\right)\left(p_0^2 - \boldsymbol{p}^2 - M_q^2 - U_T^2 + 2U_T\sqrt{\boldsymbol{p}_T^2 + M_q^2}\right)$$

Thermo Dynamical Potential in Proper Time Regularization

$$\begin{aligned}
\Omega_{vac} &= iN_d \int \frac{d^4p}{(2\pi)^4} \ln[D(p)] = -iN_d \int \frac{d^4p}{(2\pi)^4} \int_{1/\Lambda^2}^{\infty} \frac{d\tau}{\tau} e^{\tau D_+(p)} - iN_d \int \frac{d^4p}{(2\pi)^4} \int_{1/\Lambda^2}^{\infty} \frac{d\tau}{\tau} e^{\tau D_-(p)} \\
&= N_d \sum_s \int_0^{\infty} \frac{d\tau}{\tau} \int \frac{d^4p_E}{(2\pi)^4} e^{\tau\left[-p_i^2 - p_z^2 - \left(\sqrt{p_T^2 + M_q^2} + sU_T\right)^2\right]} \\
&= \frac{N_d}{8\pi^2} \sum_s \int_{1/\Lambda^2}^{\infty} \frac{d\tau}{\tau^2} \int_{M_q}^{\infty} dE_T E_T e^{-\tau(E_T + sU_T)^2}.
\end{aligned}$$

Vacuum Parts of Scalar and Tensor Densities

$$\rho_s(V) = \frac{\partial \Omega_{vac}}{\partial M_q}, \qquad \rho_T(V) = \frac{\partial \Omega_{vac}}{\partial U_T}$$

Scalar Density

$$\rho_s = \rho_s(V) + \rho_s(D)$$

$$\rho_s(D) = N_d \sum_{s=\pm 1} \int \frac{d^3p}{(2\pi)^3}\, n(\boldsymbol{p}, s)\, \frac{M_q}{e(\boldsymbol{p}, s)} \left(1 + \frac{sU_T}{\sqrt{M_q^2 + p_T^2}}\right)$$

Medium Part

$$\rho_s(V) = -\frac{N_d M_q}{8\pi^2} \sum_s \int_{1/\Lambda^2}^{\infty} \frac{d\tau}{\tau^2} e^{-\tau(M_q + sU_T)^2}$$

Vacuum Part

Proper Time Appr.

Tensor Density

$$\rho_T(D) = \frac{N_d}{8\pi^2} \sum_{s=\pm 1} \int d^3p\, n(\boldsymbol{p}, s)\, \frac{s\sqrt{\boldsymbol{p}_T^2 + M_q^2} + U_T}{e(\boldsymbol{p}, s)}.$$

