超流動体における対称性の破れとトポロジー：^{3}Heから中性子星まで

Takeshi Mizushima

Department of Material Engineering Science, Osaka University

大阪大学 OSAKA UNIVERSITY

科研費 KAKENHI

Topological Materials Science

トポロジーが紡ぐ物質科学のフロンティア

Masatoshi Sato
(YITP, Kyoto)

James Sauls
(Northwestern)

Kazushige Machida
(Ritsumeikan)

Satoshi Fujimoto
(Osaka)

Yukio Tanaka
(Nagoya)

Muneto Nitta
(Keio)

大阪大学 OSAKA UNIVERSITY

科研費 KAKENHI

Topological Materials Science
トポロジーが紡ぐ物質科学のフロンティア

OUTLINE

- **^{3}He and Neutron Stars**

  1. ^{3}He-A: Weyl fermions & chiral anomaly
  2. ^{3}He-B: Topology & Majorana fermions

- **Topological $^3P_2$ Superfluids in Neutron Stars**

- **Nambu Sum Rule**

- **Topological Phases in Confined Superfluid ^{3}He-B**

TM, K. Masuda, and M. Nitta, arXiv:1607.07266
J. A. Sauls and **TM**, arXiv:1611.07273 (PRB in press)
TM, Y. Tsutsumi, T. Kawakami, M. Sato, M. Ichioka, K. Machida, JPSJ **85**, 022001 (2016)

Cold Universe

Temperature [K]

10^{12} Degeneracy of dense nuclear matter

10^{9} BCS condensation in neutron stars

neutron SF & proton SC in NS interiors

$T_{\rm c} \sim 0.001 T_{\rm F}$

10^{6}

10^{3} Degeneracy of electrons in solid states

Superconductivity in me $T_{\rm c} \sim 0.001 T_{\rm F}$

10^{0} Degeneracy of liquid ^{3}He

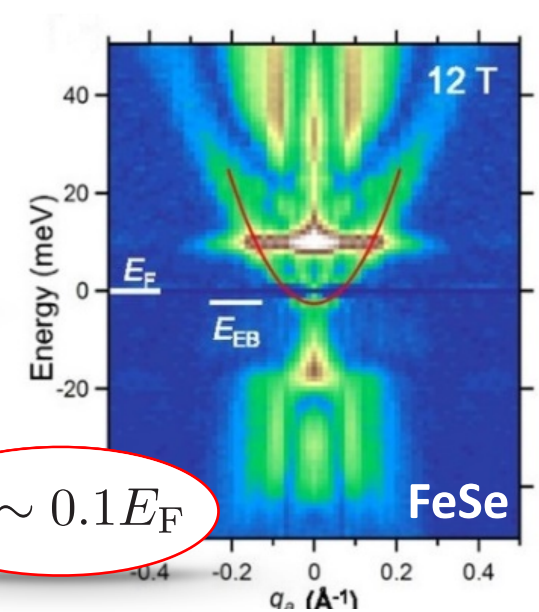
10^{-3} Superfluidity of liquid ^{3}He

$T_{\rm c} \sim 0.001 T_{\rm F}$

10^{-6}

10^{-9} Fermion condensation in cold a $T_{\rm c} \sim 0.1 T_{\rm F}$

Lowest manmade temperature (450 pK @ MIT)



$\Delta \sim 0.1 E_{\rm F}$

Kasahara *et al.*, PNAS (2014)

^{3}He & Neutron Stars

1957 BCS theory

1960 Anderson-Morel
Generalization of BCS

1963 Balian-Werthamer
most symmetric *p*-wave

1970 Spin fluctuation: Layzer-Fay

1972 Discovery of SF in ^{3}He

1980 Observation of amplitude Higgs

1982 Exotic SC in U-compounds

1959 Migdal: prediction of BCS in NS

$$\Delta \sim 1\ \mathrm{MeV}$$

1966 Suppression of s-wave SF: Wolf

1967 Pulsar discovered: Hewish & Bell

1969 Pulsar Glitches observed in Vela
SF in NS: Baym, Pethick, & Pines

1970 $^3P_2$: Tamagaki & Hofferberg-Glassgold-Richardson-Ruderman

1982 Thouless-Kohmoto-Nightingale-Nijs (1985 Kohmoto)

1984 Geometric (Berry) phase

1986 Weyl fermions in ^{3}He-A: Volovik

1988 Topology in ^{3}He-B: Salomaa-Volovik

1997 Observation of chiral anomaly(?)

Topology of SF phases in NS?

2011 $^3P_2$ in NS core(?): Page *et al.*

2008- Topological periodic table

Pairing Symmetry

	Spin	Orbital	Candidate
singlet *s*-wave	Odd	Even	Many metals, Fe-based compounds (multiple gaps)
singlet *d*-wave	Odd	Even	High-Tc cuprates, $CeCoIn_5$, URu_2Si_2
triplet *p*-wave	Even	Odd	^3He, Sr_2RuO_4(?), UBe_{13}(?), UCoGe, $Cu_xBi_2Se_3$(?), … & Neutron stars(?)
triplet *f*-wave	Even	Odd	UPt_3

Several candidate materials for topological or Weyl superconductors

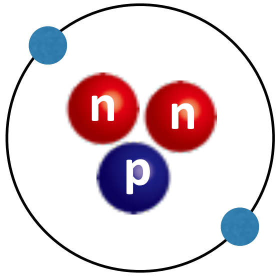
but their pairing symmetries are still controversial…

^3He is one of established topological & Weyl SC/SF

Normal ^{3}He & Dense Nuclear Matter

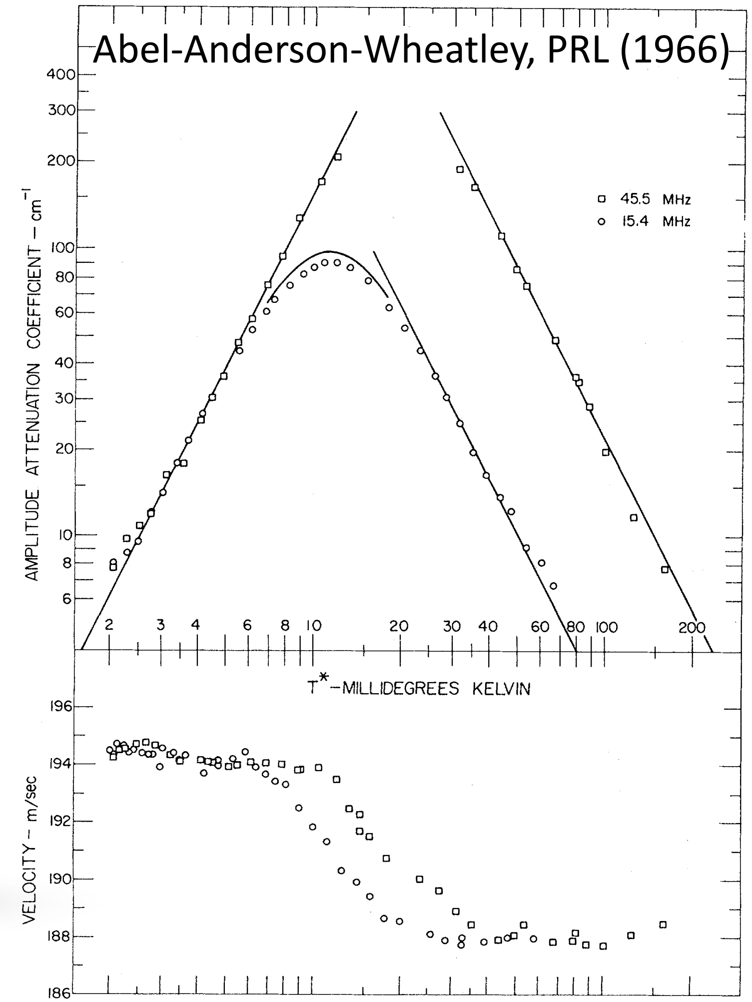
^{3}He atom

spin 1/2 fermion



$S_{el} = 0$, $S_n = 1/2$

Crossover from 1st sound to zero sound



Normal ^{3}He: Fermi liquid w/ large effective mass

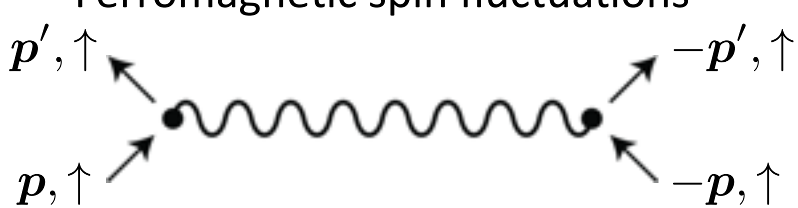
$$m^*/m_0 = 1 + F_0^s/3 = 3 \sim 6$$

$$\chi = \chi_0 \left(\frac{m^*/m_0}{1 + F_0^a} \right)$$

corrections from the internal fields

$F_0^a = -0.75$ strong ferromagnetic exchange int. very close to Stoner instability

Ferromagnetic spin fluctuations



Layzer-Fay, Int. J. Magn. **1**, 135 (1971)

$$V(\boldsymbol{p}, \boldsymbol{p}') = \frac{F_0^a}{1 + F_0^a \chi(\boldsymbol{p}, \boldsymbol{p}')} < 0$$

spin triplet *p*-wave is the dominant pairing channel
subdominant *f*-wave pairing

Order Parameter for Spin-triplet Superfluids

Spin triplet (L=1)
p-wave (S=1) OP

$$\begin{pmatrix} \Delta_{\uparrow\uparrow}(\boldsymbol{k}) & \Delta_{\uparrow\downarrow}(\boldsymbol{k}) \\ \Delta_{\downarrow\uparrow}(\boldsymbol{k}) & \Delta_{\downarrow\downarrow}(\boldsymbol{k}) \end{pmatrix} = \begin{pmatrix} -d_x(\boldsymbol{k}) + i d_y(\boldsymbol{k}) & d_z(\boldsymbol{k}) \\ d_z(\boldsymbol{k}) & d_x(\boldsymbol{k}) + i d_y(\boldsymbol{k}) \end{pmatrix}$$

$$J = S + L = 0, 1, 2$$

$$A_{\mu i}$$

spin momentum

$$d_\mu(\boldsymbol{k}) = A_{\mu i}\hat{k}_i$$

most symmetric p-wave pairing

$^3P_2$ pairing

$$A_{\mu i} = E_0 \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}_{\mu i} + \begin{pmatrix} 0 & V_1 & V_2 \\ -V_1 & 0 & V_3 \\ -V_2 & -V_3 & 0 \end{pmatrix} + \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{12} & T_{22} & T_{23} \\ T_{13} & T_{23} & T_{33} \end{pmatrix}$$

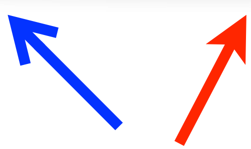
Scalar (J=0) | antisymmetric (J=1) | traceless symmetric (J=2)

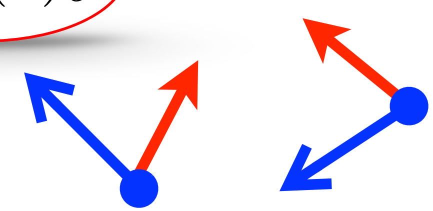
Balian-Werthamer (1963)

$$\mathrm{SO}(3)_S \times \mathrm{SO}(3)_L \times \mathrm{U}(1) \longrightarrow \mathrm{SO}(3)_J$$

$$\mathrm{SO}(3)_{\boldsymbol{L}-\boldsymbol{S}} \times \mathrm{U}(1)$$

spin orbital





Spontaneous spin-orbit symmetry breaking: Emergence of spin-orbit interaction

Superfluid ^{3}He

Symmetry of Normal ^{3}He $G = \mathrm{SO}(3)_{\boldsymbol{S}} \times \mathrm{SO}(3)_{\boldsymbol{L}} \times \mathrm{U}(1)$

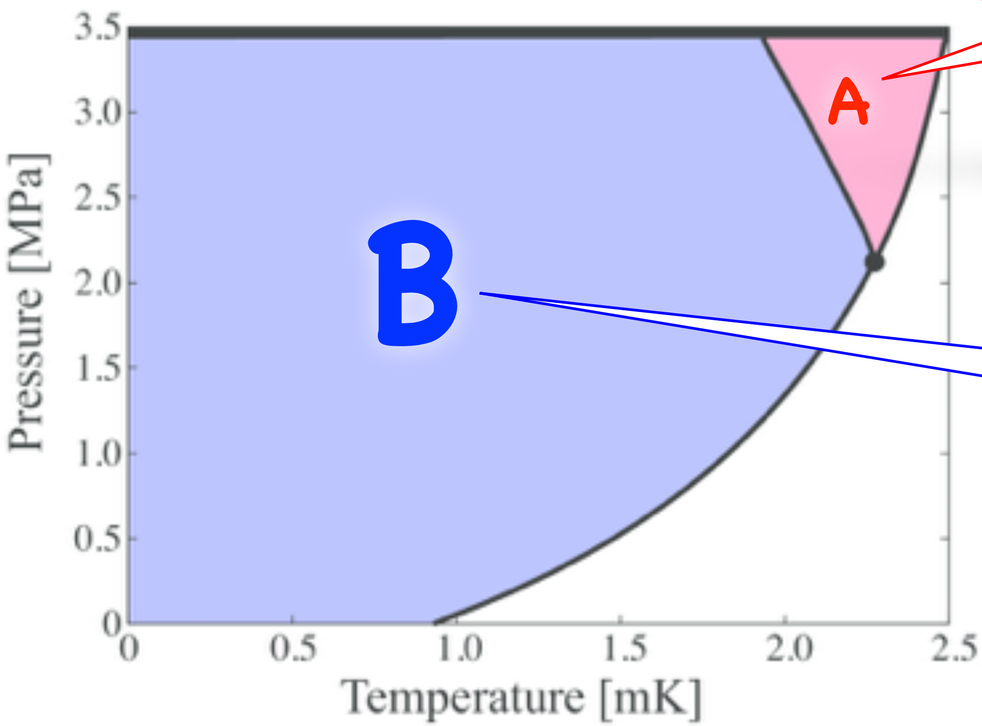
spin rotation momentum

Spin triplet *p*-wave order parameter

$$\Delta_{ab}(\boldsymbol{k}) = i(\boldsymbol{\sigma}\sigma_y)_{ab} \cdot \boldsymbol{d}(\boldsymbol{k})$$

$$d_\mu(\boldsymbol{k}) = A_{\mu i} k_i$$

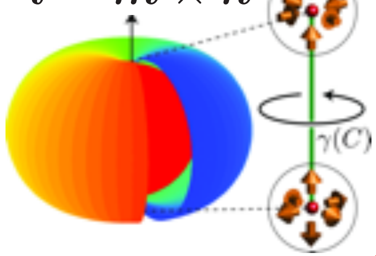
spin momentum



Anderson-Brinkman-Morel (ABM) state

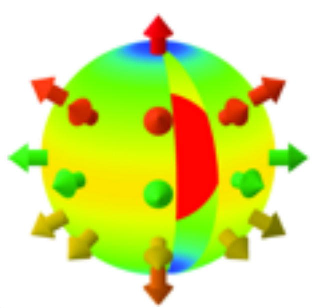
$$A_{\mu i} = \Delta_{\mathrm{A}} \hat{d}_\mu (\hat{\boldsymbol{m}} + i\hat{\boldsymbol{n}})_i$$

$\hat{\boldsymbol{l}} = \hat{\boldsymbol{m}} \times \hat{\boldsymbol{n}}$



$S_z = 0 \quad L_z = 1$

Balian-Werthamer (BW) state



$$A_{\mu i} = \Delta_{\mathrm{B}} \delta_{\mu i}$$

$$\mapsto J = 0$$

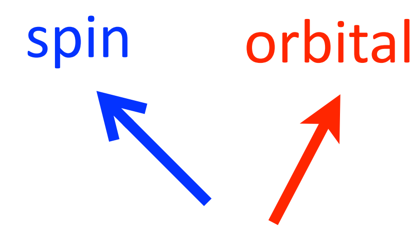
$$(k_x - ik_y)|S_z = 1\rangle + k_z|S_z = 0\rangle + (k_x + ik_y)|S_z = -1\rangle$$

Symmetry Breaking in Superfluid ^{3}He

<u>Symmetry group of ^{3}He</u>

neglect small residual interaction (dipole int.)

$$G = \mathrm{SO}(3)_{\boldsymbol{S}} \times \mathrm{SO}(3)_{\boldsymbol{L}} \times \mathrm{U}(1)$$

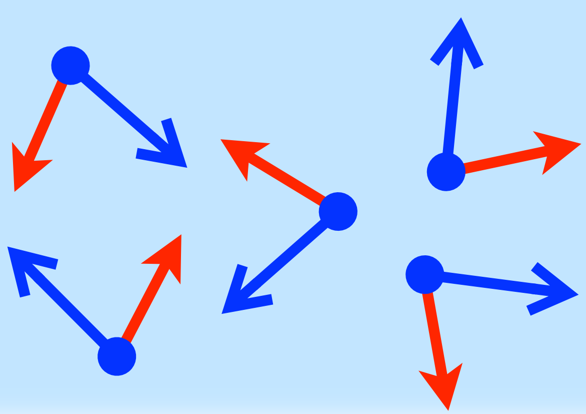


B-phase

$$H = \mathrm{SO}(3)_{\boldsymbol{L}+\boldsymbol{S}}$$

$$S + L = 0$$

<u>Spin-orbit locked phase</u>

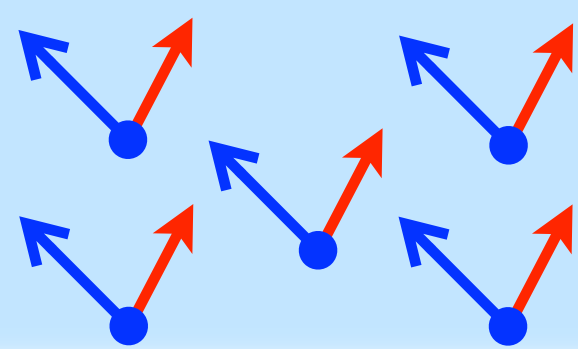


A-phase

$$H = \mathrm{U}(1)_{L_z-\phi} \times U(1)_{S_z}$$

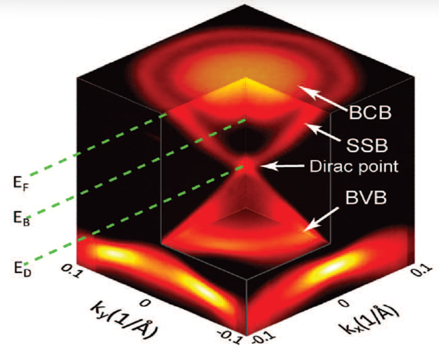
$$L_z = +1, S_z = 0$$

<u>Spin & Orbital states are ordered</u>



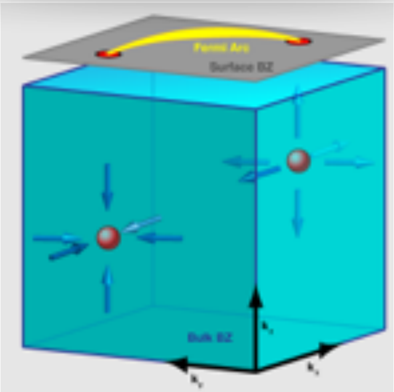
^{3}He: Paradigm for Topological Phenomena

