カラー超伝導における渦

2012年11月20日@千葉工業大学

Muneto Nitta(新田宗土)

(Keio U./慶應義塾大学)

Topological Quantum Phenomena in Condensed Matter with Broken Symmetries

Keio University
1858
CALAMVS
GLADIO
FORTIOR

共同研究者

青＝ハドロン・原子核、緑＝素粒子、赤＝物性

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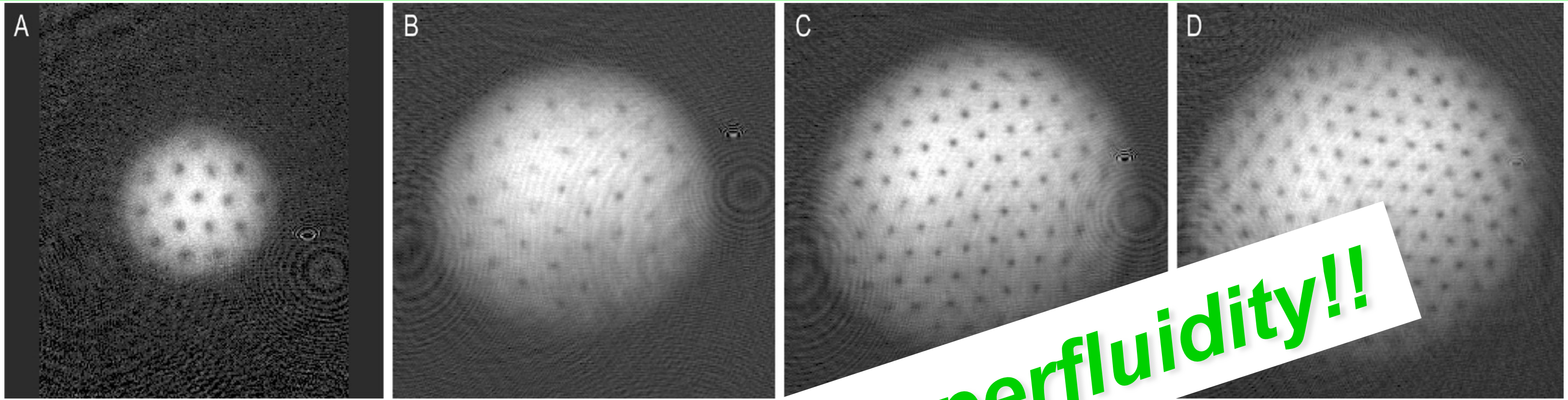
山本直希（INS, Seatle）, **安井繁宏**（KEK）, **板倉数記**（KEK）,
広野雄士（東大/理研）, **金澤拓也**, **藤原高徳**（茨城大）,
福井隆裕（茨城大）, **Walter Vinci**（Pisa）, **Mattia Cipriani**（Pisa）

1. with Nakano,Matsuura, Phys.Rev.D78:045002,2008 [arXiv:0708.4096]
2. with Eto, Phys.Rev.D80:125007,2009 [arXiv:0907.1278]
3. with Eto,Nakano, Phys.Rev.D80:125011,2009 [arXiv:0908.4470]
4. with Eto,Yamamoto, Phys.Rev.Lett.104:161601,2010 [arXiv:0912.1352]
5. with Yasui,Itakura, Phys.Rev.D81:105003,2010 [arXiv:1001.3730]
6. with Hirono,Kanazawa, Phys.Rev.D83:085018,2011 [arXiv:1012.6042]
7. with Eto,Yamamoto, Phys.Rev.D83:085005,2011 [arXiv:1101.2574]
8. with Fujiwara,Fukui,Yasui, Phys.Rev.D84:076002,2011 [arXiv:1105.2115]
9. with Hirono, Phys.Rev.Lett.109:062501,2012 [arXiv:1203.5059]
10. with Vinci,Cipriani, Phys.Rev.D86:085018, 2012 [arXiv:1206.3535]
11. with Cipriani,Vinci, arXiv:1208.5704

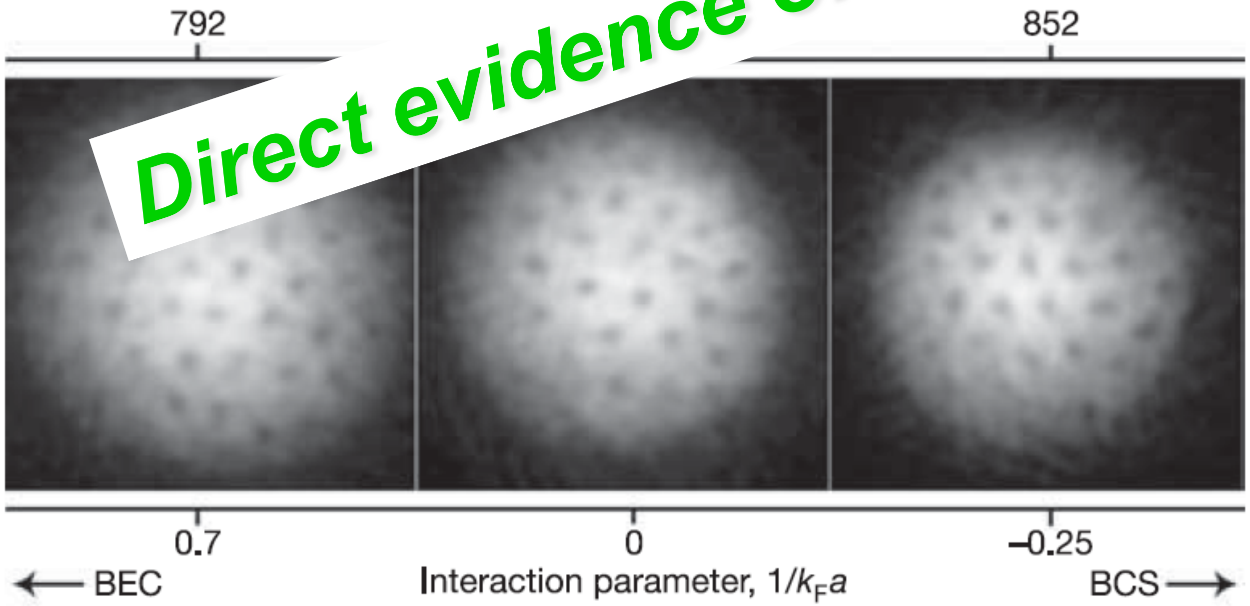
Vortex nucleation in superfluid under rotation



Ultracold atomic BEC

Abo-Shaeer, Ra… …etterle, Science 292, 476-479 (2001)

Direct evidence of superfluidity!!