Strong Cut-Off and Regularization Dependence

Energy Cut-Off, Proper-Time disturb **Spin-Polarization (SP)**

Momentum Cut-Off, Effective Potential Method **support** **SP**

Ignoring Vacuum Parts of **T-Density** in this work

Dynamical Quark Mass in Spin-Saturated System

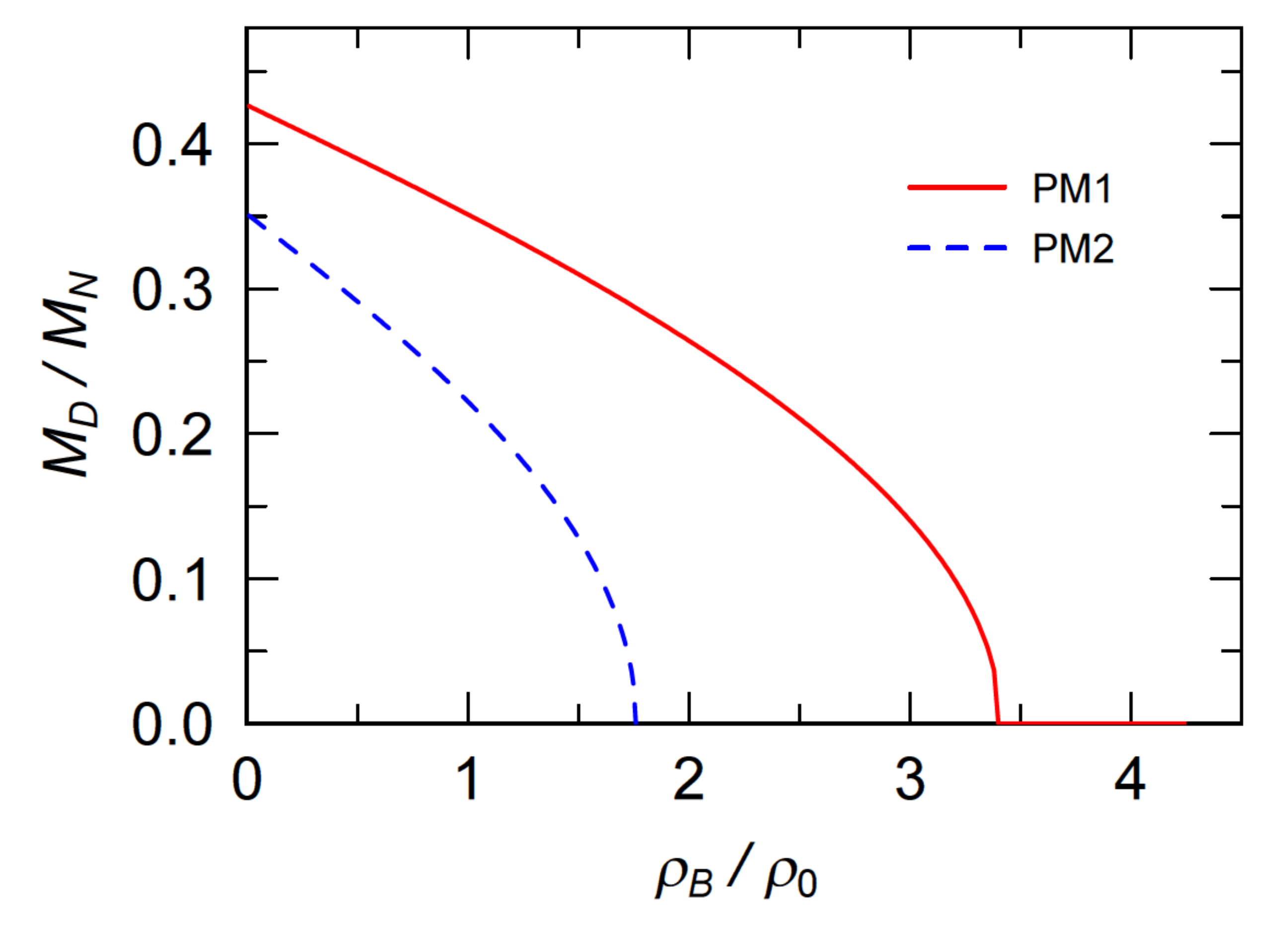
Parameters

PM1 $(G_S\Lambda^2 = 6, \Lambda = 850\ \mathrm{MeV})$

PM2 $(G_S\Lambda^2 = 6.35, \Lambda = 660.37\ \mathrm{MeV})$

Zero Current Mass $m = 0$

E. Nakano and T. Tatsumi, PR D71, 114006 (05).



Phase-Transition

Parameters

$$\text{PM1}\ (G_S\Lambda^2 = 6, \Lambda = 850\ \text{MeV})$$

$$\text{PM2}\ (G_S\Lambda^2 = 6.35, \Lambda = 660.37\ \text{MeV})$$

Zero Current Mass ***m*** **= 0**

E. Nakano and T. Tatsumi, PR D71, 114006 (05).

Hartree Equation

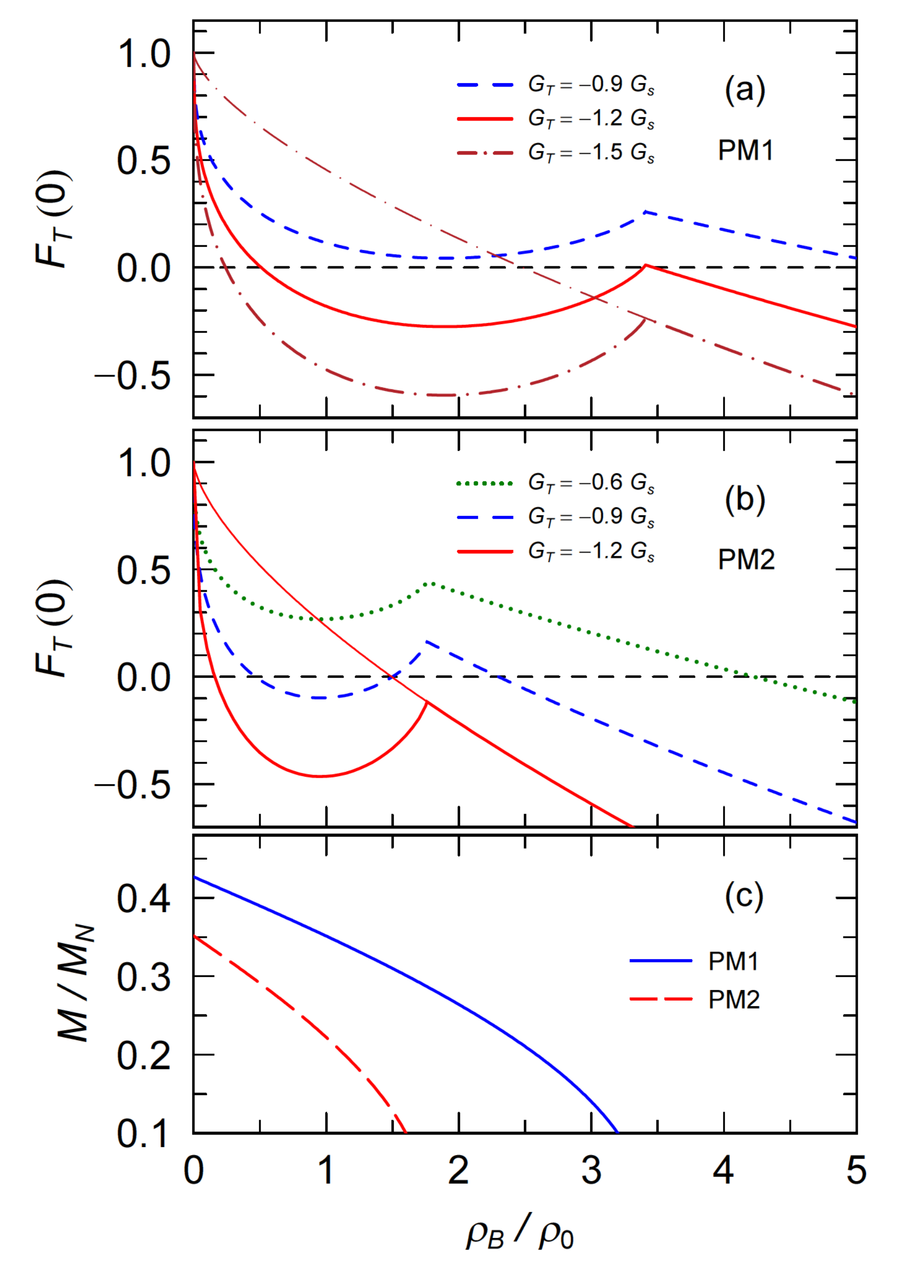
$$F_T(U_T) = 1 - \frac{G_T\rho_T}{U_T} = 0.$$