TI w/ surface Dirac fermions



Time/Inversion sym. breaking

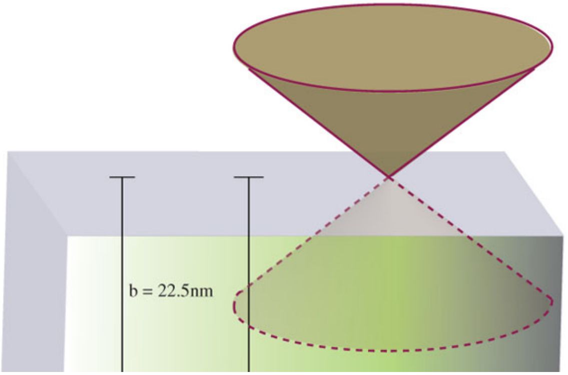
Weyl semi-metals



Particle-hole symmetry

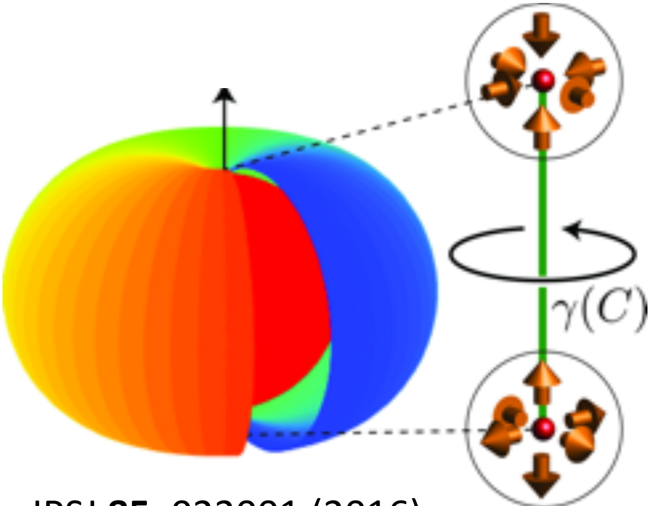
^{3}He-B: Surface Majorana fermion

Salomaa-Volovik (88);
Schnyder-Ryu-Furusaki-Ludwig (08); Qi-Hughes-Zhang (09);
Volovik (09); Chung-Zhang (09); Nagato-Higashitani-Nagai (09), …



^{3}He-A: Weyl Superfluid

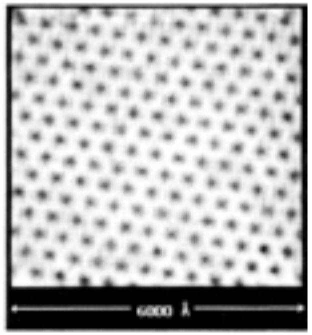
Volovik, JETP Lett **43**, 551 (1986);
Combescot and Dombre, PRB **33**, 79 (1986)



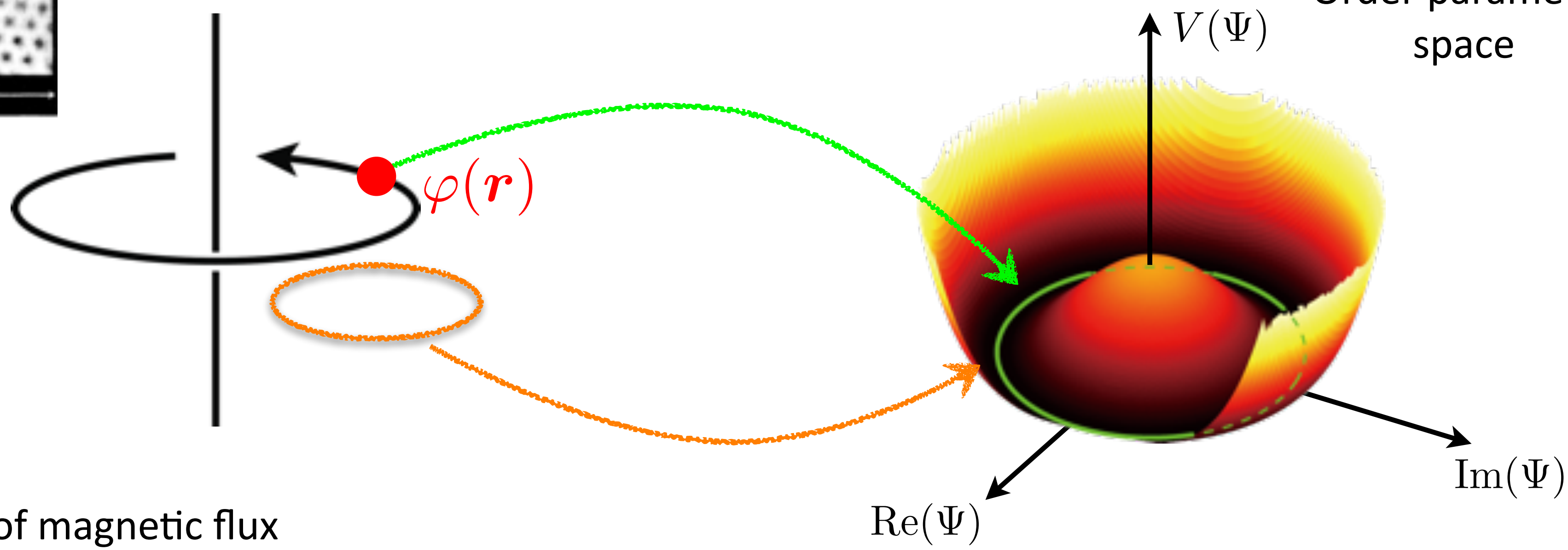
Review: **TM**, Y. Tsutsumi, T. Kawakami, M. Sato, M. Ichioka, K. Machida, JPSJ **85**, 022001 (2016)

Topology in Real Space

$$\Psi(\boldsymbol{r}) = |\Psi(\boldsymbol{r})| e^{i\varphi(\boldsymbol{r})}$$



magnetic flux
(vortex)



Order parameter
space

Quantization of magnetic flux

$$\Phi = \oint \boldsymbol{A} \cdot d\boldsymbol{r} = \frac{hc}{2e} w$$

$$\boldsymbol{A}(\boldsymbol{r}) = -i\frac{\hbar c}{2e}\frac{\nabla \Psi}{\Psi} = -i\frac{\hbar c}{2e}\frac{d\varphi(\theta)}{d\theta}\hat{\boldsymbol{e}}_\theta$$

winding number: Topological invariant

mapping from **real space** to **order parameter space**

$$\pi_1[U(1)] = \mathbb{Z}$$

⇒ classification of ordered states

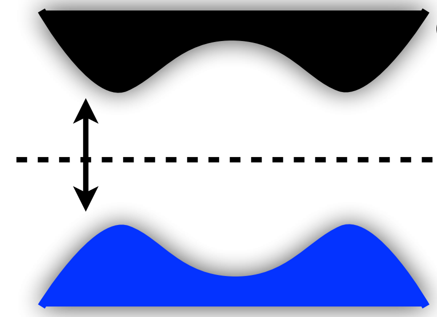
e.g., N. D. Mermin, Rev. Mod. Phys. **51**, 591 (1979)

Topology in Momentum Space

Minimal Hamiltonian: 2x2 Hermitian matrix

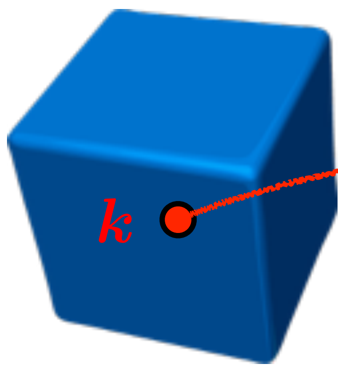
$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} m_0 + m_z & m_x - i m_y \\ m_x + i m_y & m_0 - m_z \end{pmatrix} \mapsto \sum_j \hat{m}_j(\boldsymbol{k})\sigma_j$$

uniquely parameterized with unit vector (=S^2)

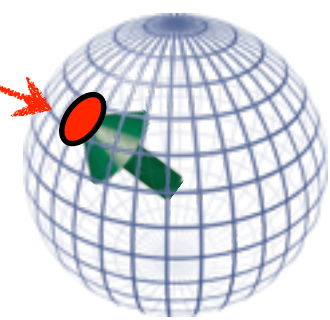


$\pi_d(S^2)$

***d*-dim. Brillouin zone**



target space S^2

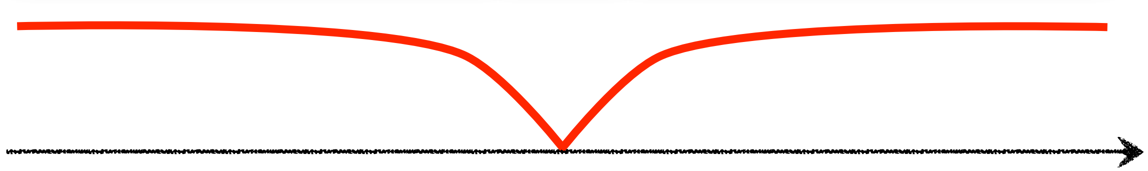


the map of ***k***-points to the Hilbert space (S^2)

topologically nontrivial

trivial

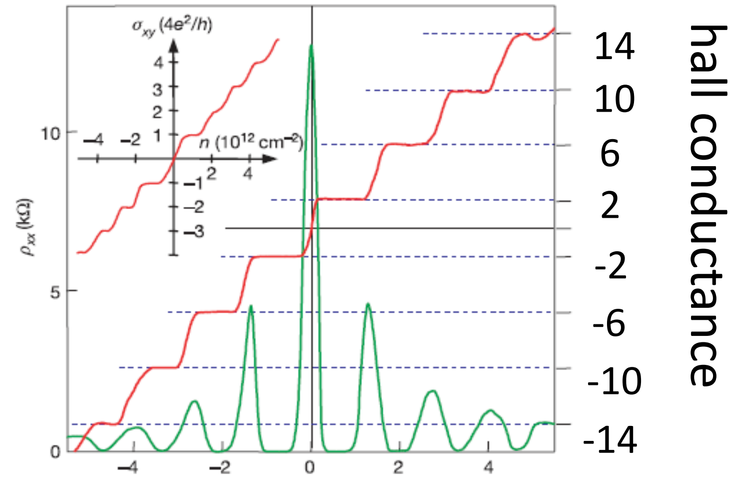
fermion excitation



Quantum Hall States

Quantization of Hall conductance

$$\sigma_{xy} = \frac{e^2}{h}\nu$$



Bulk topology

Conduction band

E_F

Valence band

ν = Chern number (winding number)
topological invariant in ***k***-space (BZ)

Thouless-Kohmoto-Nightingale-Nijs, PRL **49**, 405 (1982)
Kohmoto, Ann. Phys. **160**, 343 (1985)

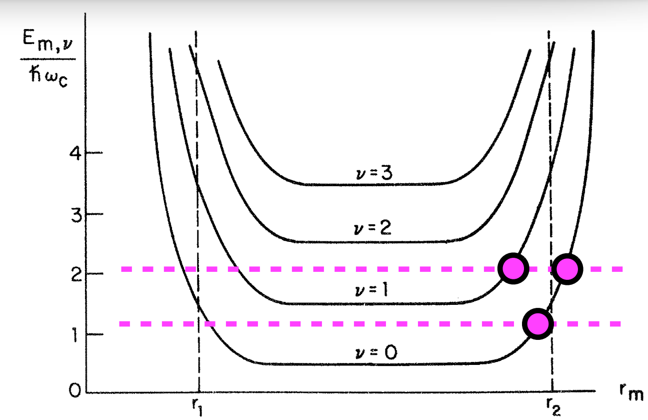
New classification of quantum states

Edge states

Halperin, PRB **25**, 2185 (1982)

2-dim electrons under a high field
with **"boundaries"**

ν = # of gapless edge states

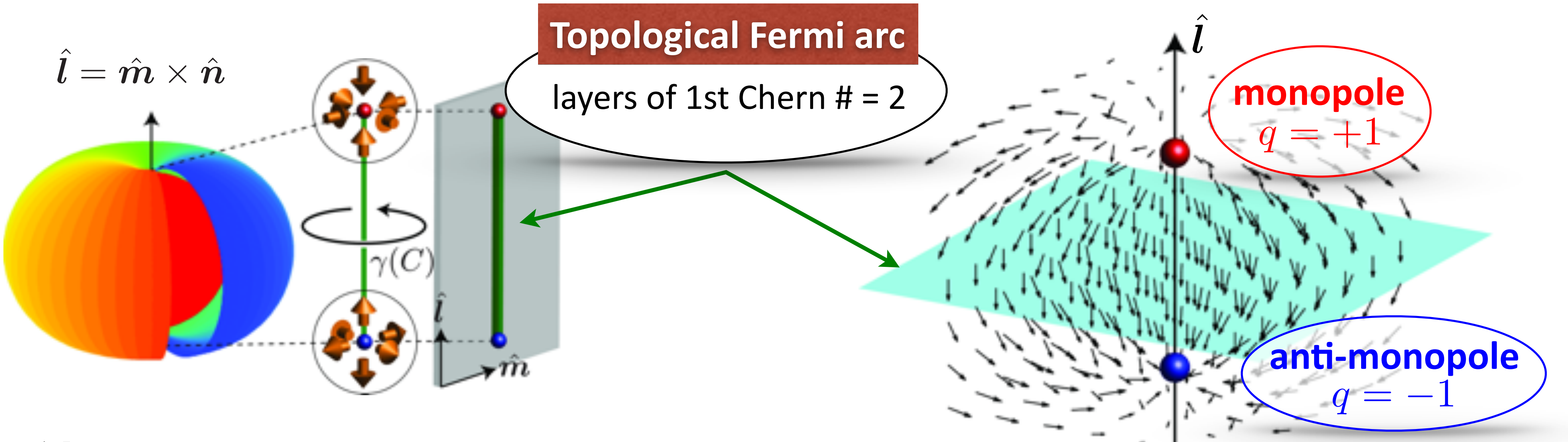


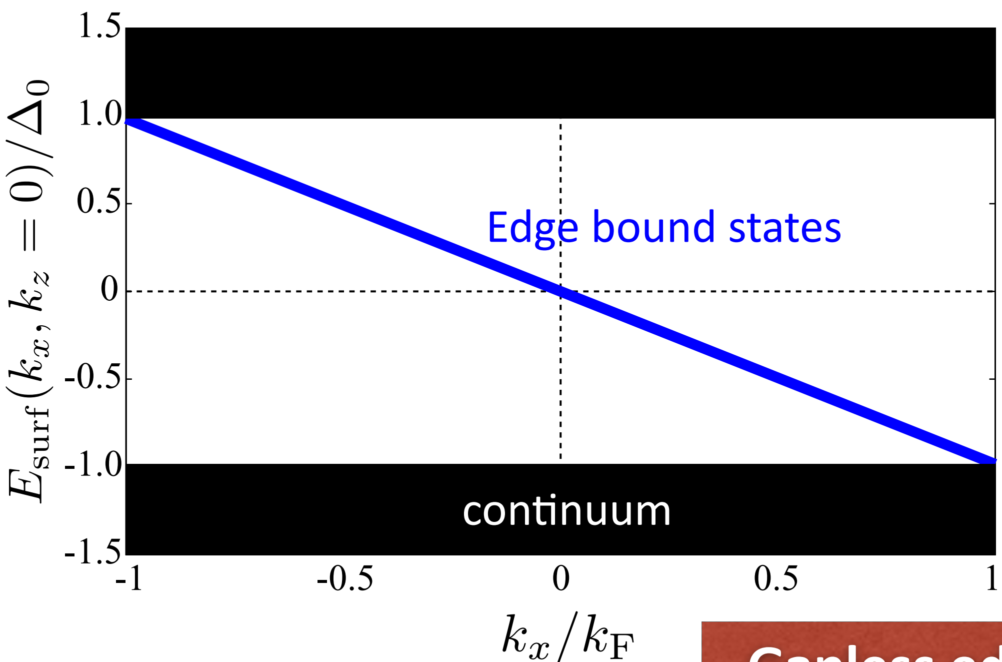
Bulk-edge correspondence: topological invariant = # of gapless edge states

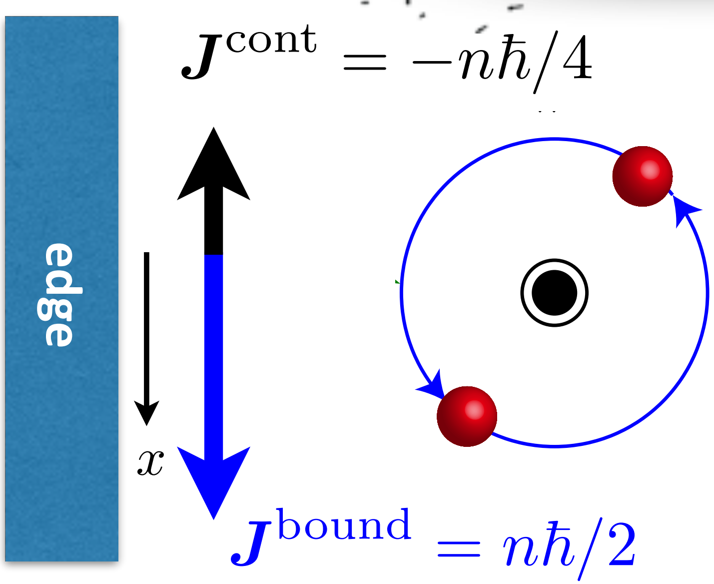
Hatsugai, PRL '93; PRB '93;

Berry Curvature & Edge States in ^{3}He-A

$\boldsymbol{d}(\boldsymbol{k}) = \Delta_{\mathrm{A}} \hat{\boldsymbol{z}}(\hat{\boldsymbol{m}} + i\hat{\boldsymbol{n}}) \cdot \boldsymbol{k}$ Berry curvature $[\boldsymbol{\Omega}_n(\boldsymbol{k})]_\mu = \epsilon_{\mu\nu\eta} \langle \partial_{k_\nu} u_n(\boldsymbol{k}) | \partial_{k_\eta} u_n(\boldsymbol{k}) \rangle$

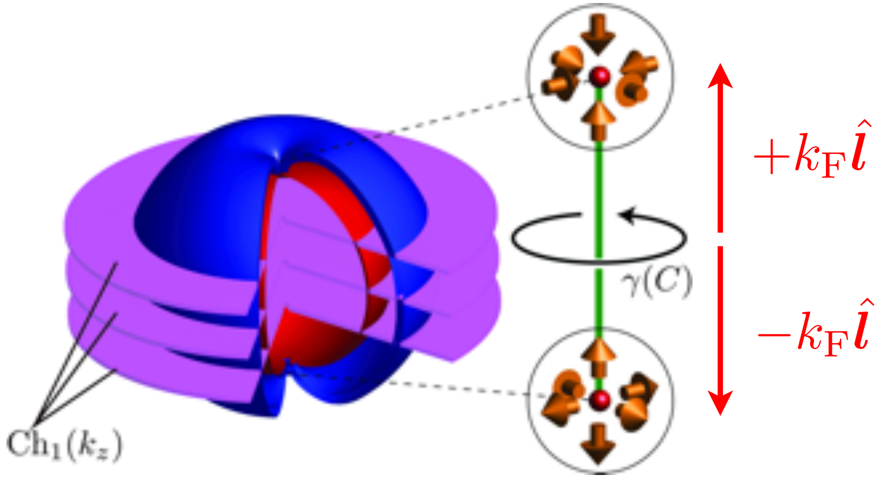






Gapless edge states carry macroscopic mass current

Weyl Fermions in ^{3}He-A



Bogoliubov quasiparticles around point nodes

Volovik (86); Combescot and Dombre (86)

$$\mathcal{H}(\boldsymbol{k}) = e^{\mu}_{j}\tau^{j}(k_{\mu} - k_{\mathrm{F}}\hat{l}_{\mu})$$

$$(e^{\mu}_{1}, e^{\mu}_{2}, e^{\mu}_{3}) = \left(\frac{\Delta}{k_{\mathrm{F}}}\hat{m}_{\mu}, \frac{\Delta}{k_{\mathrm{F}}}\hat{n}_{\mu}, v_{\mathrm{F}}\hat{l}_{\mu}\right)$$