BEC/BCS Crossover

Zwierlein, Abo-Shaeer, Schirotzek, Schunck & Ketterle *Nature* 435, 1047-1051 (23 June 2005)

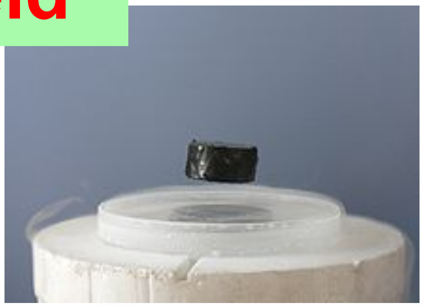
Superconductors in magnetic field

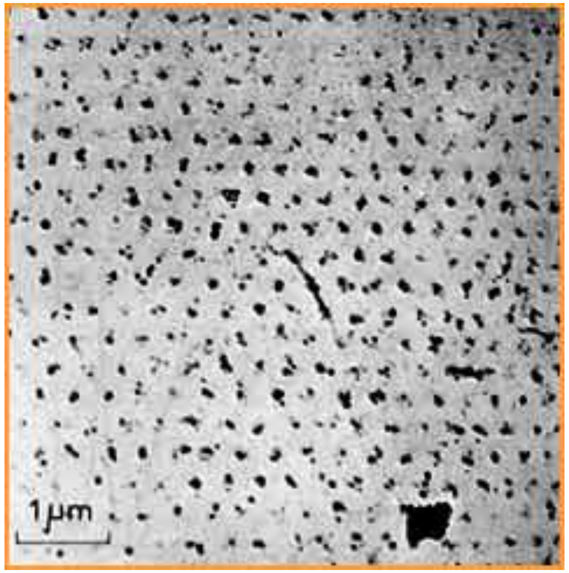
Vortex Matter

http://ltp.phys.titech.ac.jp/vormat.htm

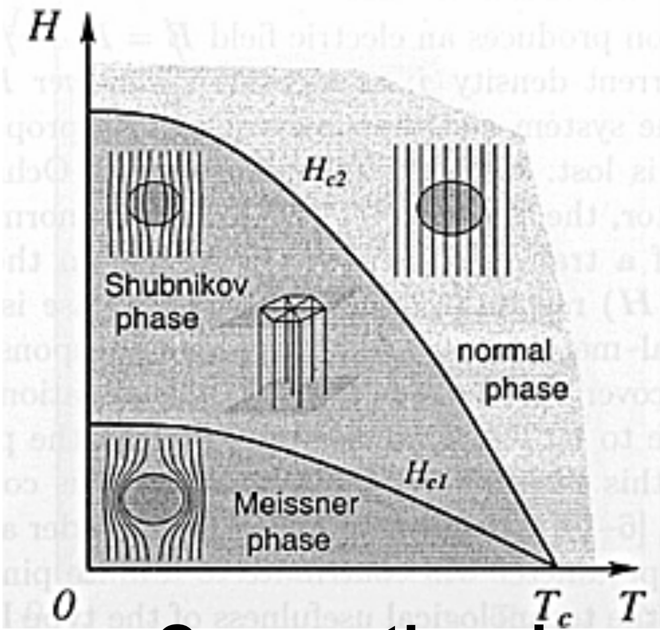
1. Vortex Core States
2. Vortex Lattice、Glass, Liquid
3. Vortex Dynamics

Vortex
Flux tube

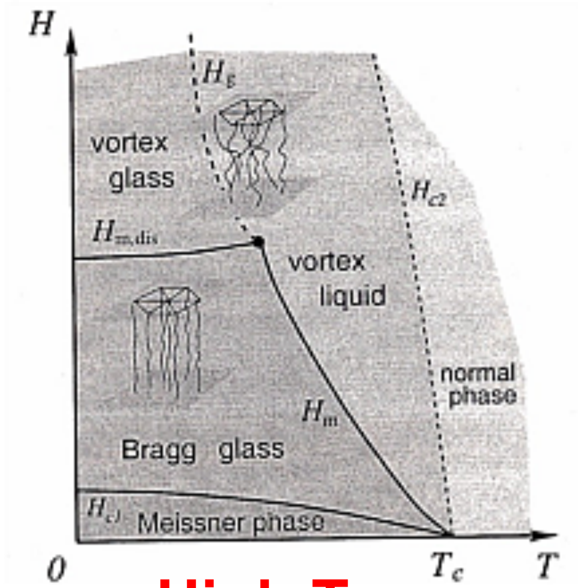




Abrikosov vortex lattice



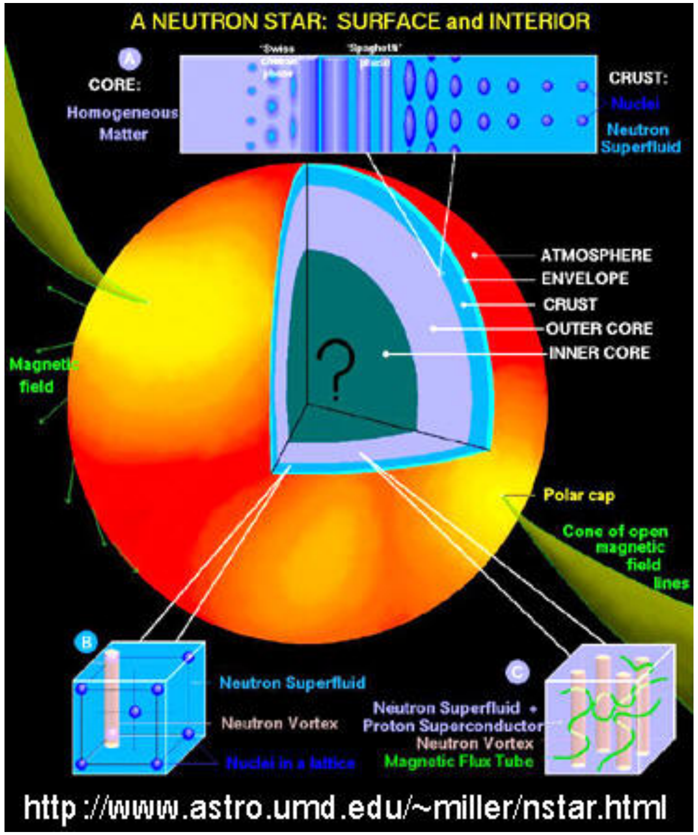
Conventional supercond



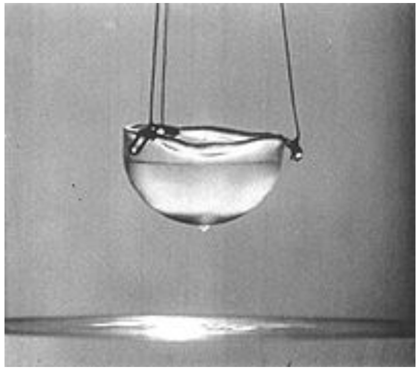
High Tc supercond

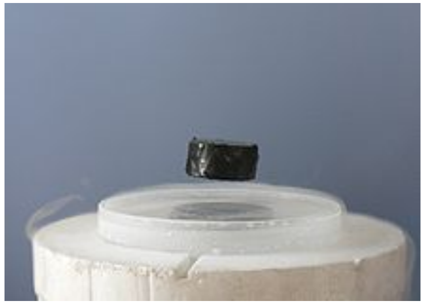
Neutron Stars

Core Nuclear matter



Neutron superfluid





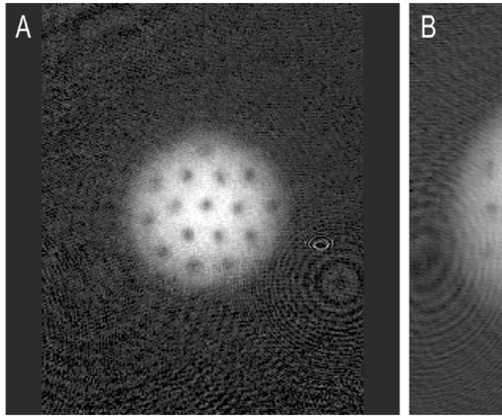
Proton super-conductor

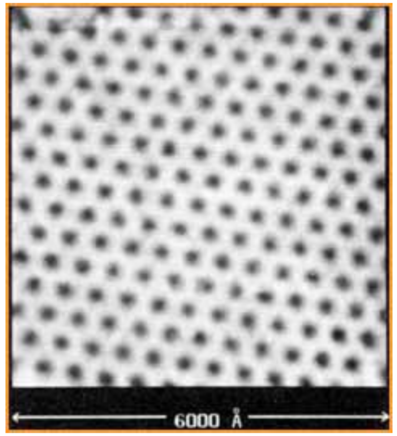
Rotation

Magnetic field

Baym&Pines ('60s)
Anderson&Itoh ('75)