$$F_T(U_T) \to 1 \ \text{when}\ U_T \to \infty$$

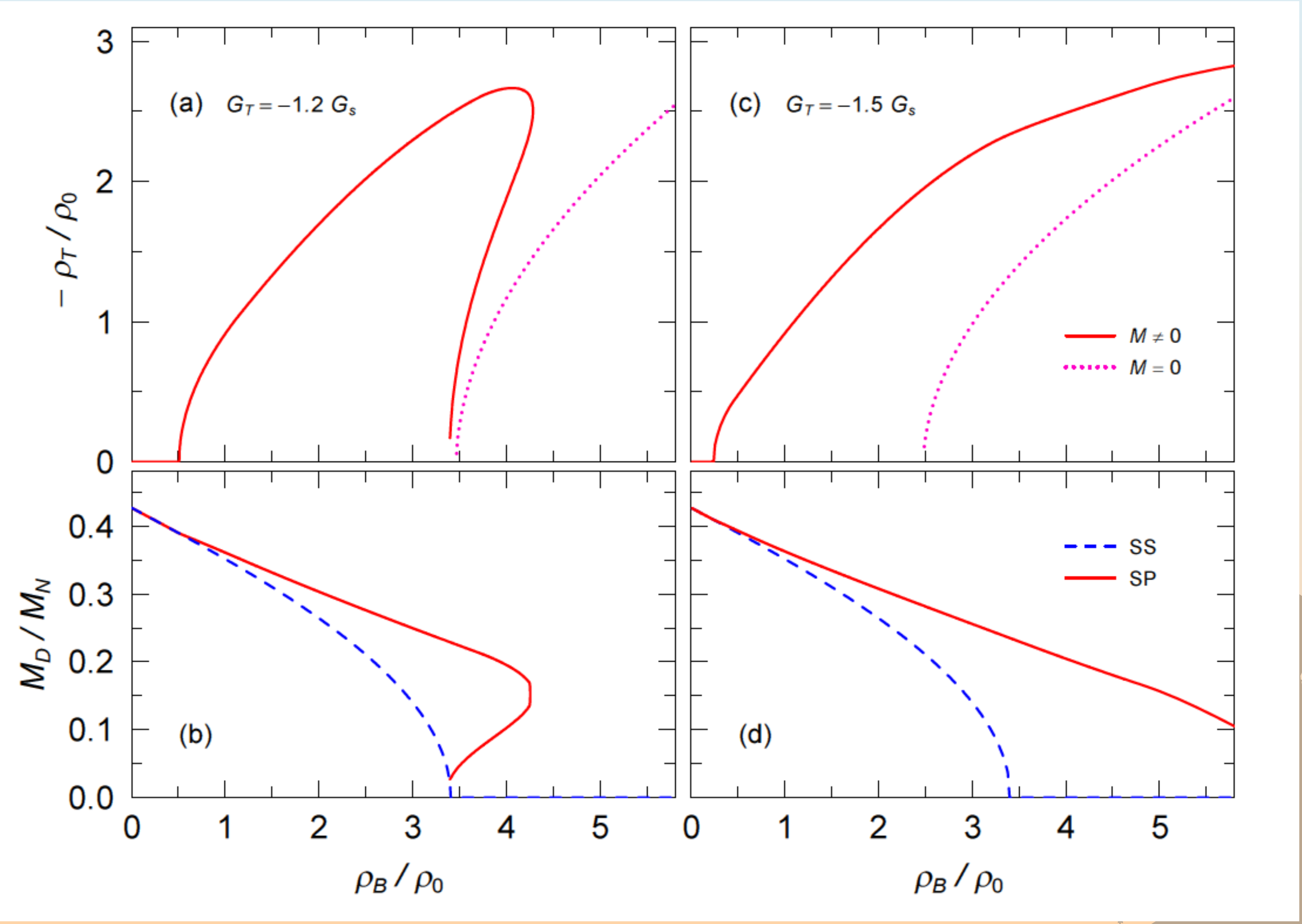
Spontaneous SP Condition

$$F_T(0) \ < \ 0$$

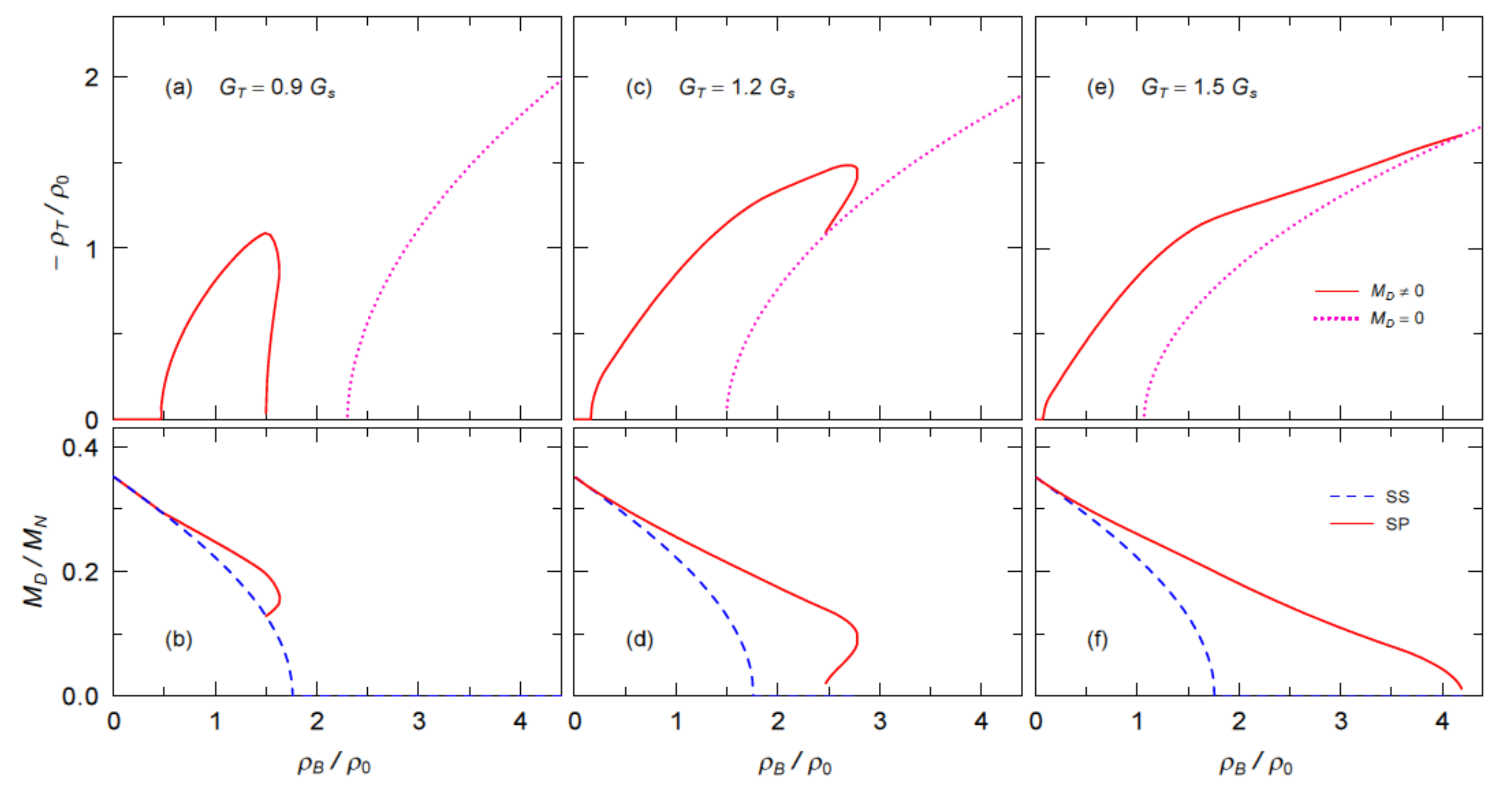
$$1 + \frac{G_T N_d}{2\pi^2}\left\{p_F E_F + \frac{M_q^2}{2}\ln\left(\frac{E_F + p_F}{E_F - p_F}\right)\right\} \leq 0$$