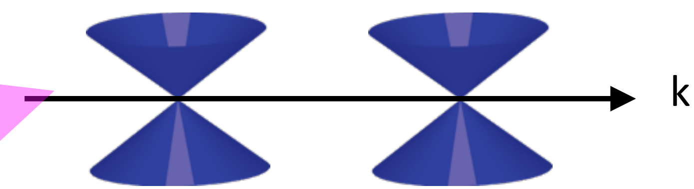
vielbein: dislocation/defect in spatial coordinates induces gauge field (*e.g.*, Sumiyoshi-Fujimoto, PRL (16))

Weyl fermions

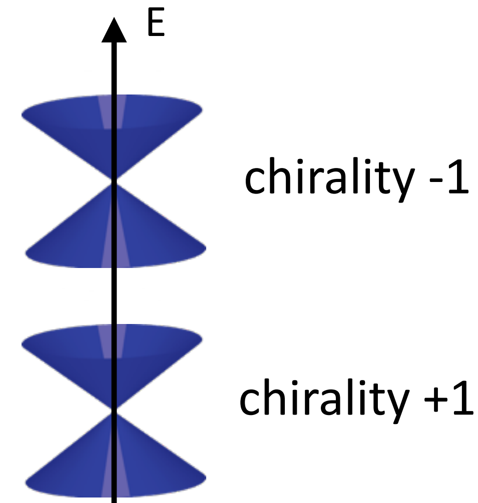
$$\mathcal{H} = \pm\boldsymbol{\sigma}\cdot\boldsymbol{k}$$

massless Dirac fermion
doubly degenerate
chirality + or -

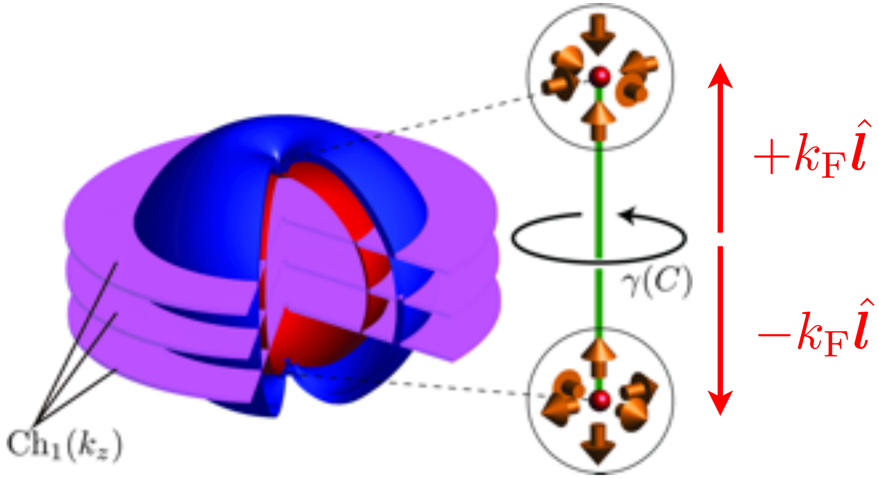
time-reversal sym. breaking



inversion sym. breaking



Weyl Fermions in ³He-A



Bogoliubov quasiparticles around point nodes

Volovik (86); Combescot and Dombre (86)

$$\mathcal{H}(\boldsymbol{k}) = e^{\mu}_{j}\tau^{j}(k_\mu - k_{\rm F}\hat{l}_\mu)$$

$$(e^\mu_1, e^\mu_2, e^\mu_3) = \left(\frac{\Delta}{k_{\rm F}}\hat{m}_\mu, \frac{\Delta}{k_{\rm F}}\hat{n}_\mu, v_{\rm F}\hat{l}_\mu\right)$$

vielbein: dislocation/defect in spatial coordinates induces gauge field (*e.g.*, Sumiyoshi-Fujimoto, PRL (16))

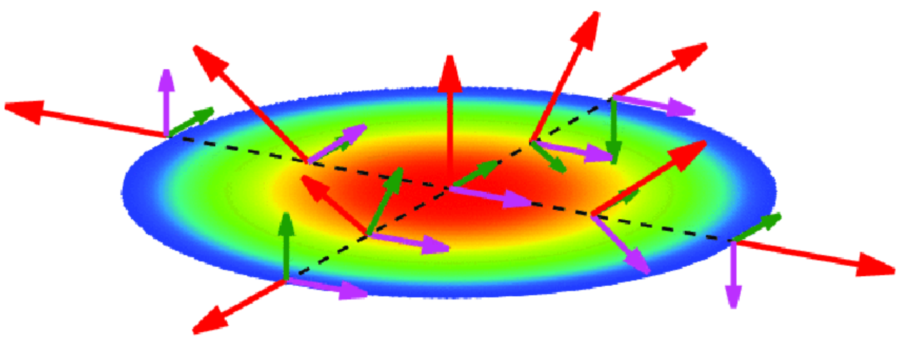
Broken Symmetry & skyrmion-vortex in ³He-A

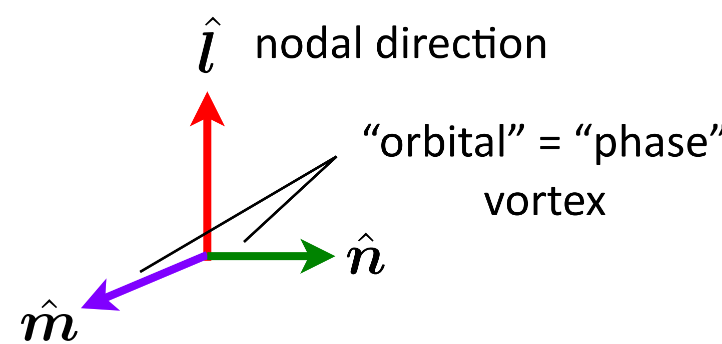
e.g., Salomaa-Volovik, RMP (87)

$$d_{\mu i} = \Delta_{\rm A} e^{i\varphi}\hat{d}_\mu(\hat{\boldsymbol{m}} + i\hat{\boldsymbol{n}})_i$$

Simultaneous gauge-orbit rotation:

phase rotation (vortex) = orbital rotation about "*l*"

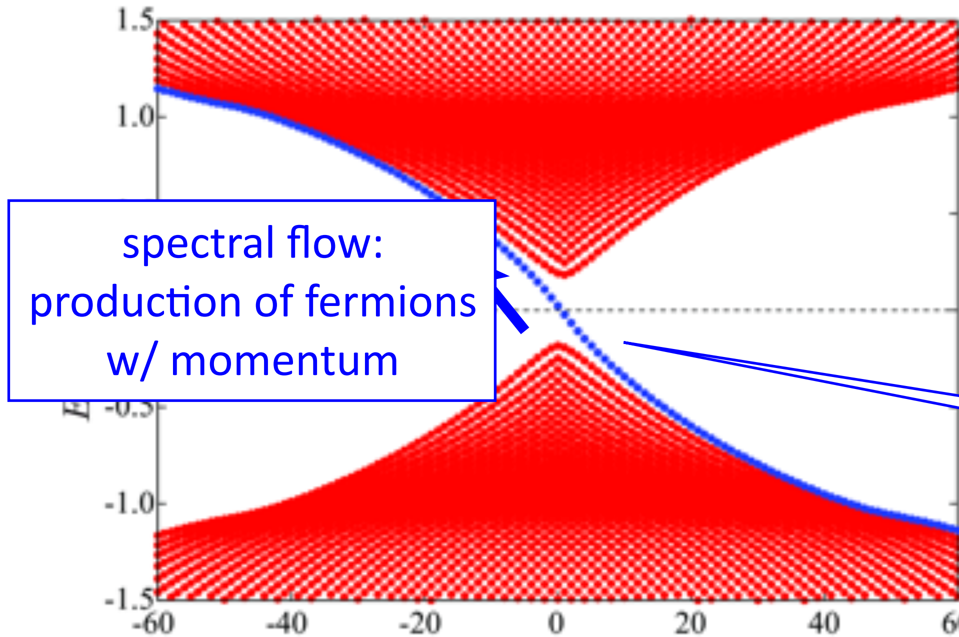




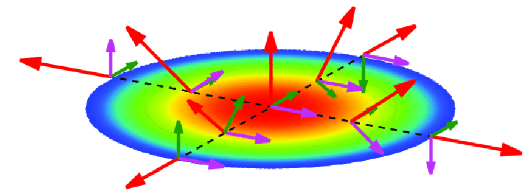
Emergent field in skyrmion-vortex (Mermin-Ho vortex)

Weyl Fermions & Anomaly in ^{3}He-A

Bogoliubov QP spectrum w/ axial momentum $k_z = \pm k_{\mathrm{F}}$



ℓ azimuthal quantum #



Weyl fermions w./
Emergent "toroidal" magnetic field

$$\boldsymbol{B}(\boldsymbol{r}) = k_{\mathrm{F}} \boldsymbol{\nabla} \times \hat{\boldsymbol{l}}(\boldsymbol{r})$$

chiral fermions

$$E_{n=0}(\ell, k_z) = -v\ell$$

Chiral Anomaly: back-action to skyrmion-vortex "**Kopnin force**"

- **Chiral anomaly:** Violation of momentum conservation in "fermions" as a consequence of real-space topology & Weyl-Bogoliubov QPs

$$\partial_t \boldsymbol{P}^{(\mathrm{F})} = \mathcal{C}_0 \int d\boldsymbol{r} \hat{\boldsymbol{l}} \left(\underline{\partial_t \hat{\boldsymbol{l}}} \cdot \underline{\boldsymbol{\nabla} \times \hat{\boldsymbol{l}}} \right) \propto \boldsymbol{E} \cdot \boldsymbol{B}$$

Exp.: Bevan *et al.*, Nature **386**, 689 (1997); Volovik, JETP Lett. **103**, 140 (2016)

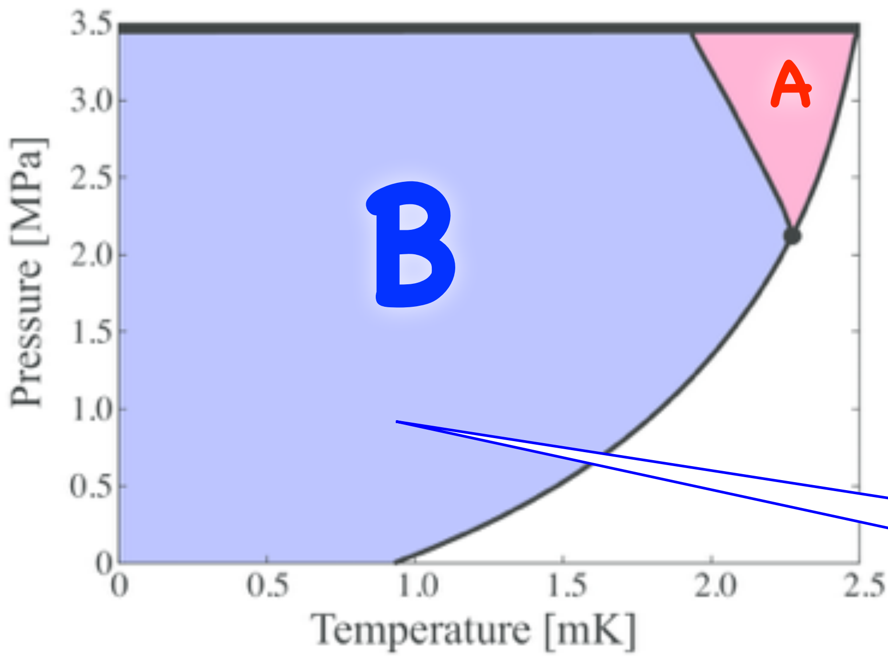
==> momentum transfer from WF to the "superfluid vacuum"

Superfluid ^{3}He-B

Symmetry of Normal ^{3}He

$$G = \mathrm{SO}(3)_{\boldsymbol{S}} \times \mathrm{SO}(3)_{\boldsymbol{L}} \times \mathrm{U}(1) \times T$$

spin rotation — momentum — time-reversal sym.



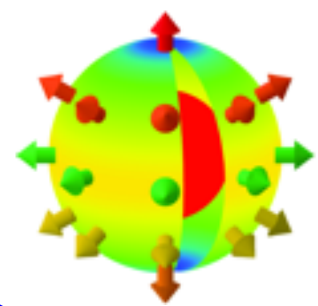
Spin triplet *p*-wave order parameter

$$S = 1 \quad L = 1$$

$$d_{\mu i}$$

spin — momentum

B: “Isotropic” BW state



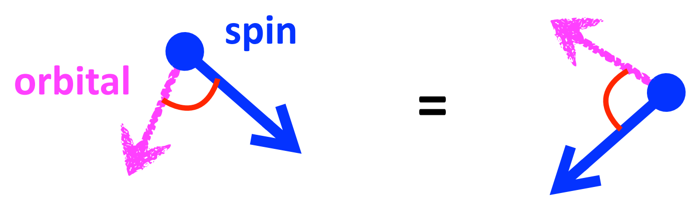
$$d_{\mu i} = \Delta_{\mathrm{B}} \delta_{\mu i}$$

$$J = J_z = 0$$

$\hat{k}_x + i\hat{k}_y$ + $\hat{k}_z$ + $-\hat{k}_x + i\hat{k}_y$

$|\downarrow\downarrow\rangle$ — $|\uparrow\downarrow + \downarrow\uparrow\rangle$ — $|\uparrow\uparrow\rangle$

Spontaneous breaking of spin-orbit symmetry



Spin-orbit coupling emergent in SF vacuum

Majorana Fermions

perspective

Majorana returns

Frank Wilczek

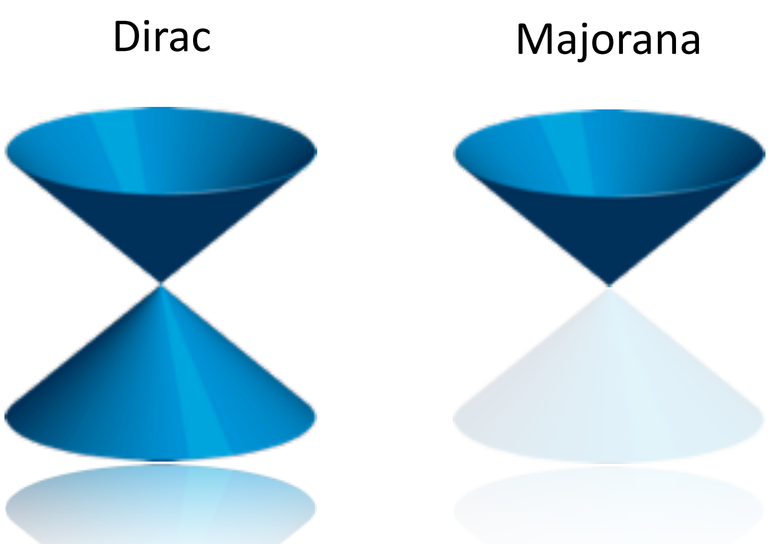
In his short career, Ettore Majorana made several profound contributions. One of them, his concept of 'Majorana fermions' — particles that are their own antiparticle — is finding ever wider relevance in modern physics.

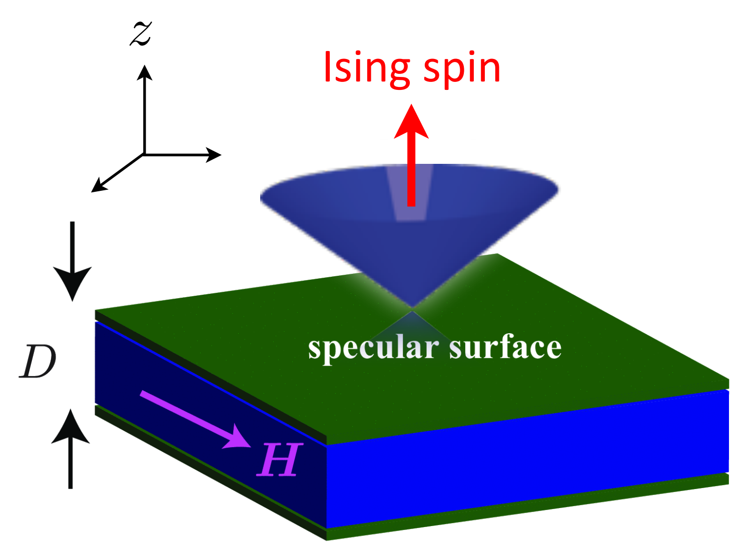
Ettore Majorana

$$\psi_{\rm M} = \mathcal{C}\psi_{\rm M}$$

Cooper pair

Majorana fermions: self-charge-conjugation



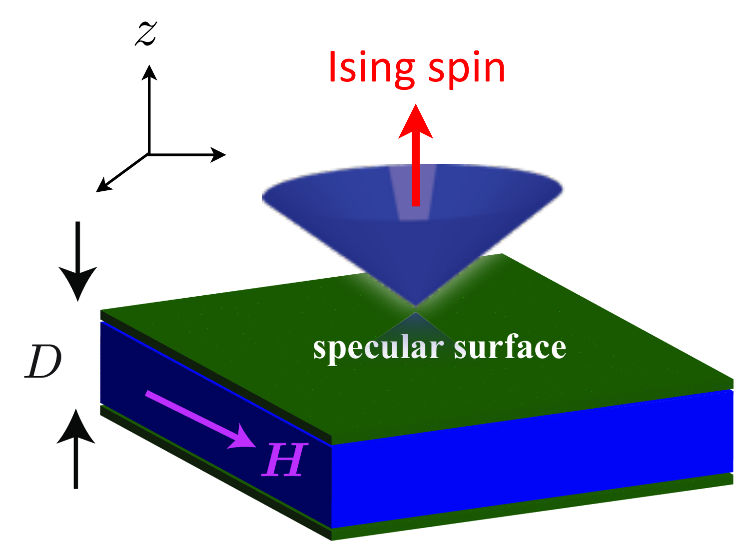


Majorana "Ising" Spin

Volovik; Chung-Zhang; Nagato-Higashitani-Nagai; TM-Sato-Machida, ...