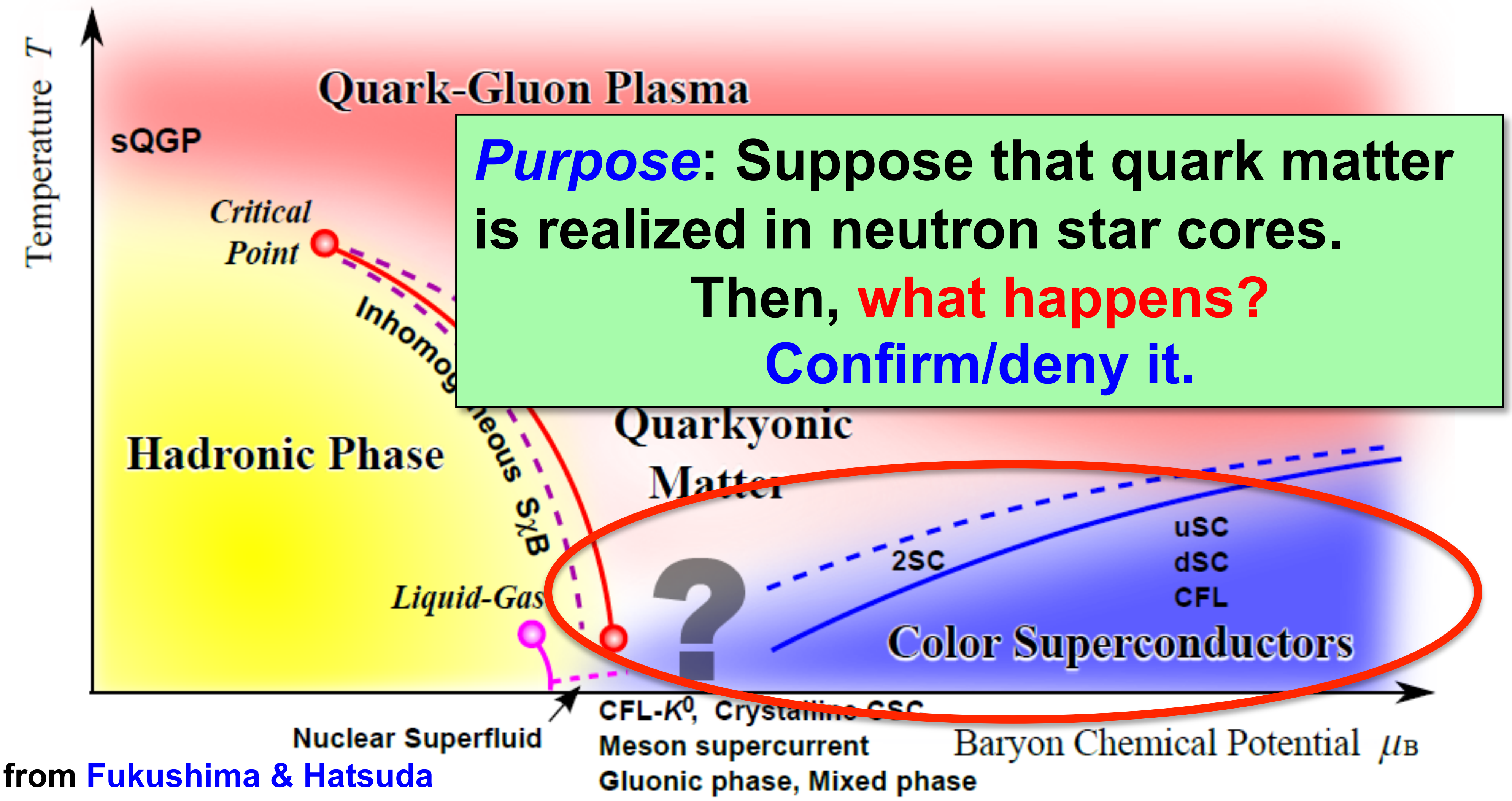
Superfluid vortices





vortices (Flux tubes)

Topic in this talk



Purpose: Suppose that quark matter is realized in neutron star cores.
Then, **what happens?**
Confirm/deny it.

from **Fukushima & Hatsuda**
Rept.Prog.Phys. 74 (2011) 014001

Quantum Chromo Dynamics (QCD)

Quark matter

quarks

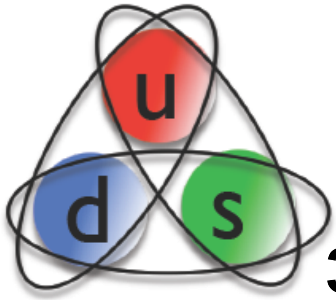
$$q^i_\alpha \quad i = \text{u,d,s flavor(global) SU(3)}$$
$$\alpha = \text{r,g,b color(gauge) SU(3)}$$

Color-flavor locked (CFL) phase

"Color superconductor"

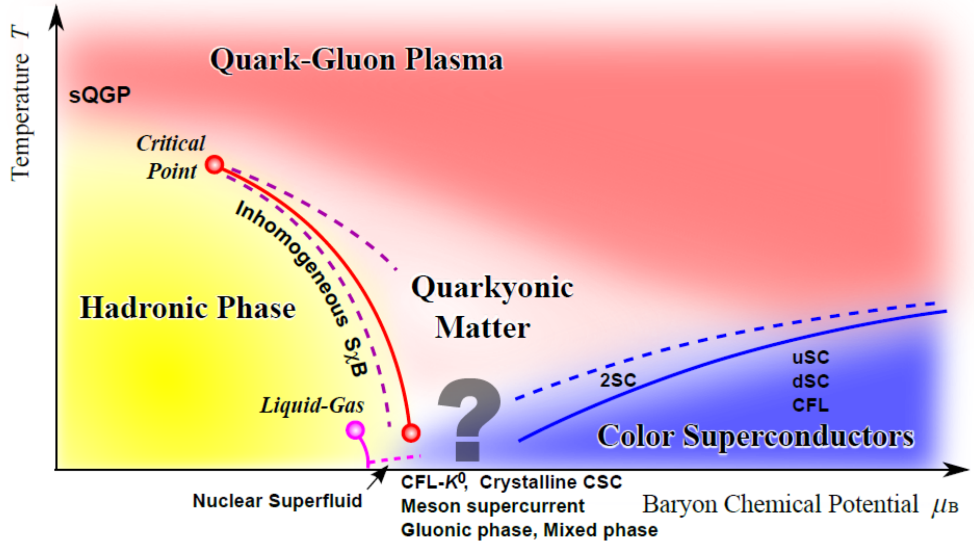
@ high density

Alford-Rajagopal-Wilczek('98)