Results of PM1



Results of PM2



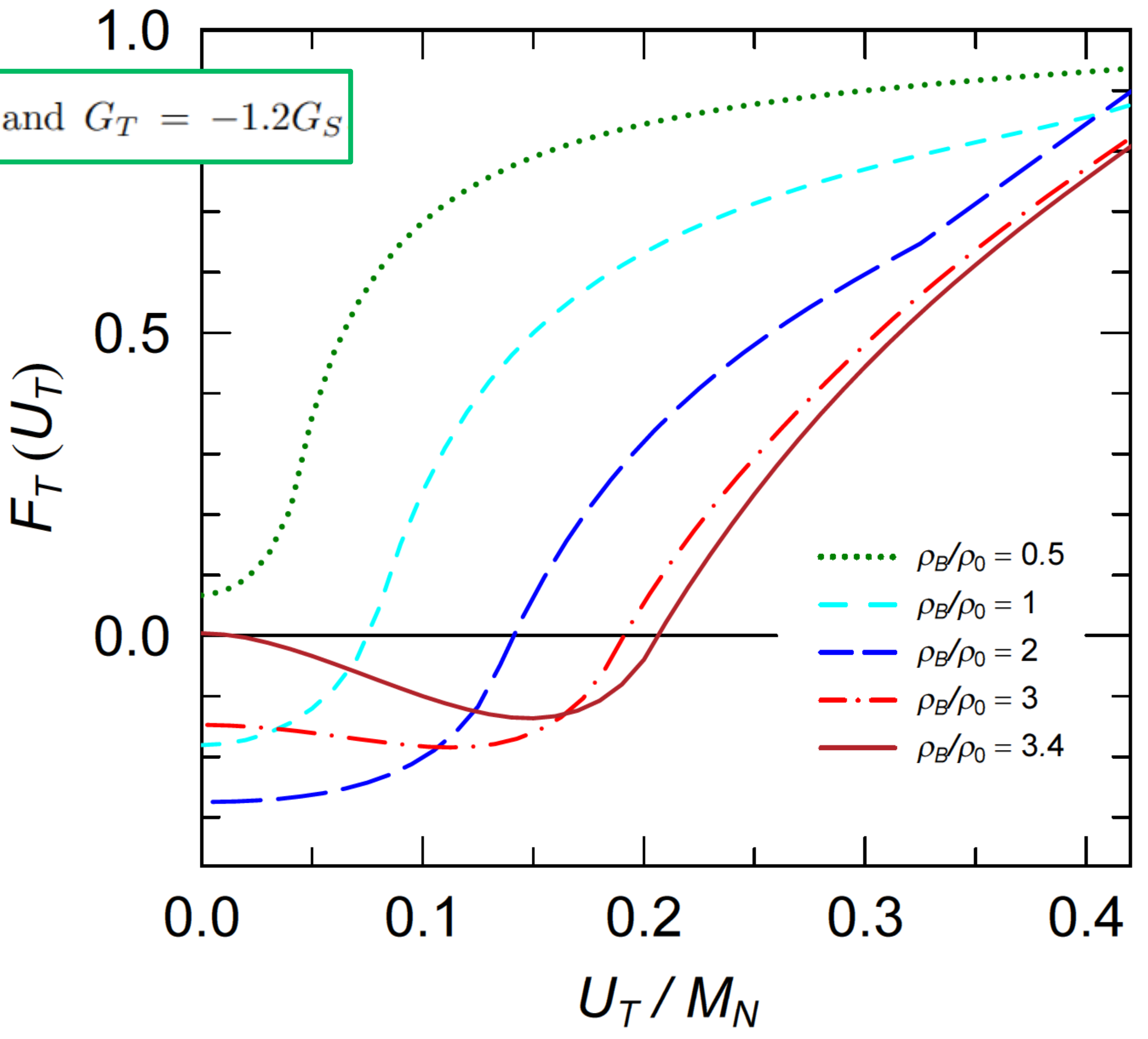
Two Kinds of SP phase

Chiral Breaking SP Phase ($M \neq 0$)