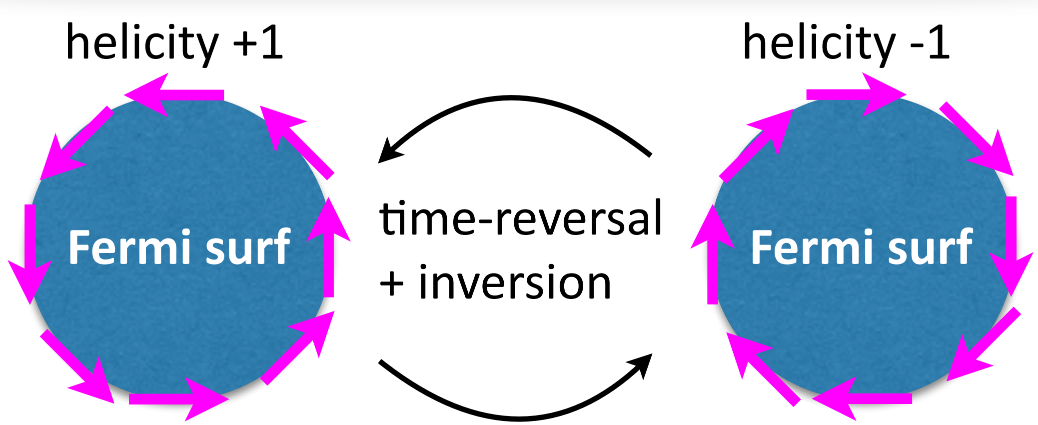
$$i\psi_{\uparrow}(\boldsymbol{r}) = \psi^{\dagger}_{\downarrow}(\boldsymbol{r})$$

$$\rho(\boldsymbol{r}) = 0 \qquad \boldsymbol{S} = (0, 0, S_z)$$

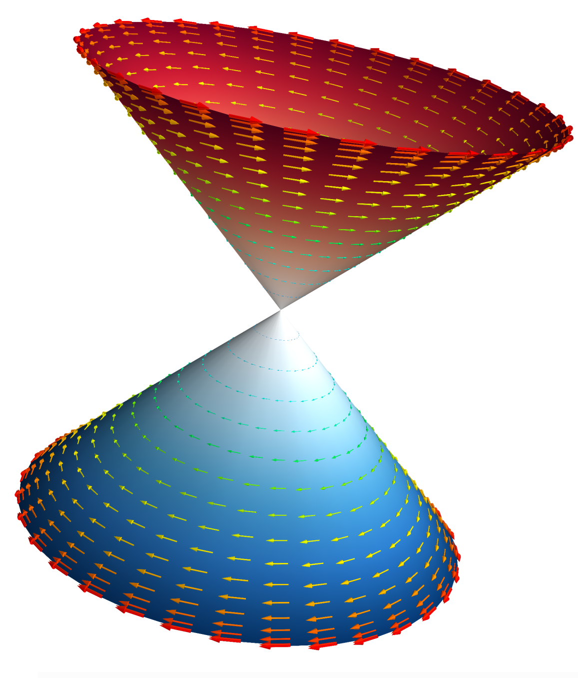


Surface MF in ^{3}He-B possess only Ising spin
==> not detectable through density fluctuation

^{3}He-B: spontaneously broken spin-orbit symmetry



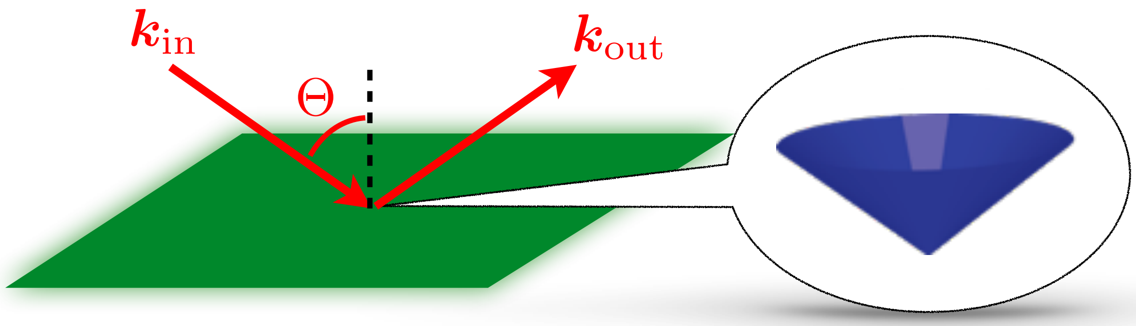
Inversion symmetry is locally broken at surface
$\Rightarrow$ opposite surface MF has opposite helicity



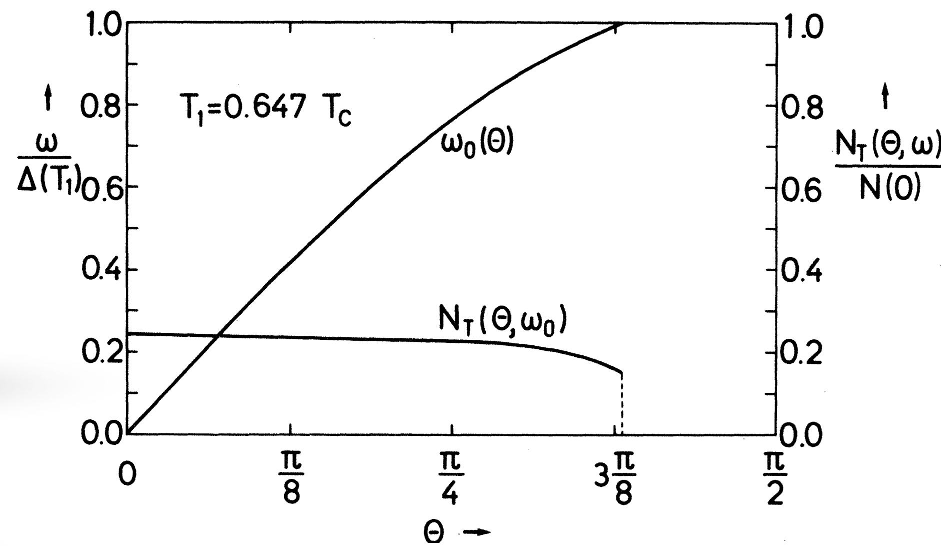
Spin current on surface

Andereev Bound States

Buchholtz and Zwicknagl, PRB **23**, 5788 (1981)

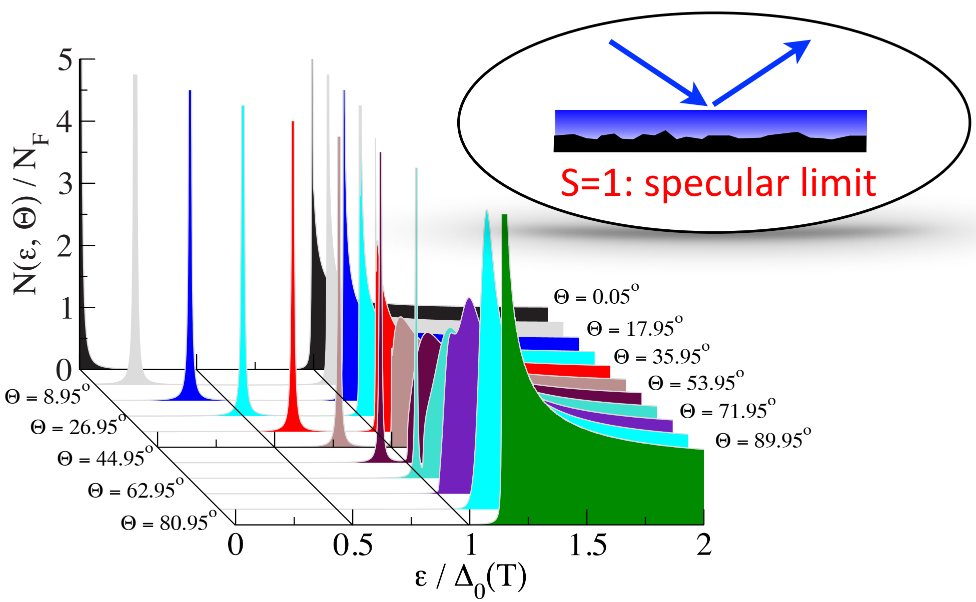


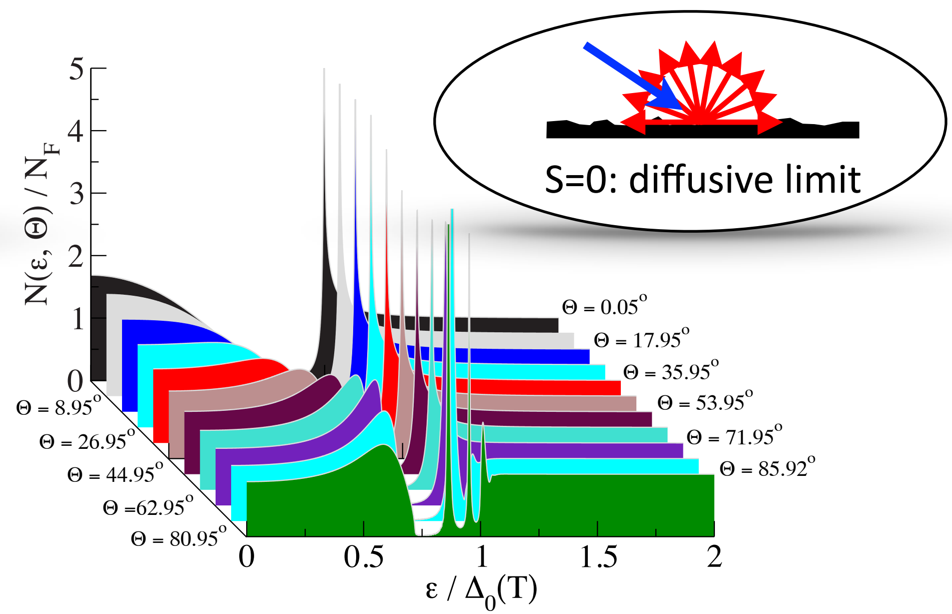
Dispersion of surface bound states



Formation of Andreev bound states

Vorontsov and Sauls, PRB **68**, 064508 (2003)



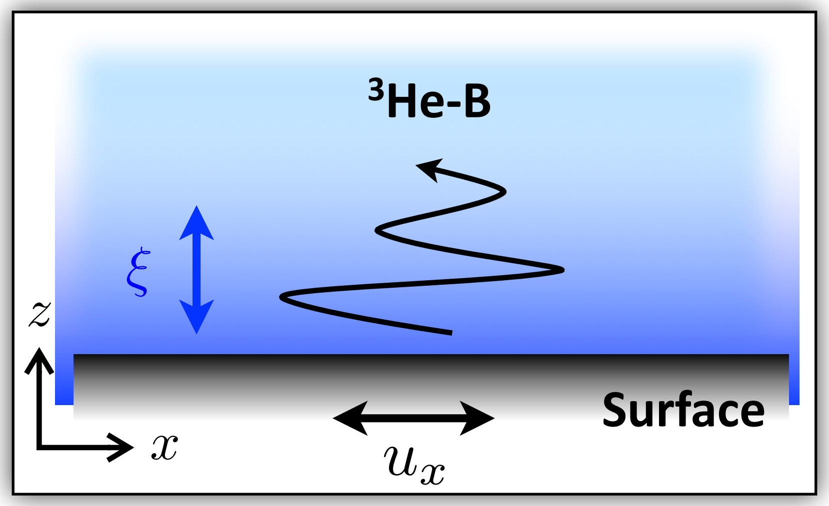


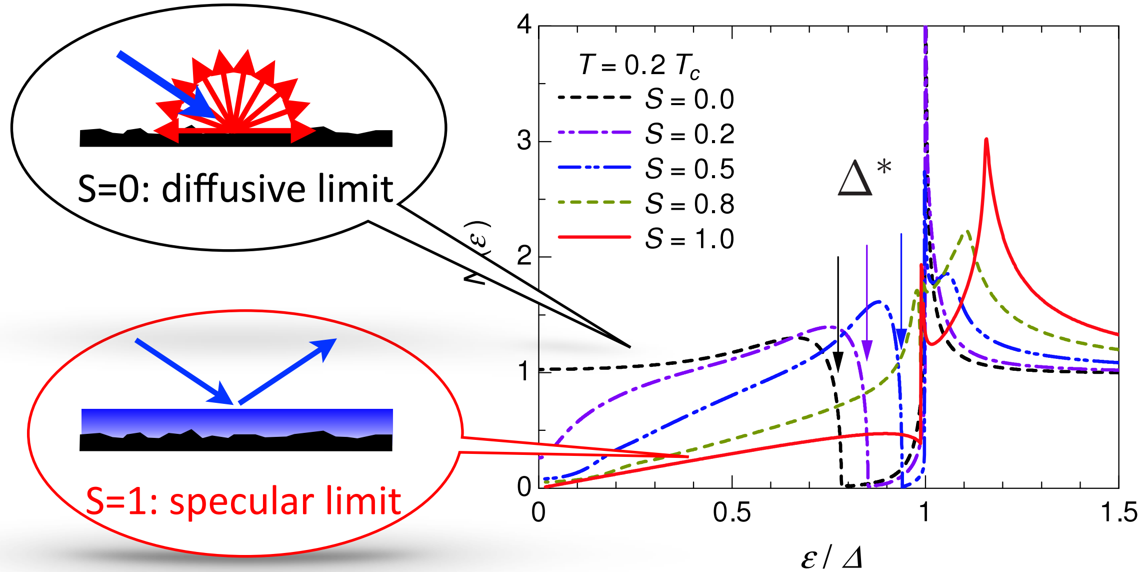
Sub-gap structure: anomalous scattering of MF in the presence of disorder (Nagato *et al*. JPSJ (11))

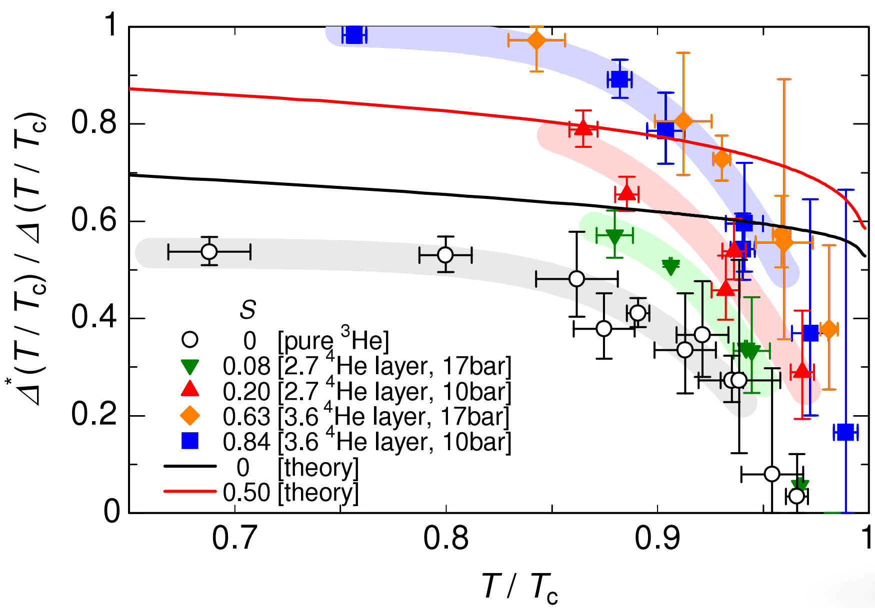
Detecting Surface States

Transverse acoustic impedance

Murakawa *et al.*, PRL **103**, 155301 (2009); JPSJ **80**, 013602 (2011)







sub-gap structure: anomalous scattering of MF in the presence of disorder (Nagato et al.)

<u>Spectroscopy of surface density of states (sub-gap structure & formation of "cone")</u>

Detection of the topological properties of surface states ?
Majorana Ising spin and mass acquisition…

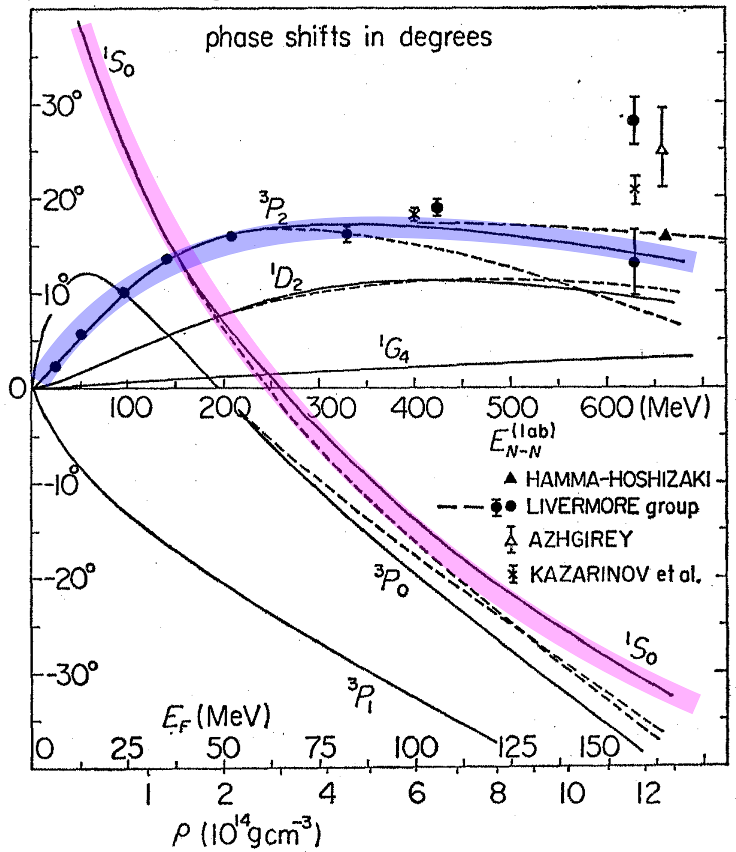
OUTLINE

- ^{3}He and Neutron Stars
  1. ^{3}He-A: Weyl fermions & chiral anomaly
  2. ^{3}He-B: Topology & Majorana fermions
- Topological $^3P_2$ Superfluids in Neutron Stars
- Nambu Sum Rule
- Topological Phases in Confined Superfluid ^{3}He-B

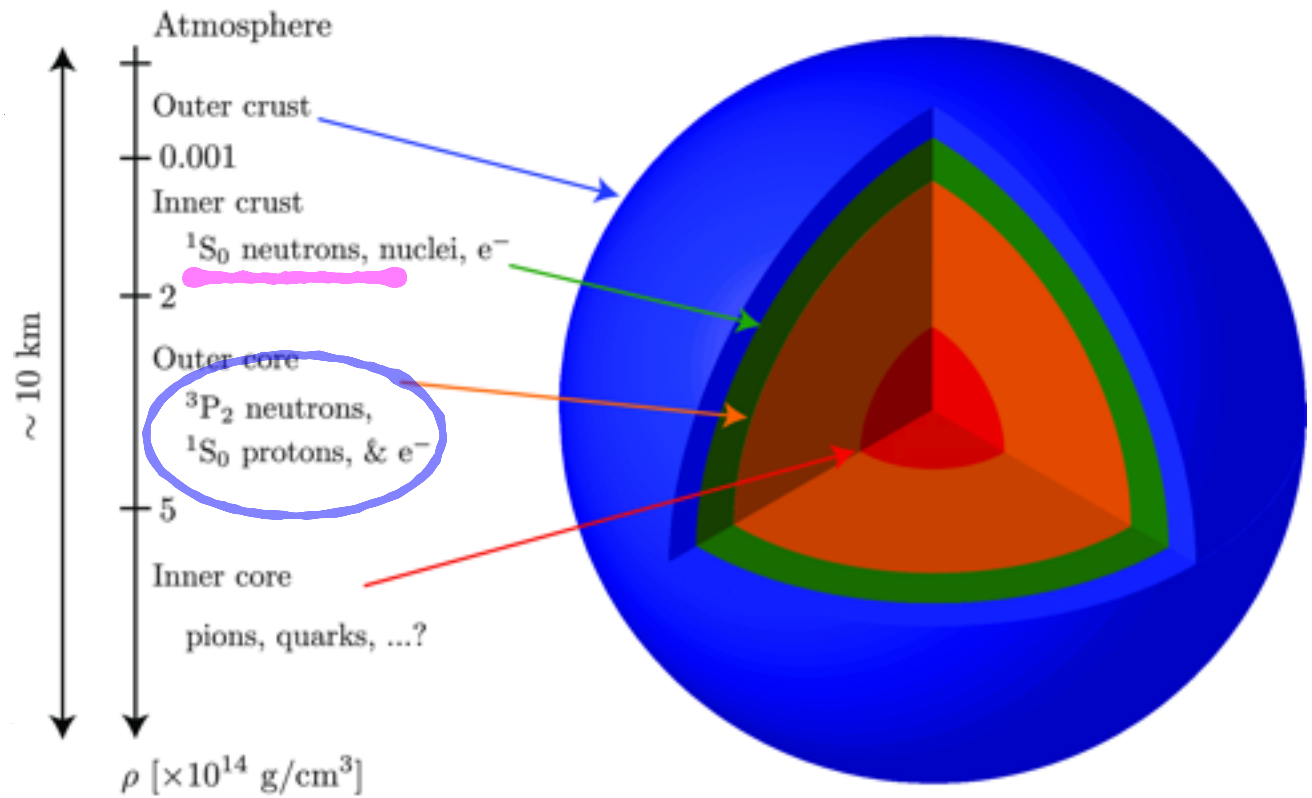
TM, K. Masuda, and M. Nitta, arXiv:1607.07266
J. A. Sauls and **TM**, arXiv:1611.07273 (PRB in press)
TM, Y. Tsutsumi, T. Kawakami, M. Sato, M. Ichioka, K. Machida, JPSJ **85**, 022001 (2016)

Superfluidity in Neutron Star Interiors

Tamagaki, PTP **44**, 905 (1970)



repulsive core + attractive *LS* force



$^3P_2$ (spin-triplet J=2) pairing

- ✓ CAS-A: 3P2 may explain the rapid cooling (Page *et al*. (2011))
- ✓ Magnetars: exotic pairing under high magnetic field ~ 10^{15}G
- ✓ Proton superconductors: Type-I or II (*e.g.*, Link (2003))?