3x3 matrix

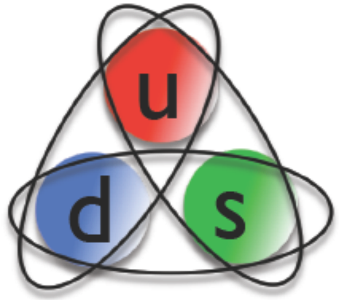
$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q^j_\beta q^k_\gamma \sim 1_{\alpha i}$$



from Fukushima & Hatsuda
Rept.Prog.Phys. 74 (2011) 014001

Color superconductivity as well as *superfluidity*

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^\beta q_k^\gamma$$

$$\alpha = 1,2,3 \ (\mathrm{r,g,b}) \quad i = 1,2,3 \ (\mathrm{u,d,s})$$

$$\Phi_{\alpha i} = \begin{pmatrix} d_{[g}s_{b]} & s_{[g}u_{b]} & u_{[g}d_{b]} \\ d_{[b}s_{r]} & s_{[b}u_{r]} & u_{[b}d_{r]} \\ d_{[r}s_{g]} & s_{[r}u_{g]} & u_{[r}d_{g]} \end{pmatrix} \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array}$$

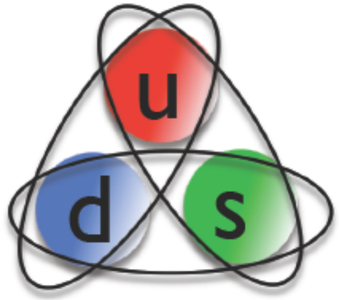
$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

$$G = U(1)_\mathrm{B} \times SU(3)_\mathrm{C} \times SU(3)_\mathrm{F}$$

$$\boxed{\Phi_{\alpha i} \to e^{i\alpha} g_\mathrm{color} \Phi_{\alpha i} g_\mathrm{flavor}}$$

Color superconductor

$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk}q_j^{\beta}q_k^{\gamma}$$



$$\alpha = 1,2,3\ (\mathrm{r,g,b}) \quad i = 1,2,3\ (\mathrm{u,d,s})$$

Ground state

$$\Phi_{\alpha i} = \begin{array}{cc} \begin{pmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{pmatrix} & \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array} \\ \begin{array}{ccc} \bar{u} = ds & \bar{d} = sb & \bar{s} = ud \end{array} & \end{array}$$

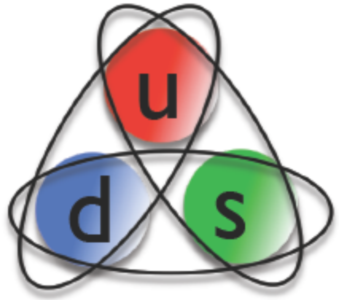
color-flavor locked (CFL)

$$G = U(1)_{\mathrm{B}} \times SU(3)_{\mathrm{C}} \times SU(3)_{\mathrm{F}} \quad \cancel{U(1)_{\mathrm{B}}}$$ **superfluidity**

$$\rightarrow H = SU(3)_{\mathrm{C+F}} \quad g_{\mathrm{color}} = g_{\mathrm{flavor}}^{-1} \quad \cancel{SU(3)_{\mathrm{C}}}$$ **color superconductivity**

Color superconductor

$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^\beta q_k^\gamma$$



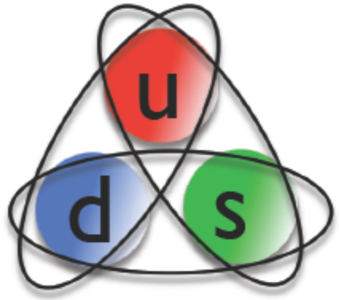
Integer quantized superfluid vortex

$$\alpha = 1,2,3\ (\mathrm{r,g,b}) \quad i = 1,2,3\ (\mathrm{u,d,s})$$

$$\Phi_{\alpha i} = \begin{array}{ccc} \left(\begin{array}{ccc} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{array}\right) & \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array} \\ \begin{array}{ccc} \bar{u} = ds & \quad \bar{d} = sb & \quad \bar{s} = ud \end{array} & \end{array}$$

Iida & Baym, Forbes & Zhitnitsky('02)

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^\beta q_k^\gamma$$

$$\alpha = 1,2,3 \;(\mathrm{r,g,b}) \quad i = 1,2,3 \;(\mathrm{u,d,s})$$

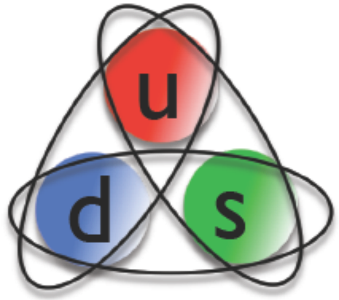
1/3 quantized vortex

$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r) e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix} \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array}$$

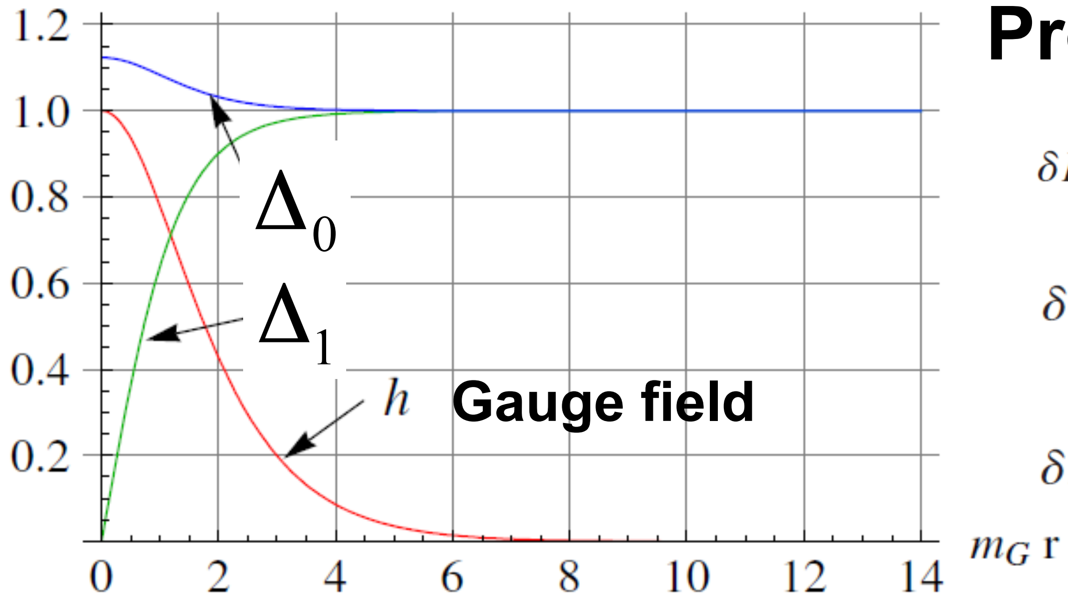
$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

Balachandran, Digal & Matsuura (BDM) (‘05)
Nakano, MN & Matsuura (‘07), Eto & MN (‘09)