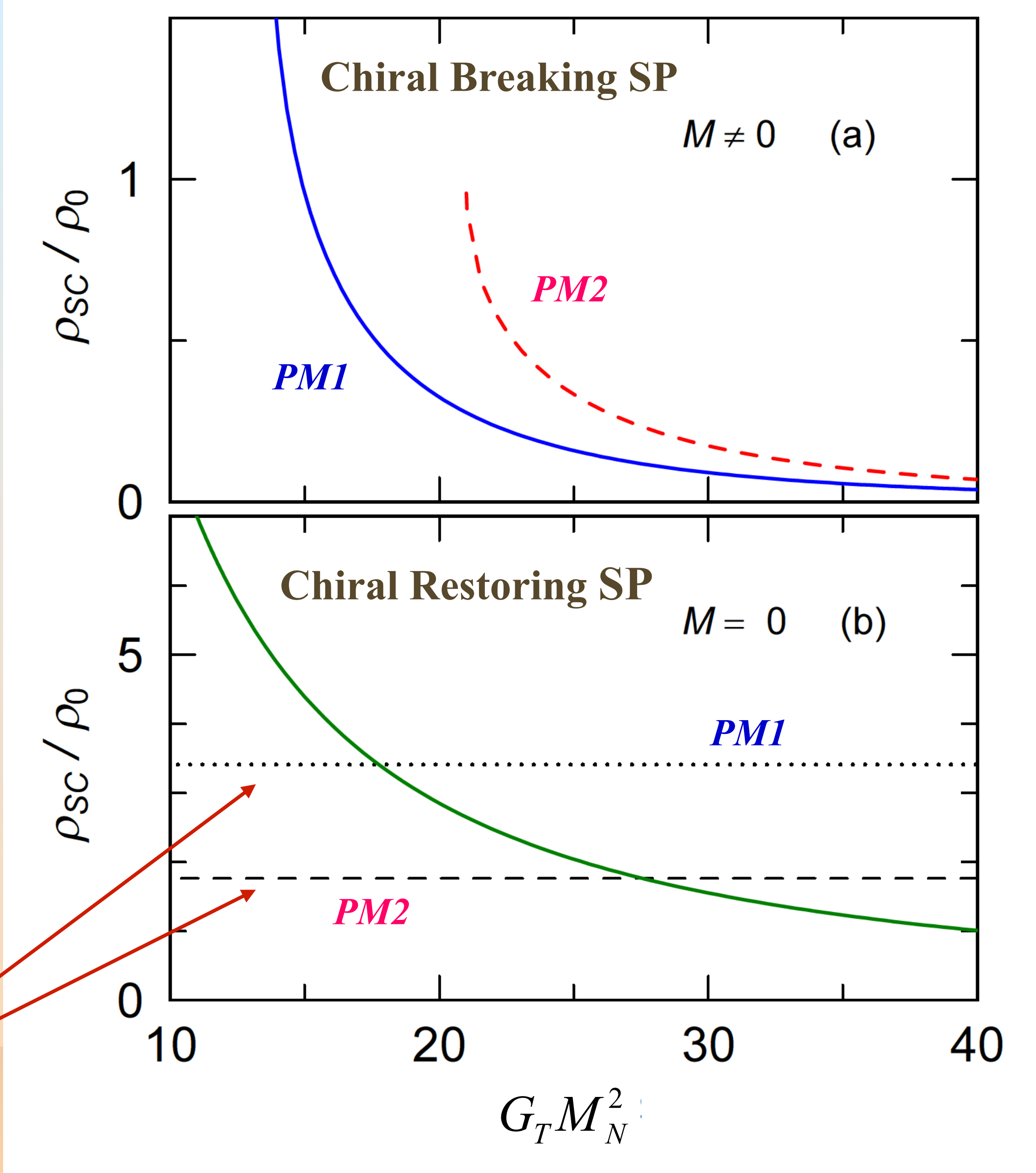
Chiral Restoring SP Phase ($M = 0$)

1st Order Transition ?

PM1 and $G_T = -1.2 G_S$



Critical Density of SP phase



Critical Density of
Chiral Phase Transition
in **Spin-Saturated** Phase

まとめ

1) ２種類のT-Typesスピン偏極

Chiral Broken Spin-Polarization (CBSP) ($M_q > 0$)

Chiral Restoring Spin-Polarzation (CRSP) ($M_q = 0$)

2) CRSPは必ず現れるが

CBSPは強い結合定数のときのみ

3) 同じ密度で両者が存在しうる ⇒ １次相転移の可能性

4) CBSP によりChiral Broken相がより高密度まで広がる

§4 Vacuum Polarization of Tensor Field

Cut-Off Formula of Momentum Distribution in Dirac Sea

$$\rho_T = -4iN_d U_T \int \frac{d^4p}{(2\pi)^4} \frac{p_0^2 + M^2 - U_T^2 + \boldsymbol{p}_T^2 - p_z^2}{[p_0^2 - e^2(\boldsymbol{p},+1)][(p_0^2 - e^2(\boldsymbol{p},-1)]}.$$

$$\rho_T(D) = N_d \sum_{s=\pm 1} \int \frac{d^3p}{(2\pi)^3} \Theta[e_F - e(\boldsymbol{p},s)] \frac{s\sqrt{\boldsymbol{p}_T^2 + M^2} + U_T}{e(\boldsymbol{p},s)},$$

$$\rho_T(V) = -N_d \sum_{s=\pm 1} \int \frac{d^3p}{(2\pi)^3} \underline{n_v(\boldsymbol{p},s)} \frac{s\sqrt{\boldsymbol{p}_T^2 + M^2} + U_T}{e(\boldsymbol{p},s)}.$$