$^3P_2$ Superfluid Phase Diagram

$$A_{\mu i}$$

spin momentum

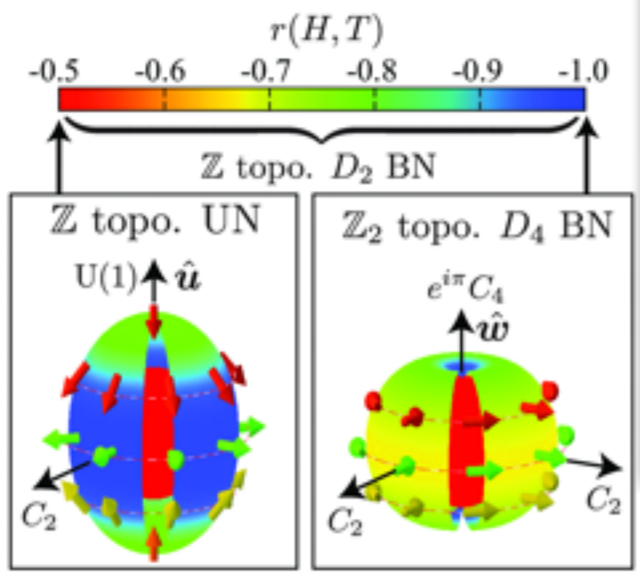
$$\xrightarrow{J=2}$$

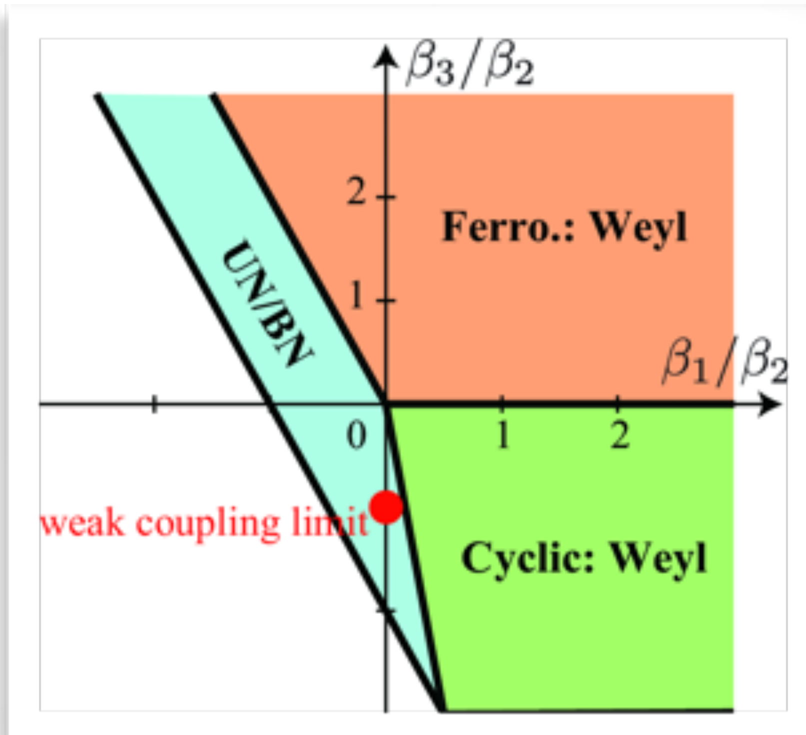
$$A_{\mu i} = A_{i\mu}$$
$$\mathrm{Tr}[A] = 0$$

$^3P_2$: symmetric traceless

Nematic phase

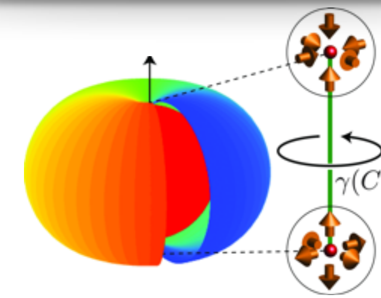
$$A_{\mu i} = \begin{pmatrix} 1 & & \\ & r & \\ & & -1-r \end{pmatrix}$$





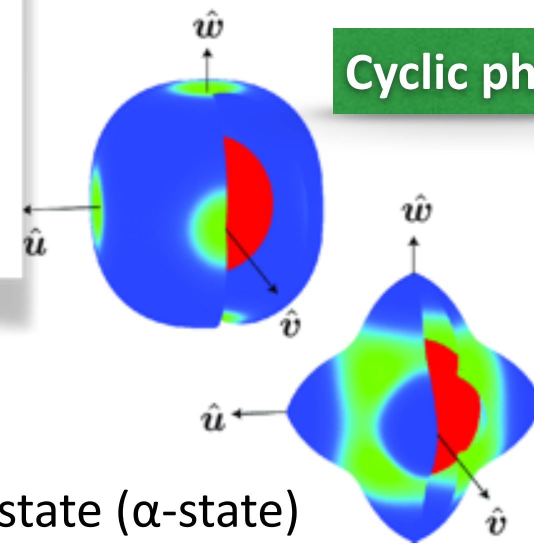
Sauls and Serene, PRD **17**, 1524 (1978)

Ferromagnetic phase



orbital ferro. & spin ferro.
= ^{3}He-A_1

Cyclic phase



Non-unitary state (α-state)

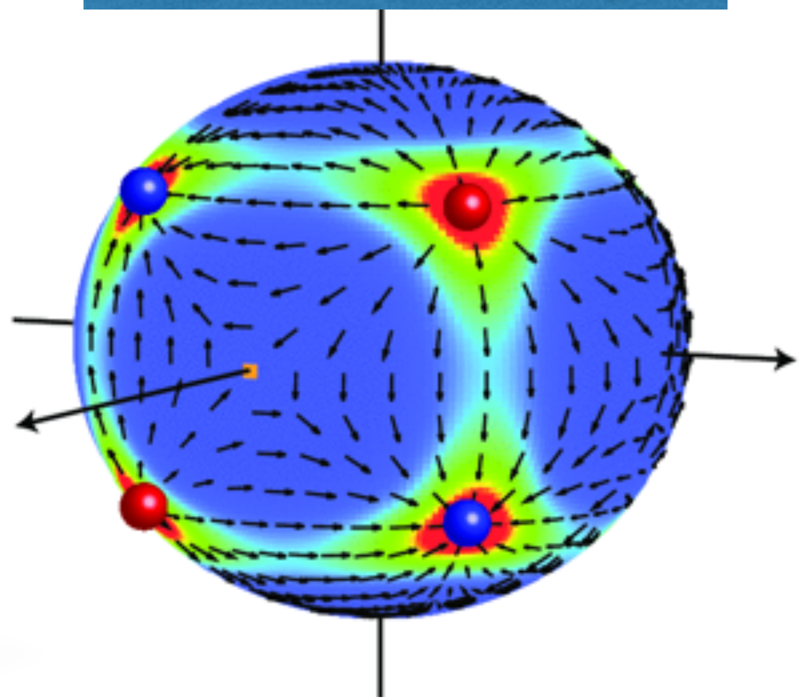
Weyl fermions in Cyclic States

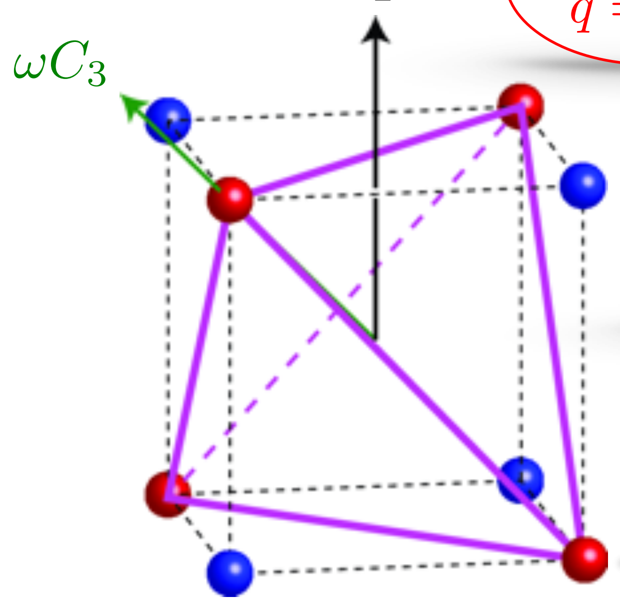
TM, K. Masuda, M. Nitta, arXiv:1607.07266

$$\boldsymbol{d}(\boldsymbol{k}) = \Delta(\hat{\boldsymbol{a}}\hat{k}_a + \omega\hat{\boldsymbol{b}}\hat{k}_b + \omega^2\hat{\boldsymbol{c}}\hat{k}_c)$$

$$\omega^3 = 1$$

Berry curvature in ***k***-space





monopole $q = +1$

anti-monopole $q = -1$

Particle-hole symmetric quartet of Weyl fermions

$$\mathcal{H}(\boldsymbol{k}) = \hat{e}^{\mu}_{a}\tau^{a}(k_\mu - qk_{\alpha,\mu})$$

monopole charge

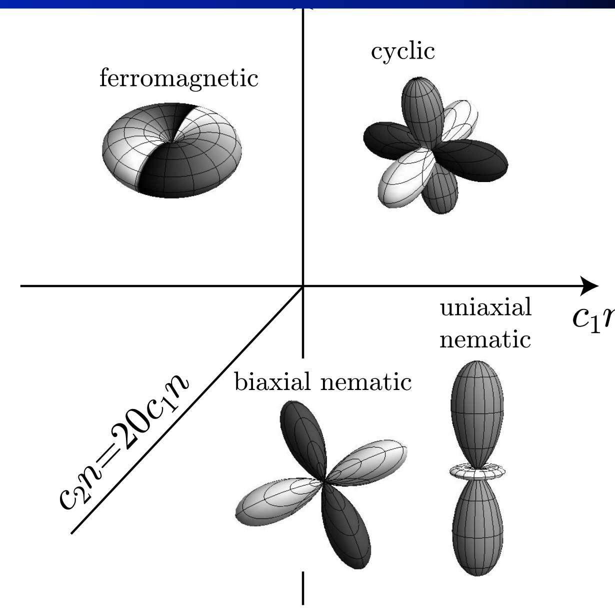
nodal position

Anomalous transport due to chiral anomaly(?)

Topological Defects in $^3P_2$ Superfluids

Symmetric traceless tensor: 5-component

$$d_{\mu i} = \begin{pmatrix} d_{xx} & d_{xy} & d_{zz} \\ & d_{yy} & d_{yz} \\ & & d_{zz} \end{pmatrix}$$ ⇒ spin-2 BEC (*e.g.*, ^{87}Rb atoms)

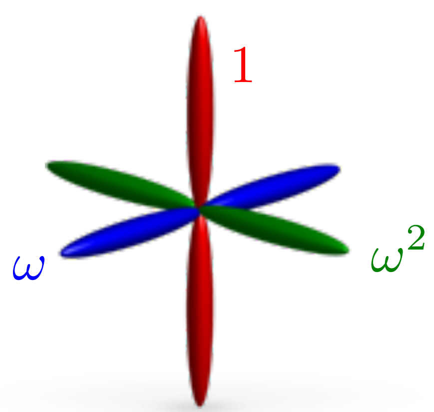


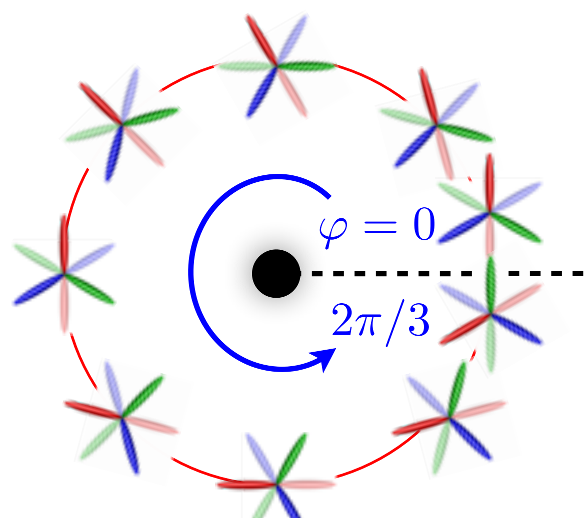
Kawaguchi and Ueda (12)

Non-Abelian fractional vortices: Non-comm. topological charge

Kobayashi, Kawaguchi, Nitta, Ueda, PRL **103**, 115301 (2009)

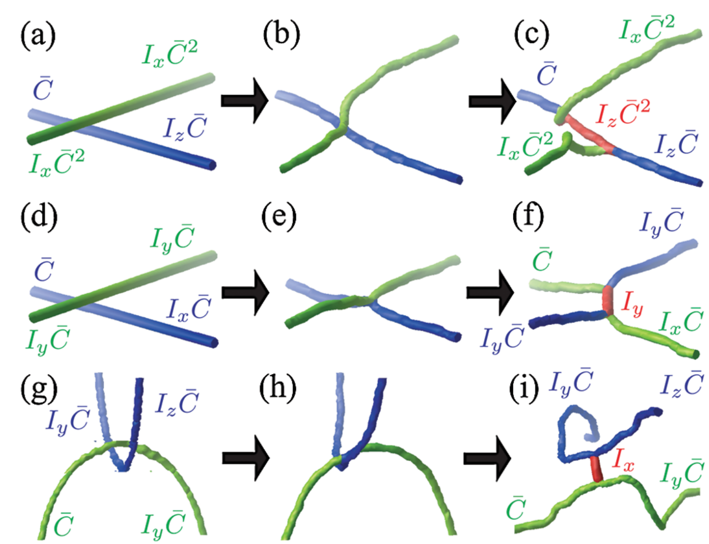
Kawaguchi and Ueda, Phys. Rep. **520**, 253 (2012)





Collision dynamics

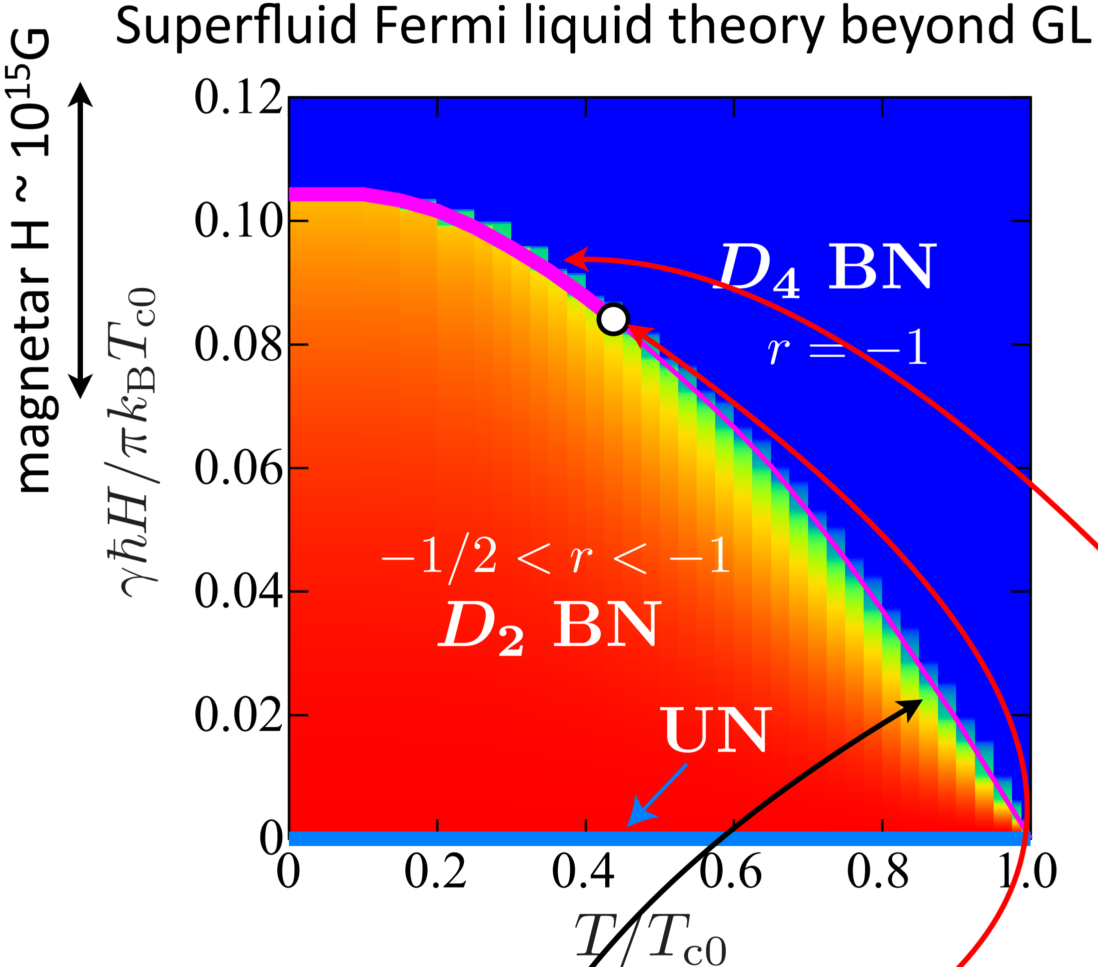
M. Kobayashi, et al., PRL (09)



Non-Abelian anyons in Non-Abelian vortex?