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$



Profiles

$F \equiv \Delta_1 + 2\Delta_0,$ **trace**

$G \equiv \Delta_1 - \Delta_0,$ **traceless**

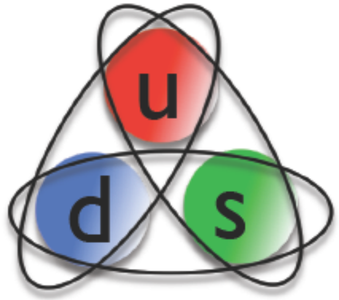
$$\delta F = q_\phi \sqrt{\frac{\pi}{2m_\phi r}} e^{-m_\phi r} + \left(-\frac{1}{3m_\phi^2 r^2} + \mathcal{O}\left(\frac{1}{(m_\phi r)^4}\right)\right)$$

$$\delta G \simeq q_\chi K_{1/3}(m_\chi r) \simeq q_\chi \sqrt{\frac{\pi}{2m_\chi r}} e^{-m_\chi r},$$

$$\delta h \simeq q_G m_G r K_1(m_G r) \simeq q_G \sqrt{\frac{\pi m_G r}{2}} e^{-m_G r}$$

Eto & MN ('09)

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \underbrace{\exp\frac{i\theta}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}}_{\text{SU(3) color}} \begin{pmatrix} \Delta_1(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

1/3 quantized

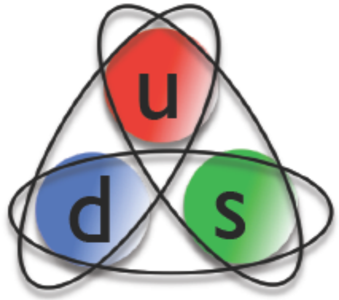
SU(3) color

→ **color flux tube**

Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

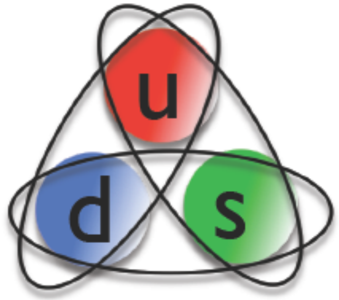
$$= \underbrace{\boxed{\exp\left(\frac{i\theta}{3}\right)}}_{\text{1/3 quantized}} \underbrace{\exp\frac{i\theta}{3}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}}_{\text{SU(3) color}} \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

→ **color flux tube**

Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) e^{i\theta} \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \underbrace{\exp\frac{i\theta}{3}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}}_{\text{SU(3) color}} \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) \end{pmatrix}$$

1/3 quantized

SU(3) color

→ **color flux tube**

Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex

$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) e^{i\theta} \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \underbrace{\exp \frac{i\theta}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}}_{\text{SU(3) color}} \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) \end{pmatrix}$$

1/3 quantized

SU(3) color

→ **color flux tube**

Comment

Non-Abelian vortices were discovered earlier in the context of Supersymmetry and String theory ('03)

Superfluid vortex

Non-Abelian vortex

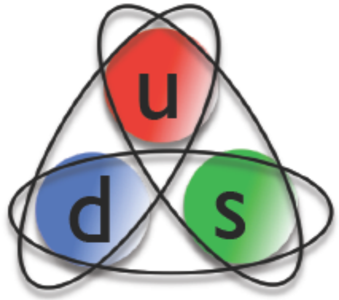
Non-Abelian vortices

Color Fluxes

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$



Abelian vortex

No flux

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$

Which are energetically favored?

Non-Abelian vortices

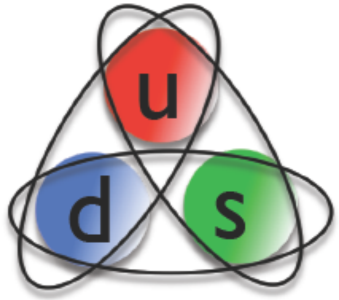
Color Fluxes

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$

Split



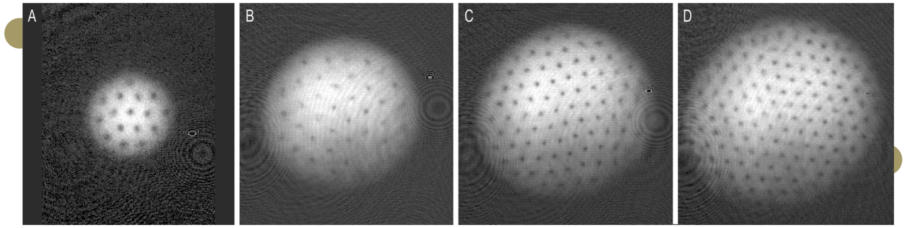
Abelian vortex

No flux

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$

$$E[\quad] = E[\quad] = E[\quad] = \frac{1}{9}E[\quad]$$

Nakano, MN & Matsuura ('07)



Abrikosov vortex lattice


Colorful vortex lattice

Colorful vortex lattice

Google画像検索
"カラフル　水玉"

In a neutron star

Total number of vortices

$$N_v \simeq 1.9 \times 10^{19}\left(\frac{1\ \mathrm{ms}}{P_{\mathrm{rot}}}\right)\left(\frac{\mu/3}{300\ \mathrm{MeV}}\right)\left(\frac{R}{10\ \mathrm{km}}\right)^2$$