Opposite Sign

Vacuum Momentum-Distribution

Energy Cut-Off : $n_v(\boldsymbol{p},s) = \Theta[\Lambda_e - e(\boldsymbol{p},s)]$

$$\rho_T(D) < 0 < \rho_T(V)$$

Momentum Cut-Off : $n_v(\boldsymbol{p},s) = \Theta(\Lambda_p - |\boldsymbol{p}|)$

$$\rho_T(V) = -N_d \sum_{s=\pm1} \int_0^{\Lambda_p} \frac{d^3p}{(2\pi)^3} \frac{s\sqrt{\boldsymbol{p}_T^2 + M^2} + U_T}{e(\boldsymbol{p},s)} = -N_d \sum_{s=\pm1} \int_0^{\Lambda_p} \frac{d^3p}{(2\pi)^3} \frac{s(E_T + sU_T)}{E_p\sqrt{1 + \frac{2sE_TU_T + U_T^2}{E_p^2}}}$$

$$\approx -N_d \sum_{s=\pm1} \int_0^{\Lambda_p} \frac{d^3p}{(2\pi)^3} \frac{s(E_T + sU_T)}{E_p}\left(1 - \frac{sE_TU_T}{E_p^2}\right) \approx -2N_dU_T \int_0^{\Lambda_p} \frac{d^3p}{(2\pi)^3} \frac{p_z^2}{E_p^3} < 0.$$

when $U_T << 1$

Relativistic Order

$$\rho_T(V) < 0$$

State Number $\Rightarrow$ $\rho_T > 0$

Dirac Spinor $\Rightarrow$ $\rho_T < 0$

Cut-Off Infinite Limit

Which effect is larger?

$\Lambda_{e,p} \to \infty$	$\rho_T(V)$
Energy Cut-Off	$\to U_T \Lambda_e^2$
Mom. Cut-Off	$\to -U_T \Lambda_p^2$

Note : $\rho_T \neq 0$ even when $m = 0$ in Momentum Cut-Off

Proper Time Regularization

$$\rho_T(V) = \frac{N_d}{4\pi^2}\Lambda^2 \int_{M-U_T}^{M+U_T} dE_T F_1\left(\frac{E_T^2}{\Lambda^2}\right) + \frac{N_d}{8\pi^2} U_T \Lambda^2 \sum_s F_2\left[\frac{(M+sU_T)^2}{\Lambda^2}\right]$$

$$F_n(x) = x\int_x^\infty \frac{d\tau}{\tau^n} e^{-\tau}$$

When $\Lambda \to \infty$

$$\rho_T(V) \approx \frac{N_d}{4\pi^2}\left\{\Lambda^2 U_T + (M_q^2 U_T - \frac{1}{3}U_T^3)\ln\frac{\Lambda^2}{|M_q^2 - U_T^2|} - \frac{1}{3}M_q^3 \ln\left(\frac{M_q+U_T}{M_q-U_T}\right)^2 + \frac{1}{3}M_q^2 U_T - \frac{5}{9}U_T^3\right\}$$

Surface Area of Dirac Sea

When $M \gg U_T$

$$\rho_T(V) \approx \frac{N_d}{4\pi^2}\left\{\Lambda^2 U_T + (M_q^2 U_T - \frac{1}{3}U_T^3)\ln\frac{\Lambda^2}{M_q^2} - M_q^2 U_T - \frac{4}{9}U_T^3\right\}$$

Infinite term

in Usual Renormalization

$$\Omega_R = \Omega_{vac} - \Omega_{counter}$$

$$\frac{\partial^2 \Omega_R}{\partial U_T^2} = finite, \quad \frac{\partial^4 \Omega_R}{\partial U_T^4} = finite, \quad \frac{\partial^4 \Omega_R}{\partial U_T^2 \partial M_q^2} = finite,$$

Condition of Spontaneous Spin-Polarization

$$1+\frac{M_q G_T}{4\pi^2}\int_{1/\Lambda^2}^{\infty}\frac{d\tau}{\tau^2}e^{-\tau M_q^2}(1+2M_q^2\tau)-\frac{G_T}{2\pi^2}\left\{p_F E_F+\frac{M_q^2}{2}\ln\left(\frac{E_F+p_F}{E_F-p_F}\right)\right\}\leq 0.$$

$$p_F^2 \geq \frac{\Lambda^2}{2} - \frac{2\pi^2}{G_T}.$$

When $M_q = 0$,

Surface Area of Dirac Sea

Too Strong Cut-Off Dependence

Surface Area of Dirac Sea must be removed by Renormalization

In Scalar Density, Λ is determined from M_q

Vacuum Contribution ↔ Physical Quantity

Renormalization

Related Physical Quantity $\rightarrow$ Magnetic Susceptibility χ_{M}

Lattice QCD $\rightarrow$ $\chi_{\mathrm{M}} < 0$ G. S. Bali, et al, PRD 86, 094512 (2012)

$\Rightarrow$ **Negative Spin Susceptibility**

$$\frac{\partial \Omega_{vac}}{\partial U_T^2} = \frac{\partial \rho_T}{\partial U_T} < 0$$

Renormalizaion is needed

$$\boxed{\Omega_R = \Omega_{vac}(U_T, M_q) - \Omega_C}$$

Renormalization of the Vacuum Polarization

$$\Omega_R = \Omega_{vac} - \frac{1}{2}\beta_T U_T^2, \qquad \rho_T(R) = \rho_T(V) - \beta_T U_T.$$

R1) $\dfrac{\partial^2 \Omega_R}{\partial U_T^2} = 0$ **when $M_q = 0$ (Chiral Restoring Phase)**