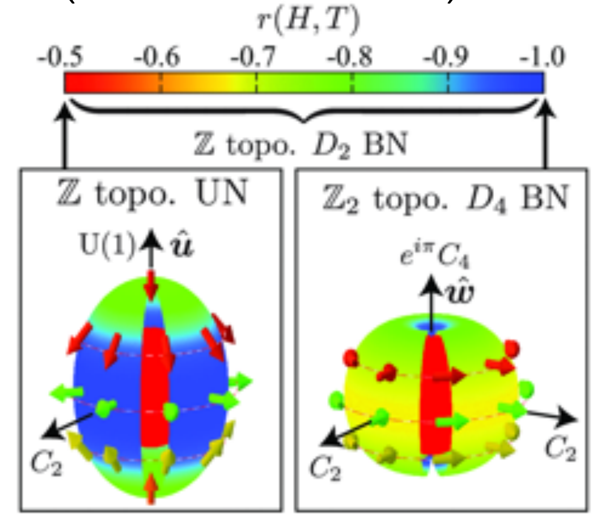
Phase Diagram under Magnetic Fields

Superfluid Fermi liquid theory beyond GL



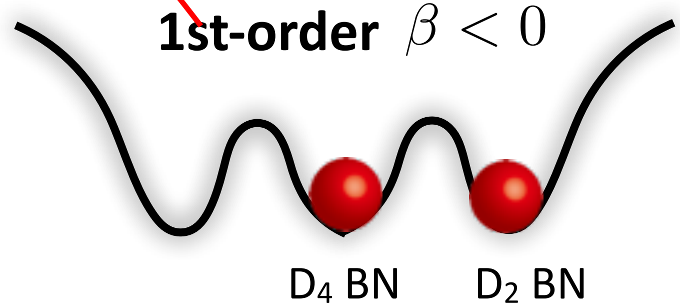
$$A_{\mu i} = \begin{pmatrix} 1 & & \\ & r & \\ & & -1-r \end{pmatrix}$$

$\tilde{r}(T,H)$



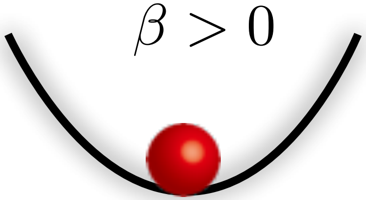
$$f(\tilde{r}) = \alpha|\tilde{r}|^2 + \beta|\tilde{r}|^4 + \gamma|\tilde{r}|^6$$

1st-order $\beta < 0$



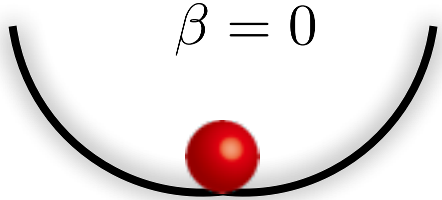
2nd-order

$\beta > 0$



tricritical point

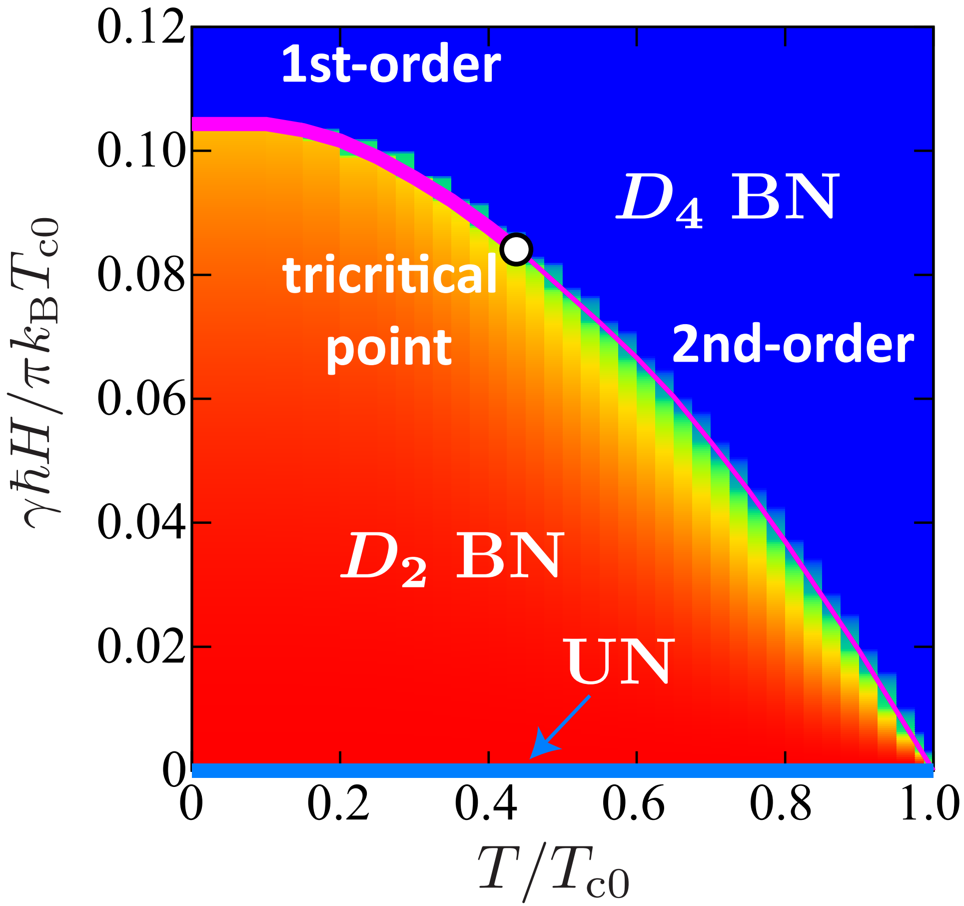
$\beta = 0$

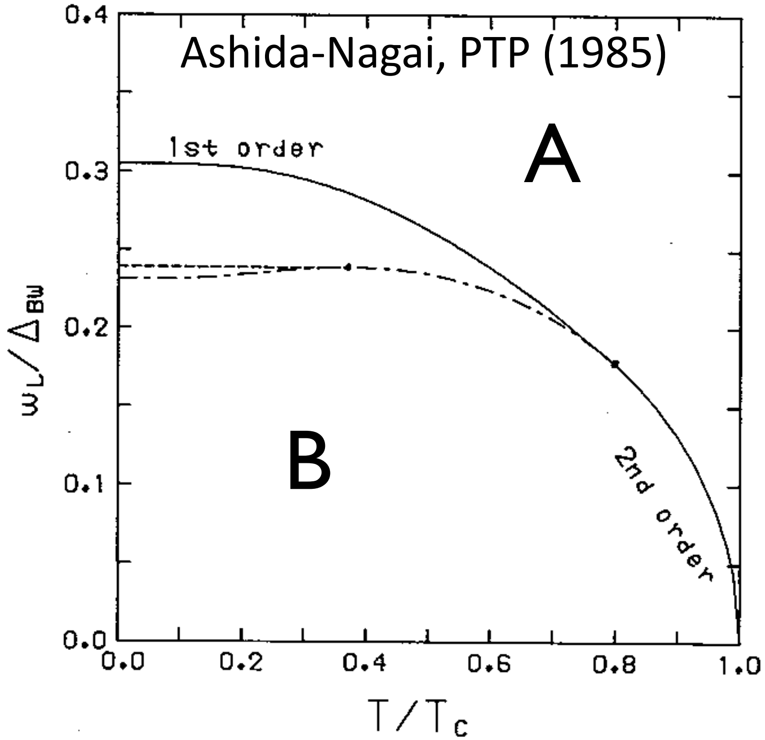


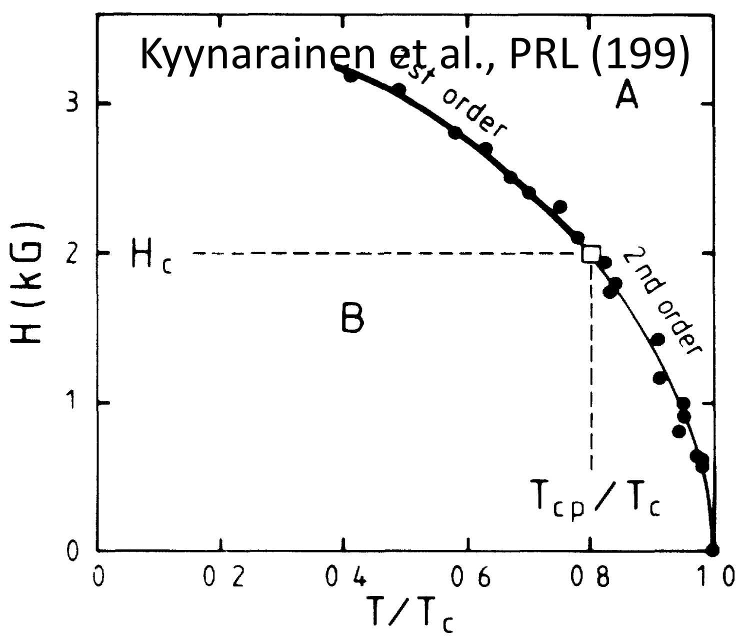
bosonic fluctuations developed at TCP?

Phase Diagram under Magnetic Fields

Superfluid Fermi liquid theory beyond GL







✓No experimental evidence of TCP in ^{3}He

✓Theoretical predictions to (in)stability of **FFLO**

✓BW/Nematic is responsible for Pauli depairing

Connection of $^3P_2$ to Solid States

TM, K. Masuda, M. Nitta, arXiv:1607.07266

O_h $\quad E_u \quad \boldsymbol{d}(\boldsymbol{k}) = \eta_1 \boldsymbol{d}_1(\boldsymbol{k}) + \eta_2 \boldsymbol{d}_2(\boldsymbol{k})$

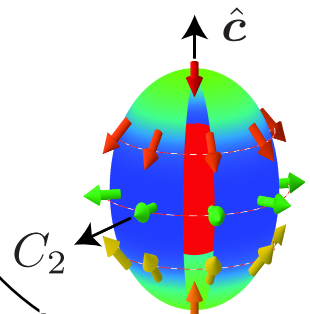
$$\boldsymbol{d}_1(\boldsymbol{k}) = 2\hat{\boldsymbol{c}}\hat{k}_c - \hat{\boldsymbol{a}}\hat{k}_a - \hat{\boldsymbol{b}}\hat{k}_b$$

$$\boldsymbol{d}_2(\boldsymbol{k}) = \sqrt{3}(\hat{\boldsymbol{a}}\hat{k}_a - \hat{\boldsymbol{b}}\hat{k}_b)$$

UBe_{13}?

uniaxial nematic

$(\eta_1, \eta_2) = (1, 0)$

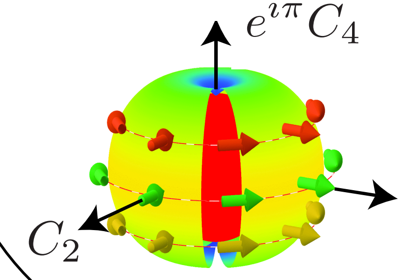


time-reversal invariant odd-parity

full gap: **BW**	Nodal: **Planar**
Z topo.	**Z_2** topo
Majorana **cone**	Majorana **Fermi arc**

biaxial D_4 nematic

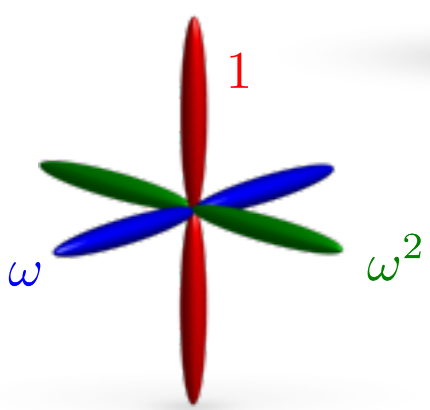
$(0, 1)$



Cyclic: time-reversal **BROKEN** odd-parity

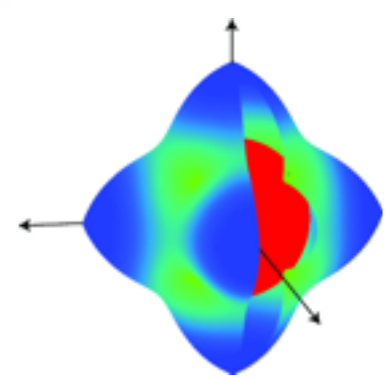
$$(\eta_1, \eta_2) = (1, \pm i)$$

non-unitary: no counterpart in ^{3}He

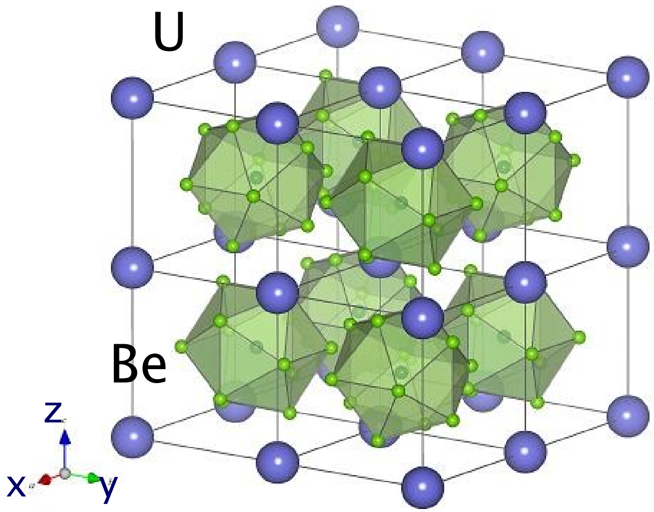


$$\boldsymbol{d}(\boldsymbol{k}) = \Delta(\hat{\boldsymbol{a}}\hat{k}_a + \omega\hat{\boldsymbol{b}}\hat{k}_b + \omega^2\hat{\boldsymbol{c}}\hat{k}_c) \qquad \omega^3 = 1$$

non-unitary state w/ full gap and 8 point nodes



Unconventional Superconductivity in Cubic Metals



Possible cubic material: **UBe_{13}**

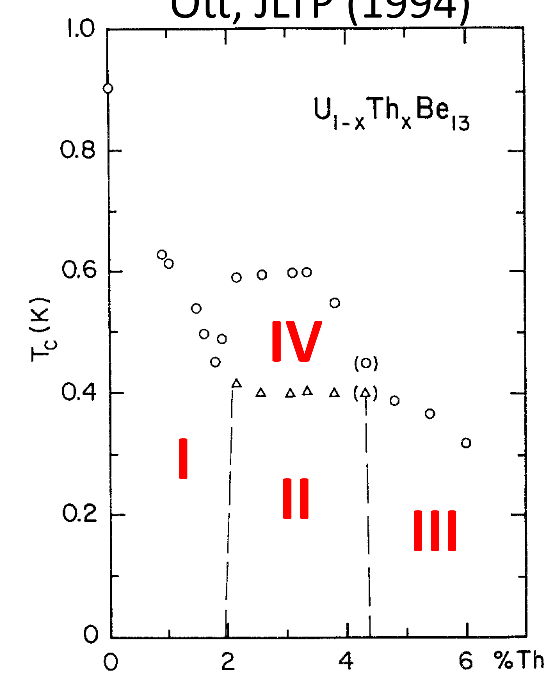
✓ Heavy fermion: $\gamma(T) = C(T)/T \sim 1$ J/molK^2

✓ Non-Fermi liquid behavior: $C(T)/T \sim -\log T$

✓ *"Unconventional" SC at Tc = 0.85 K*

full gap or point node?

Ott, JLTP (1994)



Multiple superconducting phases

$U_{1-x}Th_xBe_{13}$

Multicomponent OP: spin or orbital?

Spin singlet or triplet?

✓ NMR Knight shift unchanged ⇒ triplet?

Tien *et al.*, PRB (1989)

✓ μSR Knight shift slightly decreases at low-T ⇒ singlet?

Sonier *et al.*, Physica B (2003)

OUTLINE

- ^{3}He and Neutron Stars
  1. ^{3}He-A: Weyl fermions & chiral anomaly
  2. ^{3}He-B: Topology & Majorana fermions
- Topological $^3P_2$ Superfluids in Neutron Stars
- **Nambu Sum Rule**
- **Topological Phases in Confined Superfluid ^{3}He-B**

TM, K. Masuda, and M. Nitta, arXiv:1607.07266
J. A. Sauls and **TM**, arXiv:1611.07273 (PRB in press)
TM, Y. Tsutsumi, T. Kawakami, M. Sato, M. Ichioka, K. Machida, JPSJ **85**, 022001 (2016)

Bosons and Fermions in ^{3}He-B

Spontaneous spin-orbit symmetry breaking: Emergence of spin-orbit interaction

- **4 Nambu-Goldstone modes + 14 massive bosonic modes**

$$\mathrm{SO}(3)_S \times \mathrm{SO}(3)_L \times \mathrm{U}(1) \longrightarrow \mathrm{SO}(3)_J \qquad J = 0$$

$\mathrm{SO}(3)_{L-S} \times \mathrm{U}(1)$ phase & spin-orbit modes

Long-lived amplitude "Higgs" modes

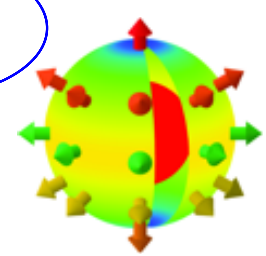
- **Topologically protected "Majorana" fermions**

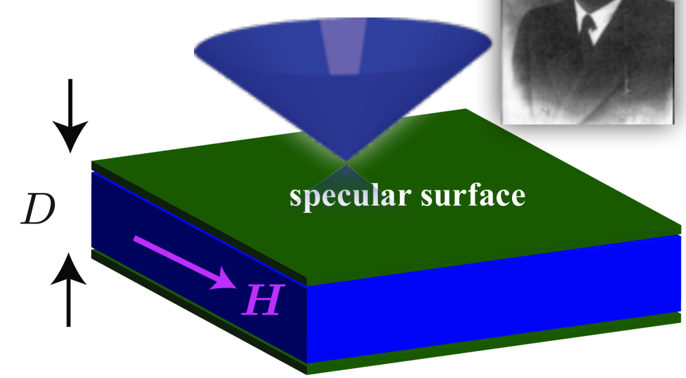
Emergent SOI is the source of nontrivial topology

^{3}He-B: DIII topological superfluid

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \varepsilon(\boldsymbol{k}) & \Delta(\boldsymbol{k}) \\ \Delta^\dagger(\boldsymbol{k}) & -\varepsilon^{\mathrm{t}}(-\boldsymbol{k}) \end{pmatrix}$$

$\boldsymbol{\sigma} \cdot \boldsymbol{k}$





Salomaa-Volovik (88);
Schnyder-Ryu-Furusaki-Ludwig (08); Qi-Hughes-Zhang (09);
Volovik (09); Chung-Zhang (09); Nagato-Higashitani-Nagai (09), ...