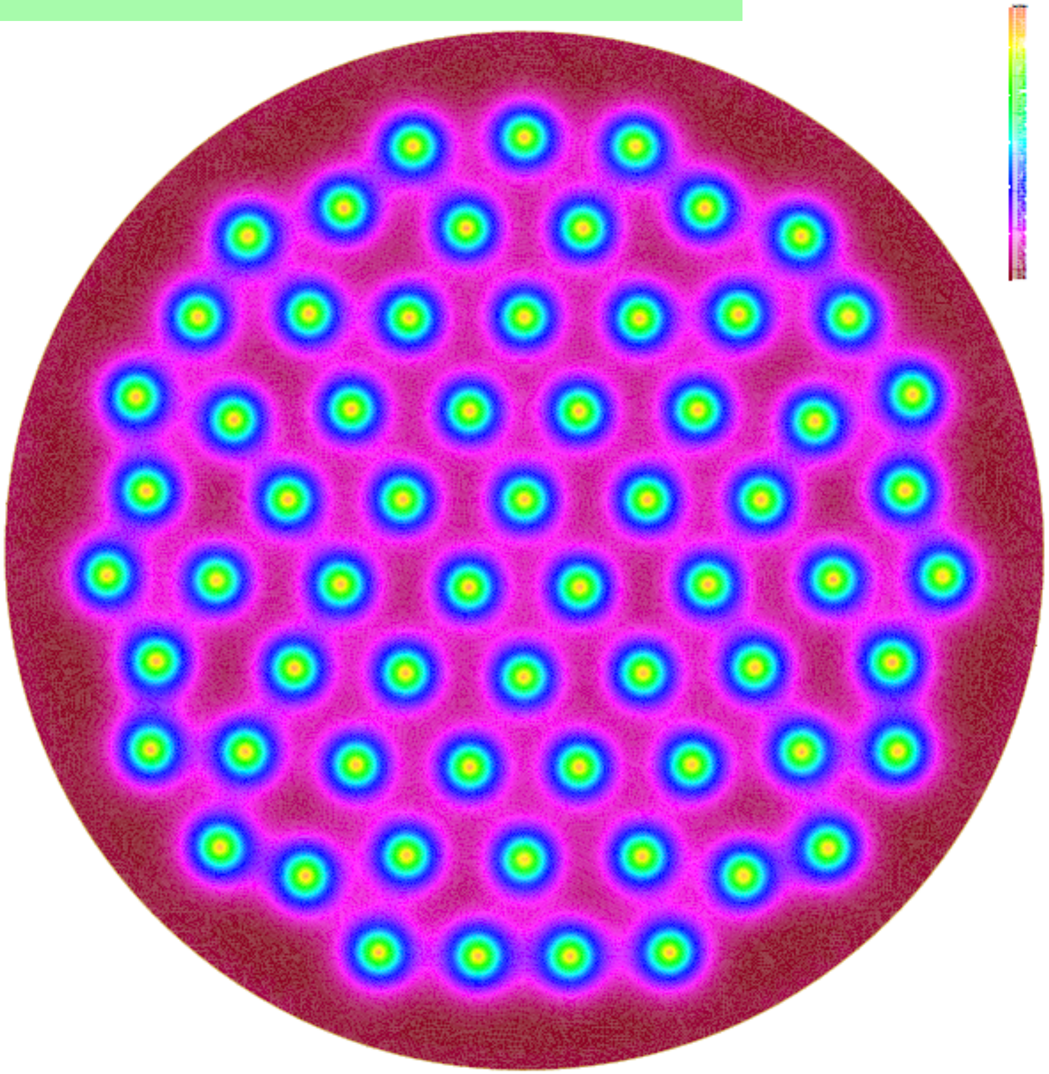
Colorful vortex lattice

Intervortex spacing

$$\ell \equiv \left(\frac{\pi R^2}{N_v}\right)^{1/2} \simeq 4.0 \times 10^{-6}\ \mathrm{m}\left(\frac{P_{\mathrm{rot}}}{1\ \mathrm{ms}}\right)^{1/2}\left(\frac{300\ \mathrm{MeV}}{\mu/3}\right)^{1/2}$$

Colorful vortex lattice

Cipriani,Vinci & MN,
in preparation



$$\Phi_{\alpha i} = \left(\begin{array}{c|cc} \Delta_1(r)e^{i\theta} & 0 & 0 \\ \hline 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{array}\right)$$

$$H = SU(3)_{\mathrm{C+F}}$$
$$\downarrow$$
$$K = [SU(2)\times U(1)]_{\mathrm{C+F}}$$

@ vortex core

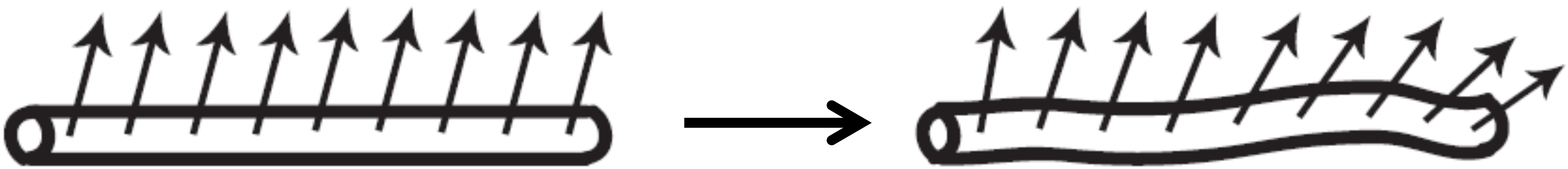
Nambu-Goldstone modes localized around the vortex

$$\boxed{\frac{H}{K} = \frac{SU(3)_{\mathrm{C+F}}}{SU(2)\times U(1)} = \mathbf{C}P^2}$$

Continuous family of solutions exists

Eto,Nakano&MN('09)

= Gapless modes propagating along the vortex line



"ground state" **1+1 dim effective theory** **fluctuations**

Orientational moduli

$$\frac{H}{K} = \frac{SU(3)_{\mathrm{C+F}}}{SU(2)\times U(1)} = \mathbb{C}P^2$$

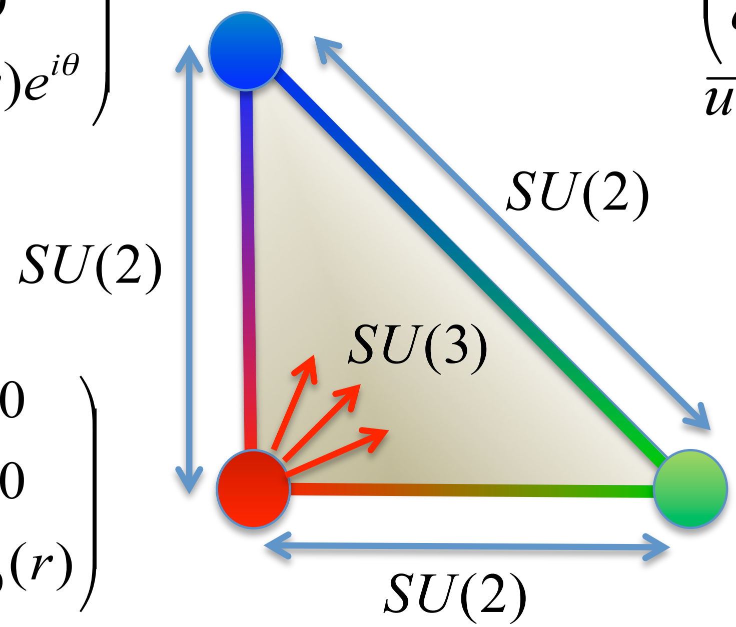
correspond to color fluxes

$$\Phi_{\alpha i} = \begin{pmatrix} d_{[g}s_{b]} & s_{[g}u_{b]} & u_{[g}d_{b]} \\ d_{[b}s_{r]} & s_{[b}u_{r]} & u_{[b}d_{r]} \\ d_{[r}s_{g]} & s_{[r}u_{g]} & u_{[r}d_{g]} \end{pmatrix} \begin{matrix} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{matrix}$$

$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

"toric diagram"

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$



$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

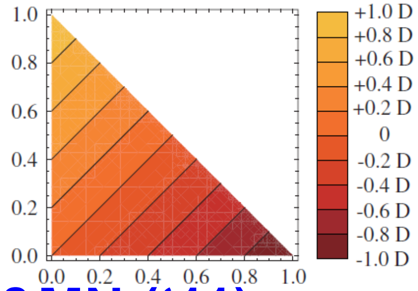
$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

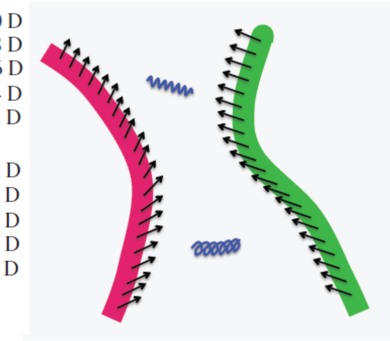
What we have found so far

1.Interaction between vortices Nakano,MN&Matsuua (‘07)

2.Inclusion of strange quark mass

decay into one kind Eto,MN&Yamamoto(‘09)





3.Interaction with quasi-particles

($U(1)_B$ phonon, gluons) Hirono,Kanazawa&MN (‘11)

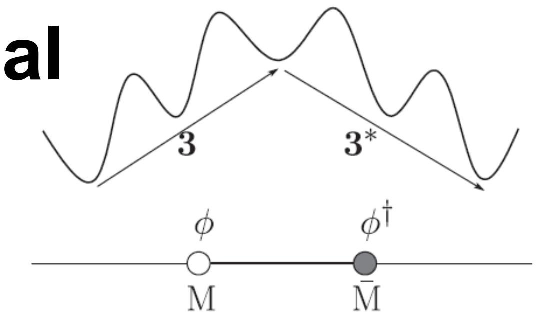
4.Interaction with $U(1)_{EM}$ gauge field