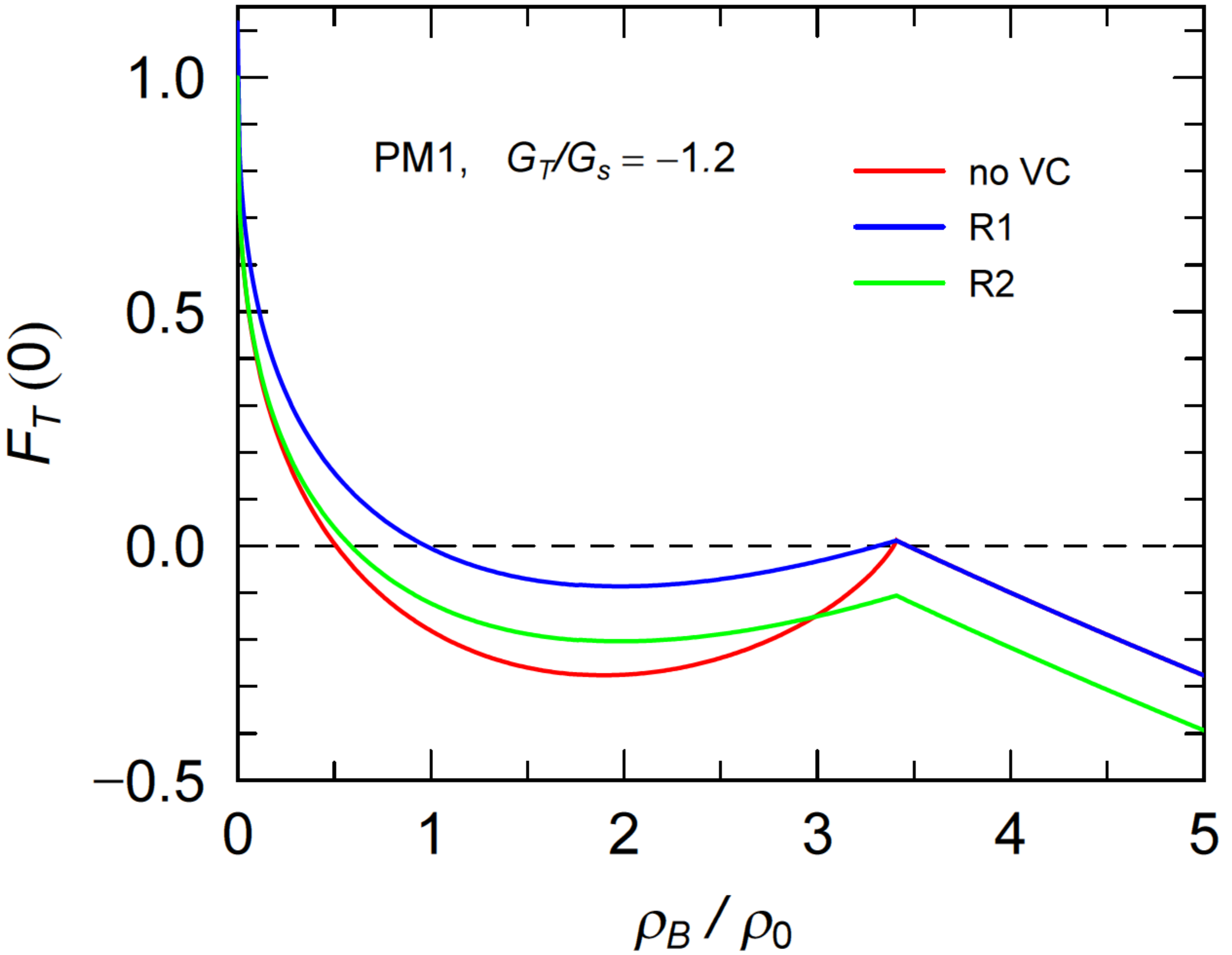
Problem: Infrared divergence

indep. of Cut-Off and Regularization

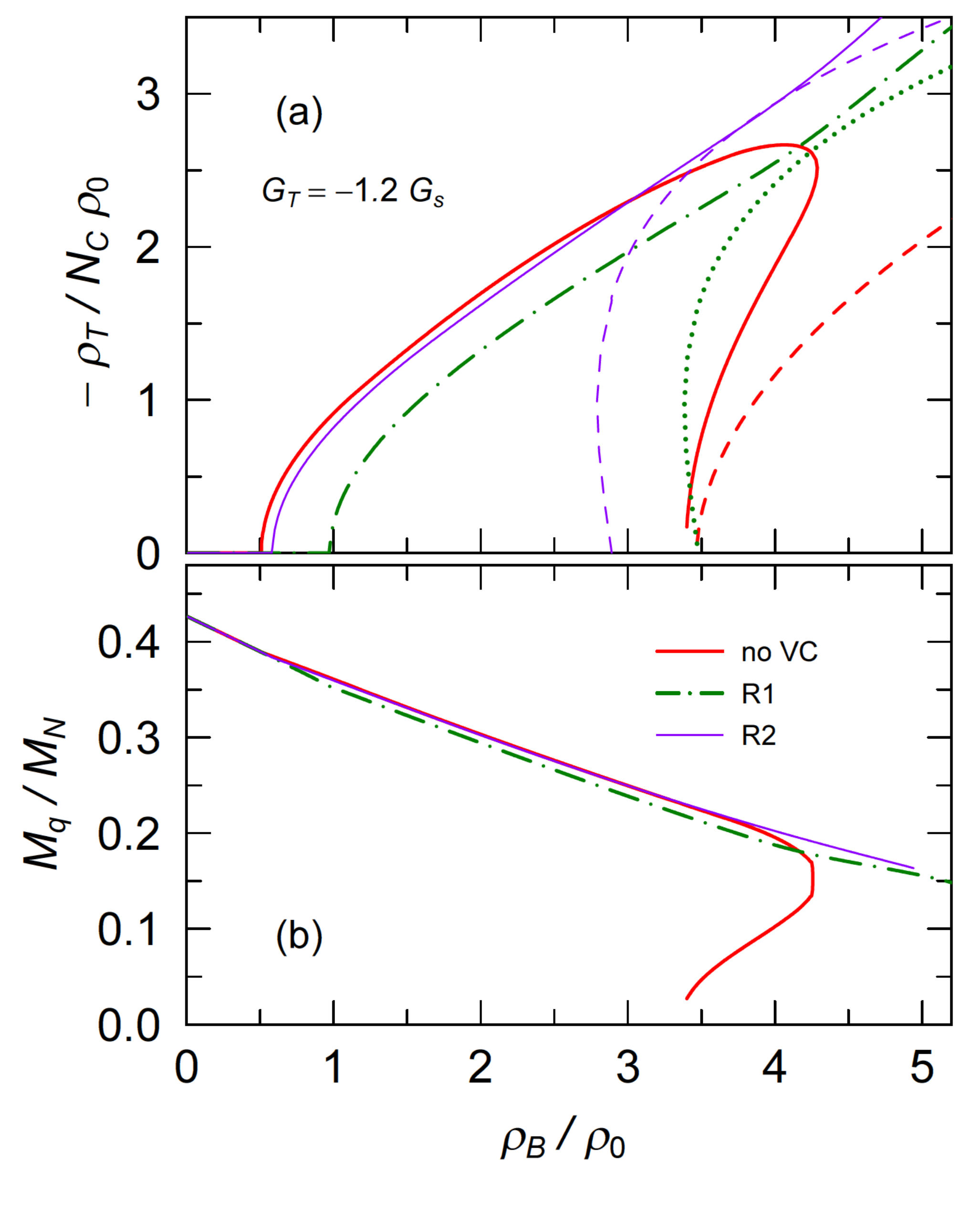
$$\frac{\partial^4 \Omega_R}{\partial U_T^4} \to \infty, \quad \frac{\partial^4 \Omega_R}{\partial U_T^2 \partial M_q^2} \to \infty \quad \text{when } M_q \to 0$$