Paradigm for interplay between bosonic excitations and topological fermions

Consequences of Spontaneous Symmetry Breaking: "Higgs"

U(1) Higgs model: $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \varphi)^\dagger D^\mu \varphi - \mathcal{V}[\varphi, \varphi^\dagger]$ Higgs, PRL (1964)

$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

$D_\mu \equiv \partial_\mu - ie A_\mu$

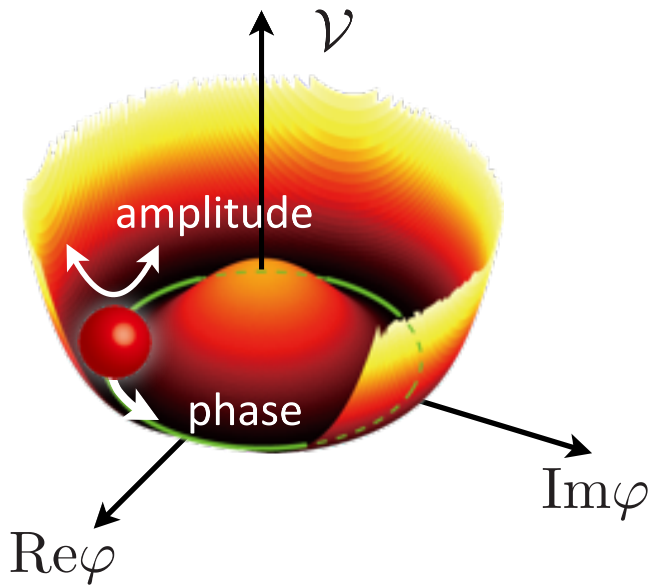
Effective theory for a vector field coupled in a gauge covariant way to a scalar field

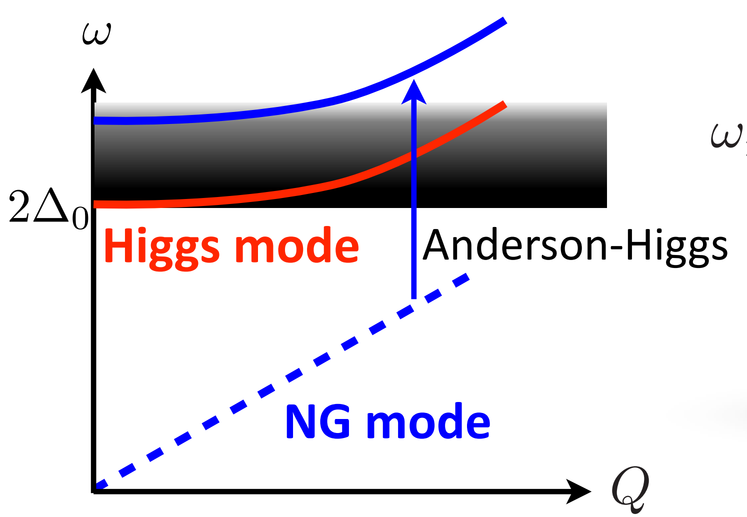
"Potential" relevant for spontaneous U(1) symmetry breaking

$$\mathcal{V} = \alpha(T) \varphi^\dagger \varphi + \frac{\beta_0}{2} \left(\varphi^\dagger \varphi\right)^2$$

$$\alpha(T) \propto -(T_c - T), \ \beta_0 > 0$$

Lorentz-invariant time-dependent GL theory





$$\omega_\pm(\boldsymbol{Q}) = \sqrt{(M_\pm)^2 + v_\pm^2 \boldsymbol{Q}^2}$$

$M^- = 0$
massless mode

$M^+ = 2\Delta_0$
massive mode

particle-hole symmetric partner

First Observations of Amplitude "Higgs" in SC/SF

Observation of a New Sound-Attenuation Peak in Superfluid ^{3}He-B

R. W. Giannetta,[a] A. Ahonen,[b] E. Polturak, J. Saunders,
E. K. Zeise, R. C. Richardson, and D. M. Lee
Laboratory of Atomic and Solid State Physics and Materials Science Center, Cornell University,
Ithaca, New York 14853
(Received 25 March 1980)

VOLUME 45, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JULY 1980

Measurements of High-Frequency Sound Propagation in ^{3}He-B

D. B. Mast, Bimal K. Sarma, J. R. Owers-Bradley, I. D. Calder,
J. B. Ketterson, and W. P. Halperin
Department of Physics and Astronomy and Materials Research Center, Northwestern University,
Evanston, Illinois 60201
(Received 10 April 1980)

VOLUME 45, NUMBER 8 PHYSICAL REVIEW LETTERS 25 AUGUST 1980

Raman Scattering by Superconducting-Gap Excitations and Their Coupling to Charge-Density Waves

SC-CDW compound $NbSe_2$

R. Sooryakumar and M. V. Klein
Department of Physics and Materials Research Laboratory, University of Illinois at Urbana-Champaign,
Urbana, Illinois 61801
(Received 24 March 1980)

Collective Modes in ^{3}He-B

TM and J.A. Sauls, in preparation

Effective Lagrangian for bosonic excitations: time-dependent GL eq

$$\mathcal{L} = \int d\boldsymbol{r} \left\{ \tau \mathrm{tr}(\partial_t \mathcal{A} \partial_t \mathcal{A}^\dagger) - \underbrace{f_{\mathrm{g}}[\partial \mathcal{A}, \partial \mathcal{A}^*]}_{\text{gradient}} - \underbrace{f_{\mathrm{b}}[\mathcal{A}, \mathcal{A}^*]}_{\text{4th bulk}} - \underbrace{f_{\mathrm{m}}[\mathcal{A}, \mathcal{A}^*]}_{\text{magnetic}} - \underbrace{f_{\mathrm{d}}[\mathcal{A}, \mathcal{A}^*]}_{\text{dipole}} \right\}$$

Equation of motion for OP fluctuations $\quad \partial_t^2 \mathcal{D}^{(c)}_{J,m} + \left[E^{(c)}_{J,m}(q)^2 \right] \mathcal{D}^{c}_{J,m} = \frac{1}{\tau} \eta^{(c)}_{J,m}$

Space-time fluctuations $\quad \mathcal{A}_{\mu i} \Rightarrow A_{\mu i} + \sum_{J,m} \mathcal{D}_{J,m}(x) t^{(J,m)}_{\mu i}$

spin orbital

BW: J=0 $\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}_{\mu i}$

spherical harmonic tensor

J=0 (scalar), 1 (vector), 2 (tensor)

Classification of 18 bosonic modes

(J, m, c)

parity under PHS

$\mathrm{c} = \pm 1$

c = + (-): fluctuation of real (imaginary) part of OP

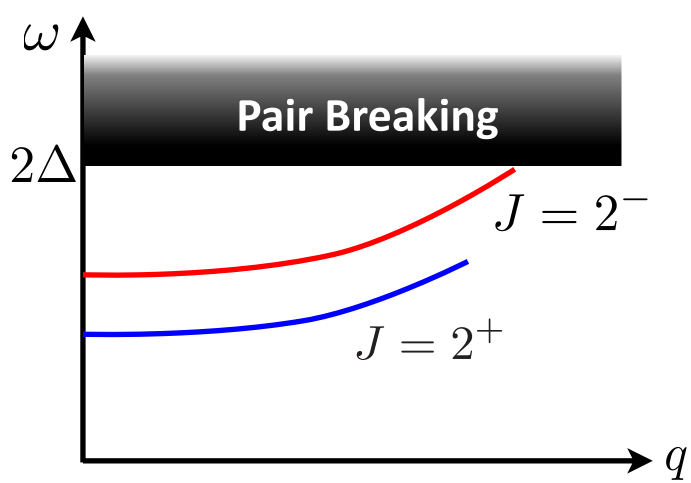
PH partners & 2J+1 multiplet in each J sector

Collective Modes in ^{3}He-B

J. A. Sauls and **TM**, arXiv:1611.07273 (PRB in press)

Mode	Symmetry	Mass	Name	
$D^{(+)}_{0,m}$	$J=0,\ \mathrm{c}=+1$	2Δ	Amplitude	
$D^{(-)}_{0,m}$	$J=0,\ \mathrm{c}=-1$	0	Phase Mode	**NG** U(1): sound
$D^{(+)}_{1,m}$	$J=1,\ \mathrm{c}=+1$	0	NG Spin-Orbit Modes	**NG** SO(3): spin
$D^{(-)}_{1,m}$	$J=1,\ \mathrm{c}=-1$	2Δ	AH Spin-Orbit Modes	
$D^{(+)}_{2,m}$	$J=2,\ \mathrm{c}=+1$	$\sqrt{\frac{8}{5}}\Delta$	2^+ AH Modes	gapped: sound/spin
$D^{(-)}_{2,m}$	$J=2,\ \mathrm{c}=-1$	$\sqrt{\frac{12}{5}}\Delta$	2^- AH Modes	gapped: sound

*dipole interaction & magnetic field are absent



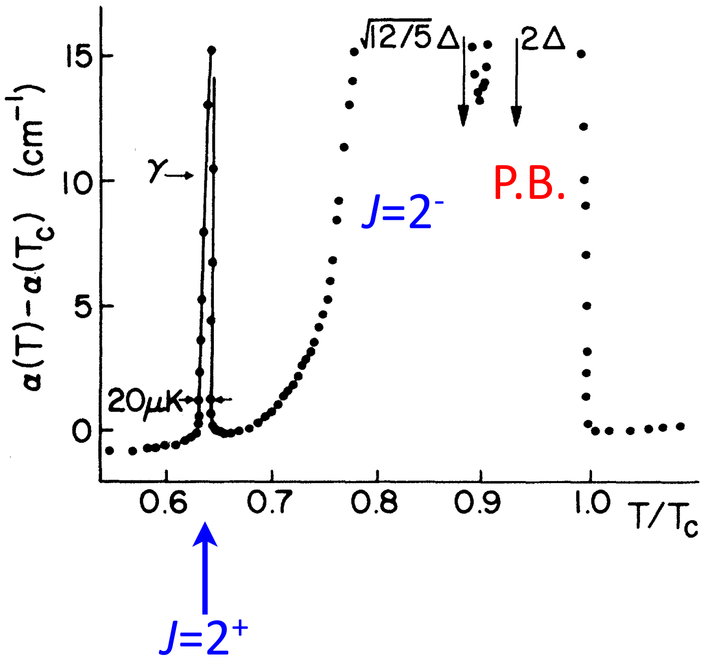
long-lived massive bosons coupled to sound waves

Vdovin, Maki, Ebisawa, Schopohl, Tewordt, Einzel, Wolfle, Nagai, Sauls, Serene, Rainer, Volovik, ...

Observations of Amplitude Higgs (Squashing) Modes

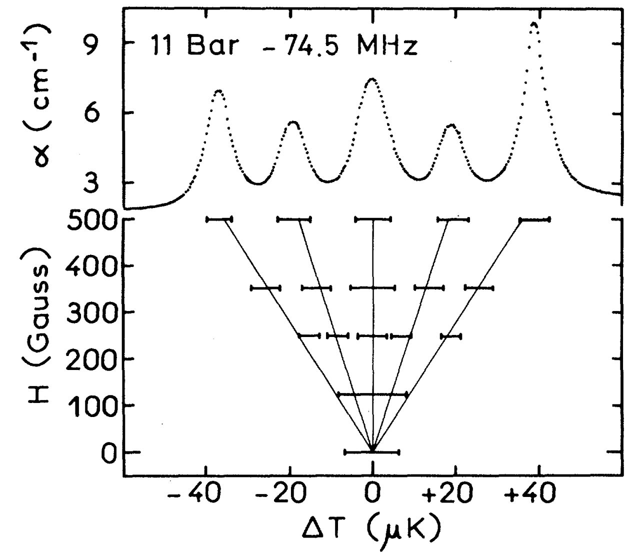
Attenuation of longitudinal sound wave

Giannett *et al.*, PRL 1980; Mast *et al.*, PRL 1980



Field-splitting of *J*=2 squashing modes

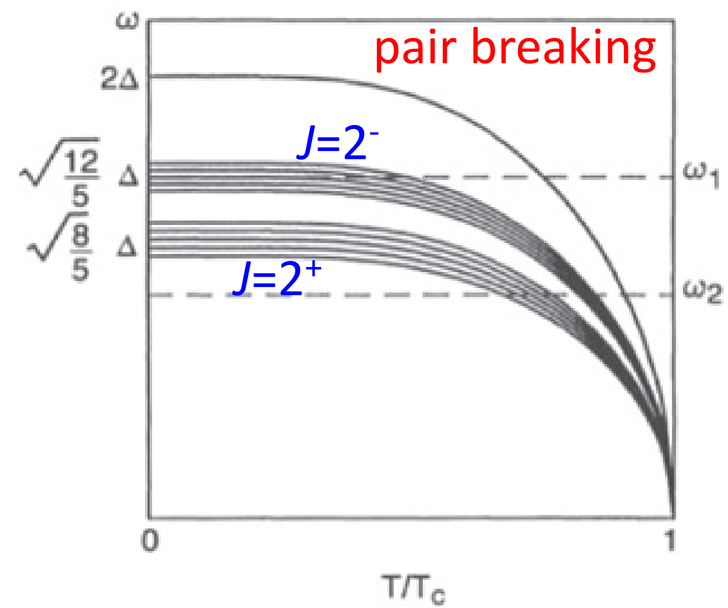
Avenel, Varoquax, and Ebisawa, PRL **45**, 1952 (1980)



Attenuation of longitudinal sound wave

$$\alpha(\omega) \propto \frac{A}{(\omega + i\delta)^2 - [\omega_{20}^-]^2} + \zeta \frac{B}{(\omega + i\delta)^2 - [\omega_{20}^+]^2}$$

PH asymmetry parameter



Fermion-Boson Mass Relations in ^{3}He-B

Nambu's "empirical observation" Y. Nambu, Physica D **15**, 147 (1985)

$$(M_J^+)^2 + (M_J^-)^2 = (2m_f)^2$$

Volovik-Zubkov
PRD (2013)

$$M_J^\pm = \sqrt{2m_f^2(1 \pm \eta^{(J)})} \qquad \eta^{(J)} \leftarrow \hat{o}^{(J)} \qquad \hat{o}^{(0)} = \boldsymbol{\sigma} \cdot \boldsymbol{p}$$

generator

$J = 0$

$M^-_{J=0} = 0$

"Phase" mode: **NG**

$M^+_{J=0} = 2\Delta_\mathrm{B}$

"amplitude" mode

$J = 1$

$M^-_{J=1} = 2\Delta_\mathrm{B}$

$M^+_{J=1} = 0$

Spin-orbit modes: **NG**

$J = 2$

$M^-_{J=2} = \sqrt{\frac{12}{5}}\Delta_\mathrm{B}$

Imaginary squashing modes

$M^+_{J=2} = \sqrt{\frac{8}{5}}\Delta_\mathrm{B}$

Real squashing modes

Nambu identity is realized in broad class of BCS type theories

Application to Top Quark Condensation

Construction of a model for dynamical electroweak symmetry breaking
using the idea from ^{3}He-B

G.E. Volovik and M.A. Zubkov, PRD **87**, 075016 (2013)

^{3}He-B
$$\left(\sqrt{\frac{8}{5}}\Delta\right)^2 + \left(\sqrt{\frac{12}{5}}\Delta\right)^2 = (2\Delta)^2$$

Re squashing   Im squashing

composite Higgs
as top quark condensates

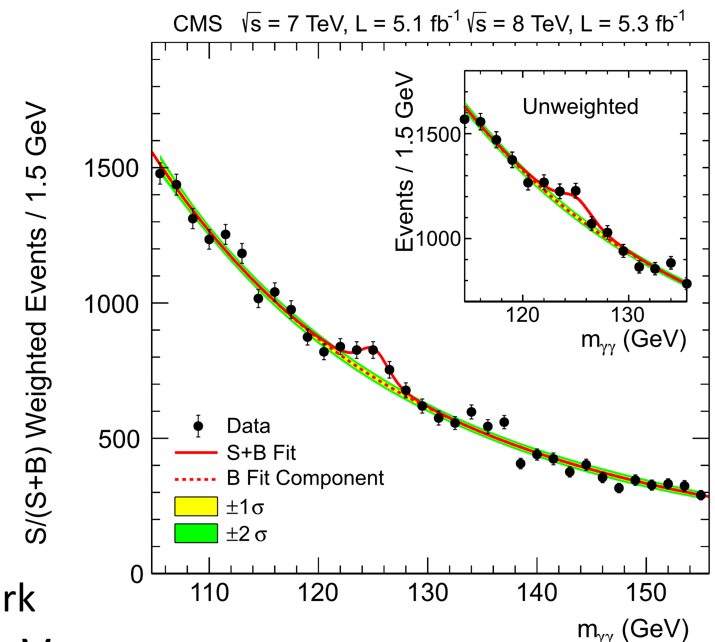
$$M_{\rm H1}^2 + {\color{red}M_{\rm H2}^2} = 4m_{\rm t}^2$$

discovered Higgs
125 GeV

Nambu partner
325 GeV

top quark
174 GeV

CMS, Phys. Lett. B **710**, 30 (2016)



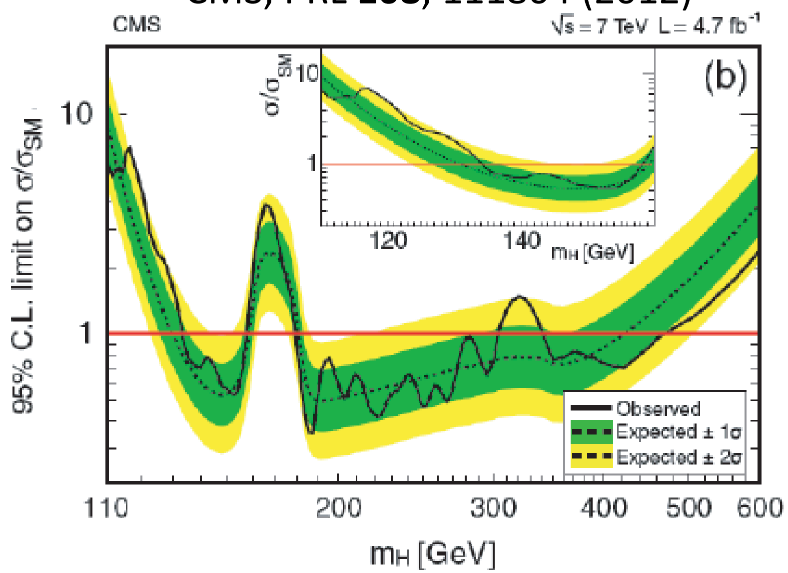
➡Nambu sum rule for a NJL-type theory of top quark condensation

➡The hint from ^{3}He-B suggests the mass of extra Higgs, ~ 325GeV

Nambu sum rule (if works) may express the mass of "extra Higgs" via quark masses

Nambu's Sum Rule & Higgs

CMS, PRL **108**, 111804 (2012)



Application to Higgs fields and top quarks

G.E. Volovik and M.A. Zubkov, PRD **87**, 075016 (2013)

top quark condensation within the Nambu-Jona Lasinio theory

$$M_{\mathrm{H1}}^2 + M_{\mathrm{H2}}^2 = 4m_{\mathrm{t}}^2$$

discovered Higgs 125 GeV

Nambu partner **325 GeV**

top quark 174 GeV

Strong coupling corrections to Nambu's fermion-boson relations?