Vinci,Cipriani&MN(‘12), Hirono&MN (‘12)

5.Quantum mechanically induced potential

monopole-anti-monopole confined by a vortex

Eto,MN&Yamamoto, Shifman&Yung (‘11)



6.Quark bound states Yasui,Itakuta&MN (’10)

Index theorem Fujiwara,Fukui,MN&Yasui (’11)

Non-Abelian statistics Yasui,Itakuta&MN(’11), Hirono,Yasui,Itakura&MN(‘12)

Interaction of vortex with quasi-particles

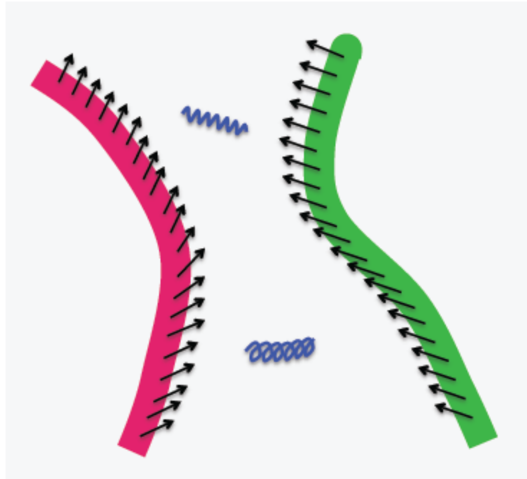
Hirono, Kanazawa & MN,
PRD83:085018,2011 [arXiv:1012.6042]

Duality transformation (electric⇔magnetic)

$$\text{massless } A_\mu \longleftrightarrow \text{massless } A_\mu$$

$$\text{massless } \phi \longleftrightarrow \text{massless } B_{\mu\nu}$$

$$\text{massive } A_\mu \longleftrightarrow \text{massive } B_{\mu\nu}$$



Interaction with massive gluons
(non-Abelian 2-form of Freedman-Townsend)

$$\boxed{S^{\mathrm{g}}_{\mathrm{int}} = \int \mathrm{d}^4x \frac{m}{2gN} B^a_{\lambda\sigma} \epsilon_{\lambda\sigma\nu\mu} \partial_\nu \left\{ H(r) \partial_\mu \theta \phi^\dagger T^a \phi \right\}}$$

Interaction with phonon
[U(1)$_B$ Nambu-Goldstone boson]

$$\boxed{S^{\mathrm{NG}}_{\mathrm{int}} = \int \mathrm{d}^4x \frac{g|\Delta|}{N} B^0_{\lambda\sigma} \epsilon_{\lambda\sigma\nu\mu} \partial_\nu \partial_\mu \theta}$$

Electric & Magnetic U(1)$_{EM}$ interaction

In the ground state,
photon-gluon mixing
occurs:

$$A_M = \cos\zeta A^{em} + \sin\zeta A^8 ;$$
$$A_0 = -\sin\zeta A^{em} + \cos\zeta A^8$$

massive

massless

$$\cos\zeta = \sqrt{\frac{e^2}{e^2 + 3g_s^2/2}} \equiv \frac{e}{g_M}$$

u: **2*e*/3, *d*: -*e*/3, *s*: -*e*/3**

$$\Phi_{\alpha i} = \begin{pmatrix} d_{[g}s_{b]} & s_{[g}u_{b]} & u_{[g}d_{b]} \\ d_{[b}s_{r]} & s_{[b}u_{r]} & u_{[b}d_{r]} \\ d_{[r}s_{g]} & s_{[r}u_{g]} & u_{[r}d_{g]} \end{pmatrix} \begin{matrix} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{matrix}$$

$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

charge matrix acting from right

$$e\begin{pmatrix} -2/3 & 0 & 0 \\ 0 & -2/3 & 0 \\ 0 & 0 & 1/3 \end{pmatrix} \sim T_8$$

Electric & Magnetic $U(1)_{EM}$ interaction

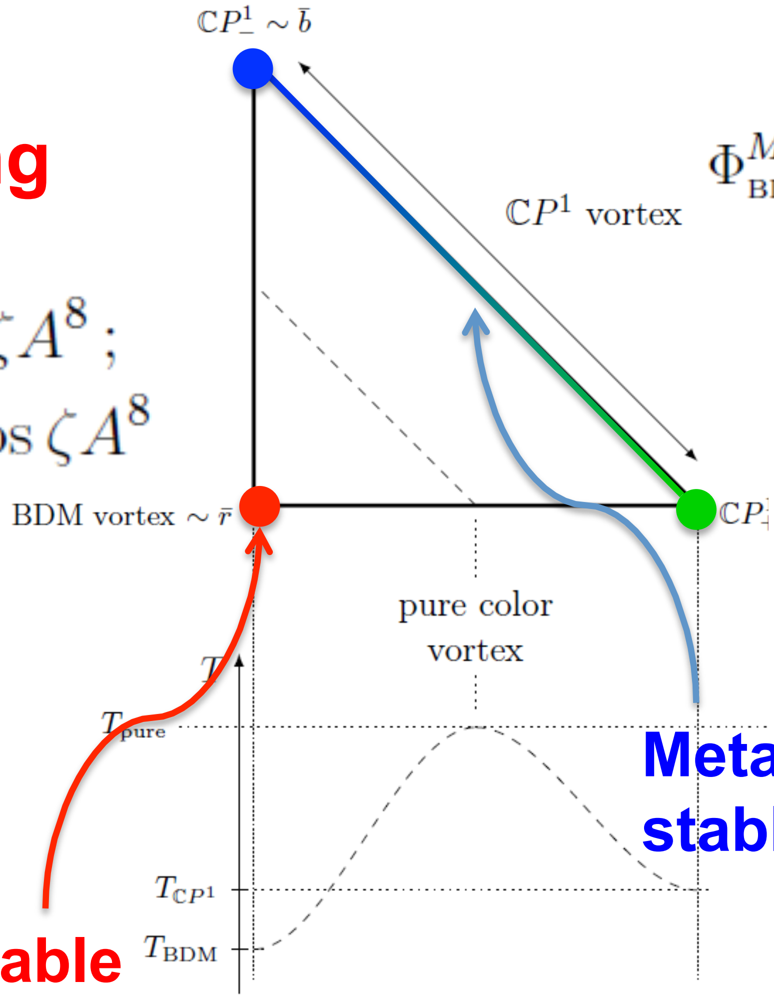
In the ground state,
photon-gluon mixing
occurs:

$$A_M = \cos\zeta A^{em} + \sin\zeta A^8 ;$$
$$A_0 = -\sin\zeta A^{em} + \cos\zeta A^8$$