R2) $\dfrac{\partial^2 \Omega_R}{\partial U_T^2} = 0$ **when $M_q = M_0$ (Vacuum)**

Phase Transition



Vacuum Contribution



真空偏極はCBSP層を高密度まで保持させる

R1 CBSP相転移を高密度へ

CRSP相転位密度は変化せず

R2 CBSPを変化させず

CRSP相転位密度を小さくする

まとめ

NJL模型で真空の寄与を決めることはできない

T密度でDirac Seaの表面積に比例する項が現れてしまう
繰り込みが必要

AV型スピン偏極では現れれない

$$\rho_A(V) \approx \frac{N_d}{\pi^2} U_A M_q^2 \ln\left(\frac{\Lambda}{M_q}\right).$$

カウンター項が必要　繰り込み点に結果は依存

格子QCD → 真空の磁気感受率が負

→ 真空の寄与はスピン偏極を増大

§5 Summary

NJL model with Scalar and Tensor Interaction

Spontaneous Spin-Polarization and Chiral Resoraion

T-Type Spin Polarization Phases

Chiral Breaking SP Phase ($M \neq 0$) when the coupling $-G_T$ is large

Chiral Restoring SP Phase ($M = 0$) with any G_T

$G_T < 0$: Iso-Scalar Spin-Polarization

$G_T > 0$: Iso-Vector Spin-Polarization

Future Problem

1) Vacuum Polarization Exact Renormalization ex. Linear Sigma Model
2) Current Quark Mass
3) Hybrid SP phase with AV and T interaction

§6 AV & T Hybrid SP Phase in Mass Zero Quark

Single Particle Energy when $M = 0$

$$(p_0^2 - \boldsymbol{p}^2)^2 - 2(p_0^2 - \boldsymbol{p}^2)(U_A^2 + U_T^2) - 4p_z^2 U_A^2 - 4\boldsymbol{p}_T^2 U_T^2 + (U_A^2 - U_T^2)^2 = 0.$$

$$e^2 = \boldsymbol{p}^2 + (U_A^2 + U_T^2) + 2s\sqrt{p_z^2 U_A^2 + \boldsymbol{p}_T^2 U_T^2 + U_A^2 U_T^2}.$$

Energy Minimum Point when $s = -1$

$$p_z = \pm U_T,\ p_T = 0 \ \text{when} \ |U_A| > |U_T|$$

AV−Type

$$p_T = \pm U_A,\ p_z = 0 \ \text{when} \ |U_A| < |U_T|$$

T−Type

U_A or U_T plays a role of Mass

$$
\begin{aligned}
\rho_A &= \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi)^3} n(e(\boldsymbol{p},s)) \frac{U_A(e^2+m^2-U_A^2+U_T^2+p_z^2-\boldsymbol{p}_T^2)}{e(e^2-\boldsymbol{p}^2-U_A^2-U_T^2)} \\
&= \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi)^3} n(e(\boldsymbol{p},s)) \frac{U_A\left[\boldsymbol{p}^2+U_A^2+U_T^2+2s\sqrt{p_z^2U_A^2+\boldsymbol{p}_T^2U_T^2+U_A^2U_T^2}-U_A^2+U_T^2+p_z^2-\boldsymbol{p}_T^2\right]}{e\left[U_A^2+U_T^2-U_A^2-U_T^2+2s\sqrt{p_z^2U_A^2+\boldsymbol{p}_T^2U_T^2+U_A^2U_T^2}\right]} \\
&= \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi)^3} n(e(\boldsymbol{p},s)) \frac{sU_A\left[p_z^2+U_T^2+s\sqrt{p_z^2U_A^2+\boldsymbol{p}_T^2U_T^2+U_A^2U_T^2}\right]}{e\sqrt{p_z^2U_A^2+\boldsymbol{p}_T^2U_T^2+U_A^2U_T^2}} \\
\rho_T &= \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi)^3} n(e(\boldsymbol{p},s)) \frac{U_T(e^2+U_A^2-U_T^2-p_z^2+\boldsymbol{p}_T^2)}{e(e^2-\boldsymbol{p}^2-U_A^2-U_T^2)}. \\
&= \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi)^3} n(e(\boldsymbol{p},s)) \frac{sU_T\left[\boldsymbol{p}_T^2+U_A^2+s\sqrt{p_z^2U_A^2+\boldsymbol{p}_T^2U_T^2+U_A^2U_T^2}\right]}{e\sqrt{p_z^2U_A^2+\boldsymbol{p}_T^2U_T^2+U_A^2U_T^2}} \qquad (2.34)
\end{aligned}
$$