J. A. Sauls and **TM**, arXiv:1611.07273 (PRB in press)

Key observation: NSR may be violated by excitation of Higgs bosons with symmetry distinct from that of the fermionic vacuum

(i) Strong-coupling feedback corrections to BCS theory: ***High-T TDGL***

(ii) Vacuum polarization & interactions in both the **p-h** (Landau) and **p-p** (Cooper) channels — "back-action" of bosonic fluctuations: ***Low-T Quasiclassical theory***

Mass Shift due to Strong Coupling Effects

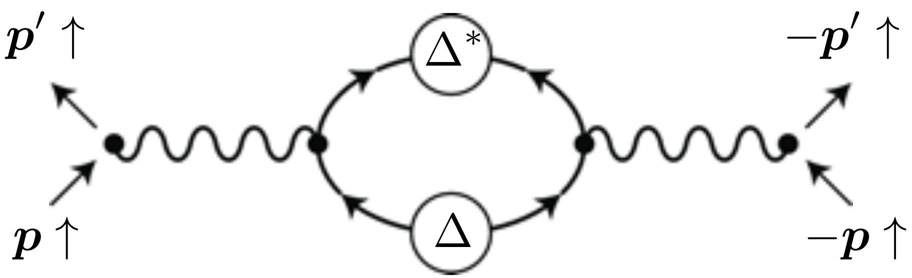
(i) Strong-coupling feedback corrections to BCS theory: ***High-T TDGL***

(ii) Vacuum polarization & interactions in both the **p-h** (Landau) and **p-p** (Cooper) channels — "back-action" of bosonic fluctuations: ***Low-T Quasiclassical theory***

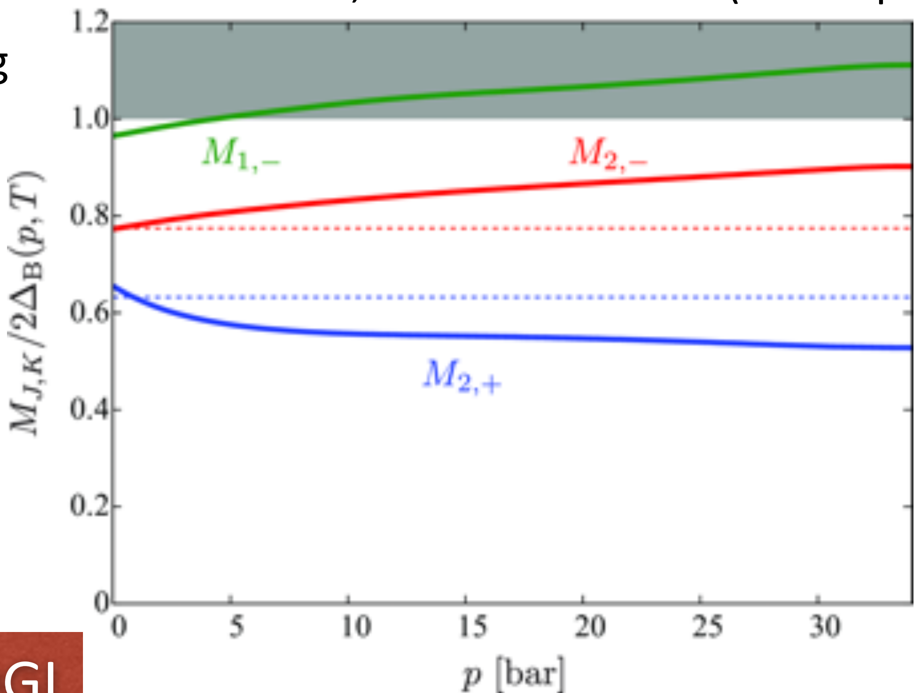
J. A. Sauls and **TM**, arXiv:1611.07273 (PRB in press)

spin-fluctuation modified by spin-fluctuation of pairing

⇒ B-to-A phase transition



Brinkman, Serene, and Anderson, PRA **10**, 2386 (1974)



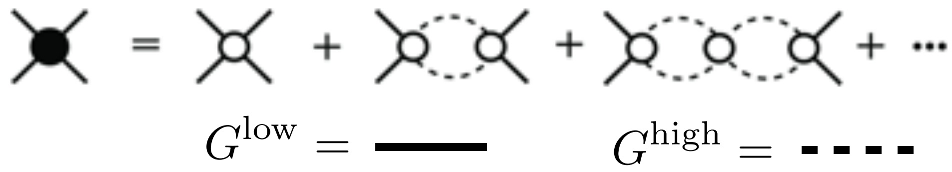
Beyond TDGL

The parent state is the Fermi liquid ground state: "Fermionic vacuum"

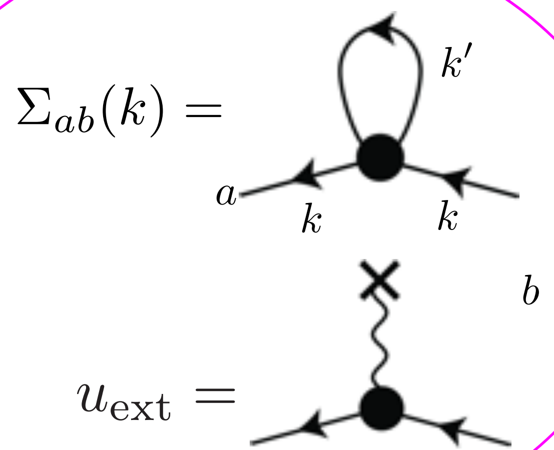
Calculation of bosonic spectrum arising from the "back-action" of the fermionic vacuum requires the theory that includes both fermonic and bosonic degrees of freedom

Superfluid Fermi Liquid Theory

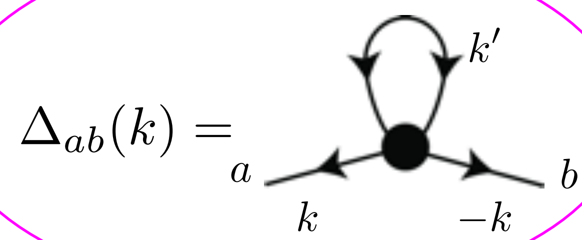
high-energy contributions are represented by phenomenological parameters



Particle-hole interactions



Particle-particle interactions



subdominant *f*-wave pairing

$$x_3 \equiv \ln(T_c^f / T_c)$$

$$F_l^{\text{s}} \qquad F_l^{(\text{a})}$$

Fermi liquid parameters (scalar & spin exchange)

Do Interactions & polarizations of the fermionic vacuum violate the sum rule?

Vacuum Polarization Corrections

Observation 1 Masses of *J*=0 and *J*=1 bosonic modes are unrenormalized by interactions

$J=0$ $\quad M_{J,-}=0$ $\quad M_{J,+}=2\Delta$

$J=1$ $\quad M_{J,-}=2\Delta$ $\quad M_{J,+}=0$

the spin NG mode acquires a mass when magnetic dipole int. is taken into account
==> Little Higgs: $M_{J,+}$ = **10kHz** << 2Δ ~ 100MHz

generalized Tsuneto fn.: fermionic self-energies

Dynamical equations for spin-triplet bosonic modes

$$\vec{d}^{(-)}(\hat{p};\omega) = -\int \frac{d\Omega_{p'}}{4\pi} V^{(1)}(\hat{p},\hat{p}') \left\{ \left[\frac{1}{2}\gamma + \frac{1}{4}(\omega^2 - 4|\Delta|^2)\bar{\lambda}(\omega)\right] \vec{d}^{(-)}(\hat{p}';\omega) + \bar{\lambda}(\omega)\vec{\Delta}(\hat{p}')\left(\vec{\Delta}(\hat{p}')\cdot\vec{d}^{(-)}(\hat{p}';\omega)\right) - \frac{1}{2}\omega\,\bar{\lambda}(\omega)\vec{\Delta}(\hat{p}')\Sigma^{(+)}(\hat{p}';\omega)\right\},$$

homogeneous equation

Bosonic fluctuations couple to fluctuation of self-energies linearly in ω

$J=0^-,\ 1^+$ Nambu-Goldstone modes

$J=0^+,\ 1^-$ $\quad (\omega^2 - 4|\Delta|^2)\,\mathcal{D}(\omega) = 0$

cannot couple to neither self-energy fluct., residual pairing (*d*-, *f*-, …), nor external fields

Vacuum Polarization Corrections

Observation 2 In J=2, the NSR is not protected against the polarizations of fermion vacuum

EOM for J=2$^-$:

$$\left[\omega^2 - (M^{(0)}_{2,-})^2\right]\mathcal{D}^-_{2,m} + \frac{8}{5}\Delta^2\mathcal{F}^-_{2,m} = \frac{4}{5}\Delta\omega\Sigma^+_{2,m}$$

bare mass — J=2 f-wave fluct — polarization of fermion vacuum

➡ Spin-fluctuation model predicts the subdominant f-wave attraction & the f-wave fluctuations can be coupled only to J=2 bosonic modes

$$d_\mu(\boldsymbol{p}) = \mathcal{D}_{\mu i}\hat{p}_i + \mathcal{F}_{\mu,ijk}\hat{p}_i\hat{p}_j\hat{p}_k \qquad T^f_{\rm c} \ll T^p_{\rm c}$$

J=2, S=1, L=1 — J=2, S=1, L=3

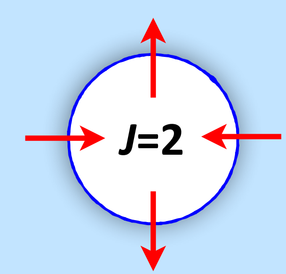
Self-energy fluctuations

$$\left[1 + \frac{1}{5}F^{\rm s}_2\lambda(\omega)\right]\Sigma^+_{2,m}(\omega) = \frac{1}{5}F^{\rm s}_2\lambda(\omega)\left(\frac{\omega}{2\Delta}\right)\left[\mathcal{D}^-_{2,m}(\omega) + \mathcal{F}^-_{2,m}(\omega)\right]$$

Pair fluctuations polarizes the J=0 condensate vacuum & generate an internal stress proportional to

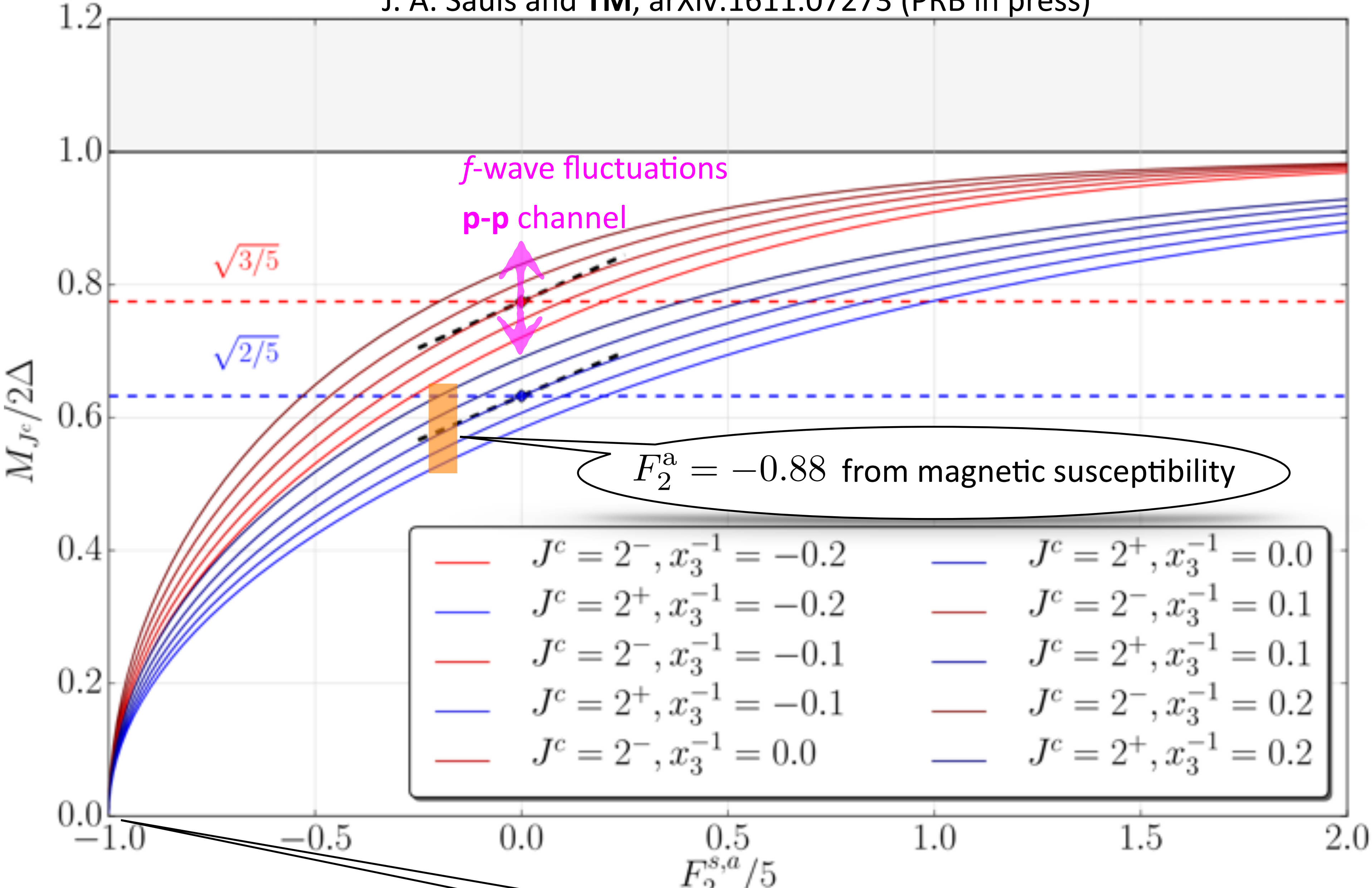
1. Fermi liquid parameter (particle-hole interaction channel)
2. time-derivative of bosonic mode amplitudes

J=0 condensate vacuum



Vacuum Polarization Corrections to Masses of J=2 Modes

J. A. Sauls and **TM**, arXiv:1611.07273 (PRB in press)



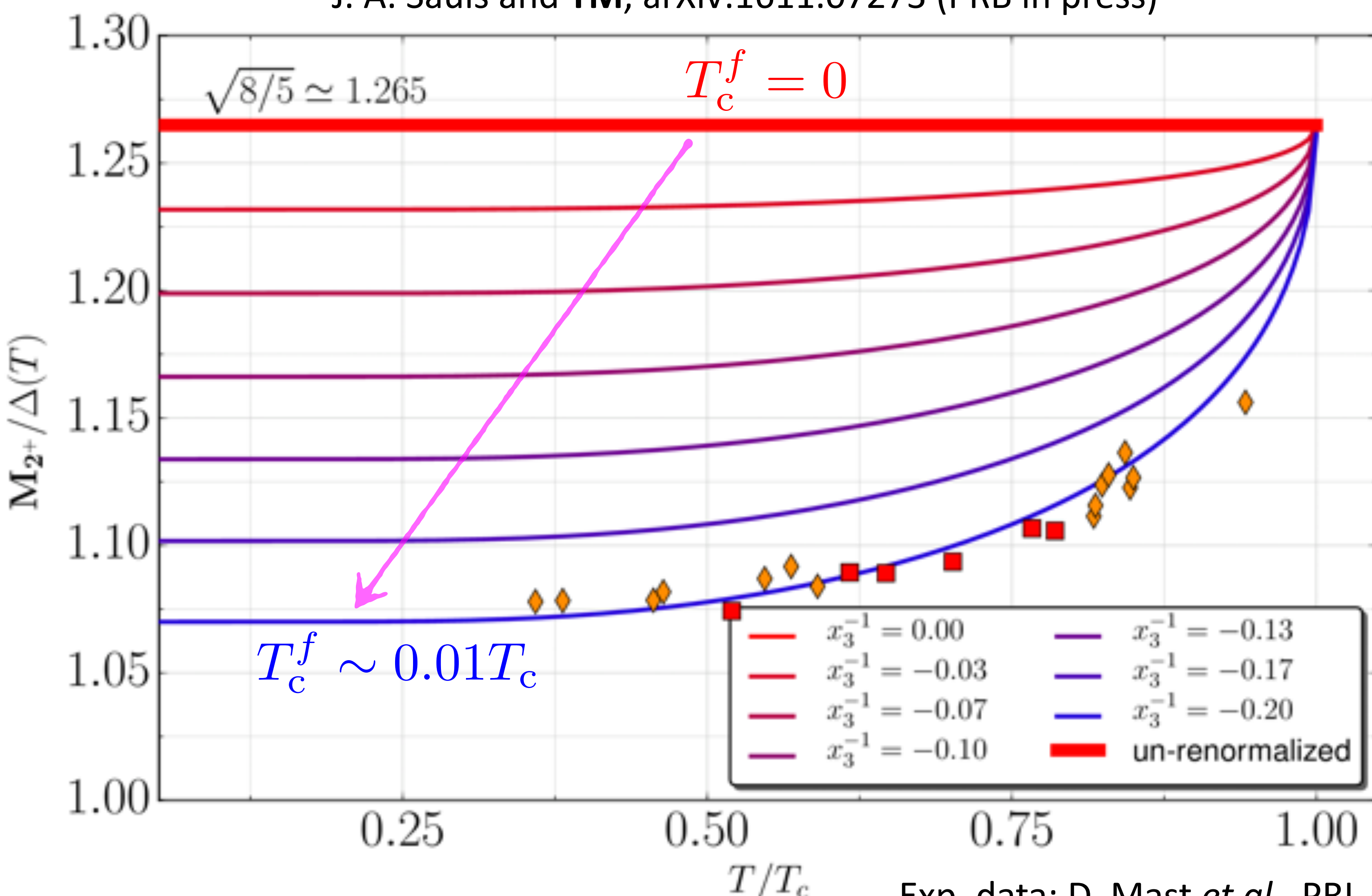
Dynamical signature of the Pomerunchuk instability of the fermionic vacuum

Mass Shift of J=2⁺ Squashing Modes in ³He-B

➡ Subdominant attractive *f*-wave interaction plays an essential role

➡ The violation of the NSR for *J*=2 modes is order of 20-30% in low temperatures

J. A. Sauls and **TM**, arXiv:1611.07273 (PRB in press)



Exp. data: D. Mast *et al.*, PRL **45**, 266 (1980)

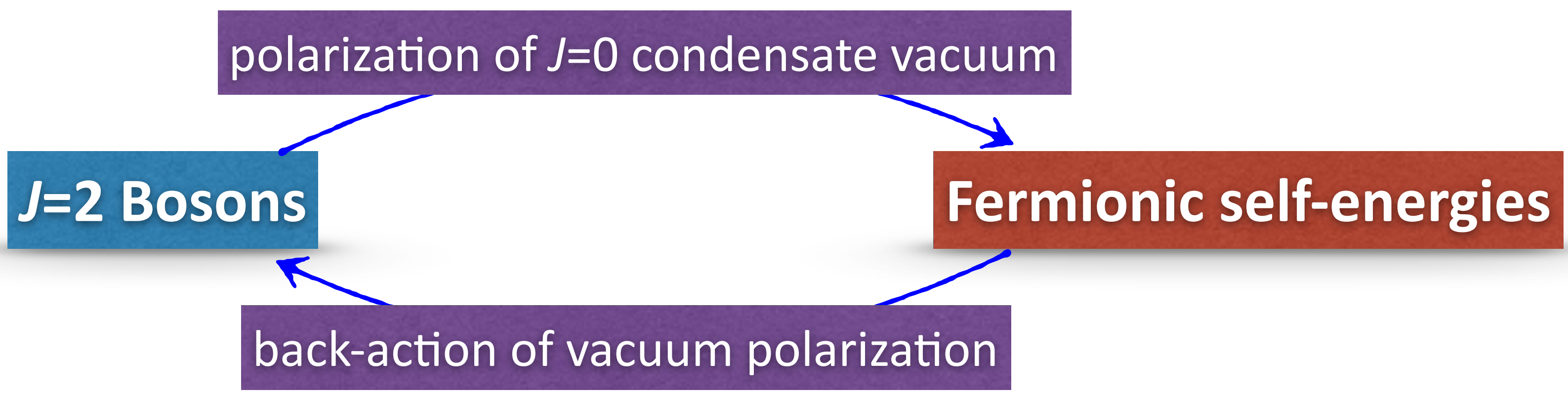
Summary

Nambu relation can be maintained in *J*=0 sector

$$(M_J^+)^2 + (M_J^-)^2 = (2m_f)^2$$

In J=0 sector, the residual interaction (dipole interaction) explicitly breaks the spin-orbit symmetry, and thus the spin-orbit NG boson acquires masses: pseudo NG bosons

Excitations of J=2 bosons generate the polarization of J=0 condensate vacuum and the back-action of vacuum polarization leads to the mass shift of *J*=2 bosons



➡bosons with the symmetry distinct from that of the vacuum may violate the Nambu's mass relation

➡The mass relation in the vacuum sector is always rigorous ? Symmetry protection of Nambu identity?

Summary

^{3}He & NS interiors: Topological aspect of unconventional SF

➡ Weyl fermions & anomaly in ^{3}He-A

$$\mathcal{H}(\boldsymbol{k}) = e^{\mu}_{j}\tau^{j}(k_{\mu} - k_{\mathrm{F}}\hat{l}_{\mu})$$

(1) l-texture: effective gauge field for Weyl fermions

(2) "Torsional" magnetic field due to l-texture

==> torsional chiral magnetic effect?

$$(e^{\mu}_{1}, e^{\mu}_{2}, e^{\mu}_{3}) = \left(\frac{\Delta}{k_{\mathrm{F}}}\hat{m}_{\mu}, \frac{\Delta}{k_{\mathrm{F}}}\hat{n}_{\mu}, v_{\mathrm{F}}\hat{l}_{\mu}\right)$$

Torsional CME in Weyl semi-metals: Sumiyoshi-Fujimoto, PRL (2016)

➡ Topology of ^{3}He-B: surface Majorana fermions ==> Ising spin & spin current

➡ $^3P_2$ in NS interiors: Nematic (~^{3}He-B), ferro. (~^{3}He-A), cyclic (~^{3}He-α)

Tricritical point & connection to superconductivity in cubic metals

Topology in confined ^{3}He-B

➡ Quantum phase transition at the critical field

Fermions: Topological phase transition & mass acquisition of surface MF

Bosons: Softening of Ising order excitation (spin-orbit pseudo-NG)

How to detect the Majorana nature of surface states?