massive

massless

$$\cos\zeta = \sqrt{\frac{e^2}{e^2 + 3g_s^2/2}} \equiv \frac{e}{g_M}$$



Stable

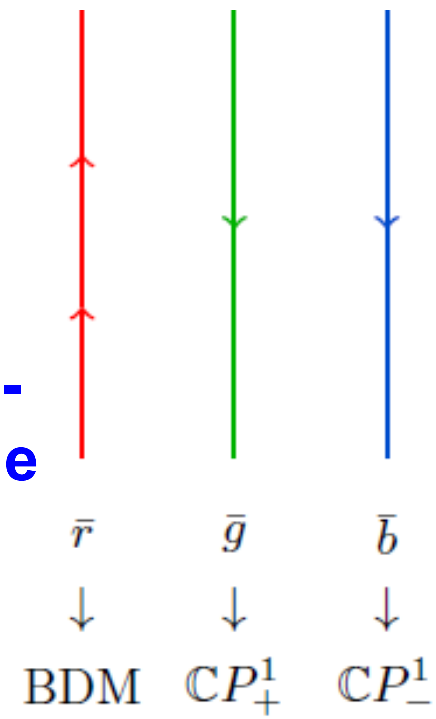
Meta-
stable

E&M induced potential

$U(1)_{EM}$ flux

$$\Phi^M_{\text{BDM}} = \oint \vec{A}^M \cdot d\vec{l} = \frac{2\pi}{g_M}$$

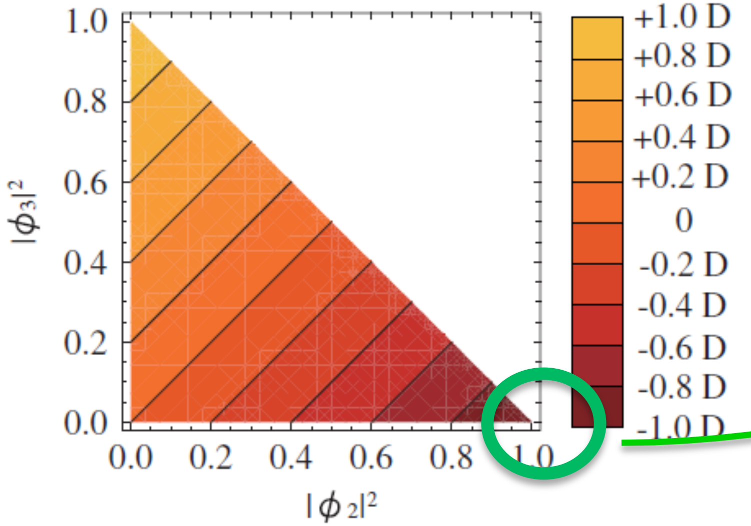
$$\Phi^M_{\mathbb{C}P^1} = -\frac{1}{2}\Phi^M_{\text{BDM}}$$



Vinci,Cipriani&MN,
PRD86:085018,2012
[arXiv:1206.3535]

Electric & Magnetic U(1)$_{\rm EM}$ interaction

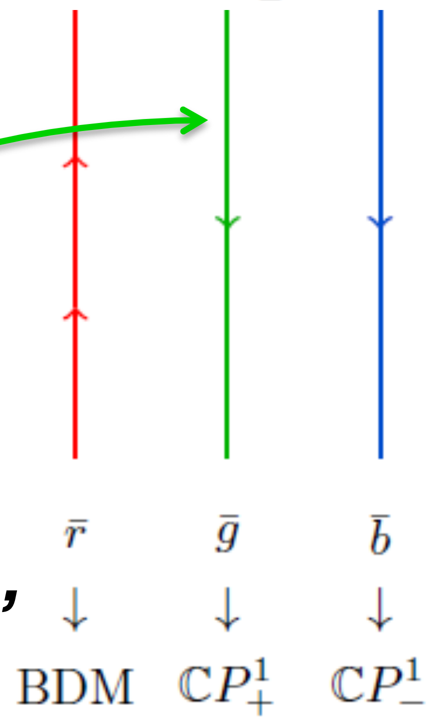
However, this potential is **washed out** by strange quark mass.



U(1)$_{\rm EM}$ flux

$$\Phi^M_{\rm BDM} = \oint \vec{A}^M \cdot d\vec{l} = \frac{2\pi}{g_M}$$

$$\Phi^M_{\mathbb{C}P^1} = -\frac{1}{2}\Phi^M_{\rm BDM}$$



"Spontaneous magnetization"

Vinci,Cipriani&MN, PRD86:085018,2012 [arXiv:1206.3535]

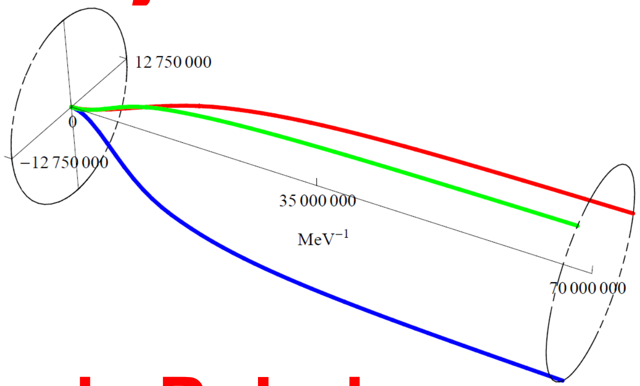
However, very small~O(1)G, which cannot be used for NS

Electric & Magnetic U(1)$_{EM}$ interaction

Other interesting phenomena

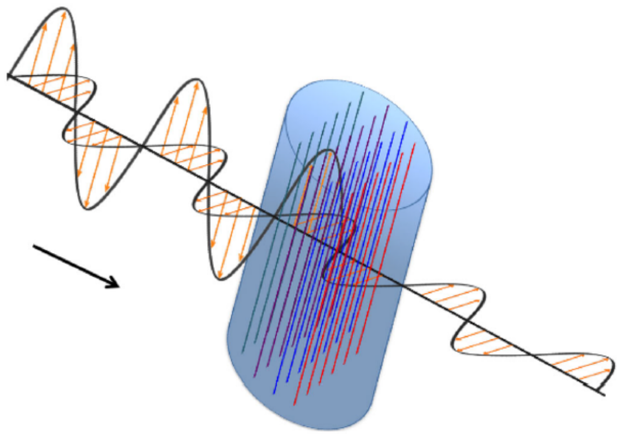
1.Colored Monopoles at Boojums

Cipriani,Vinci & MN,
arXiv:1208.5704



2. Compact Stars as Cosmic Polarizers

Hirono &MN,
Phys.Rev.Lett.109:062501,2012
[arXiv:1203.5059]



What is Boojum?

Boojum trees in Arizona

Boojum

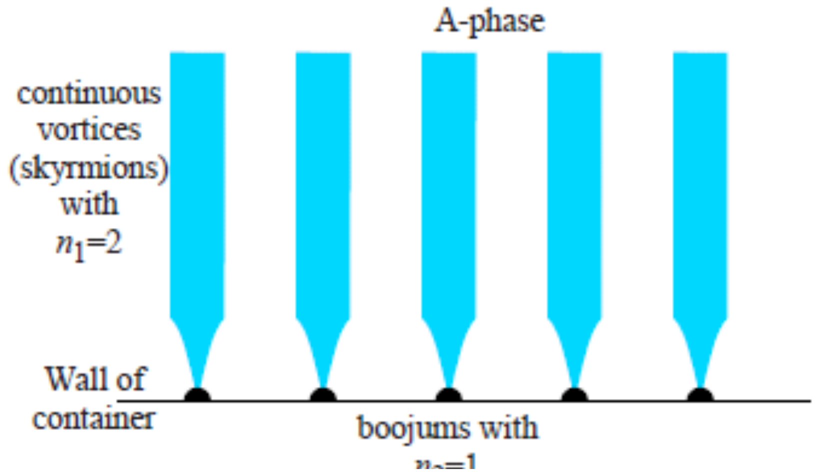
A particularly dangerous kind of Snark

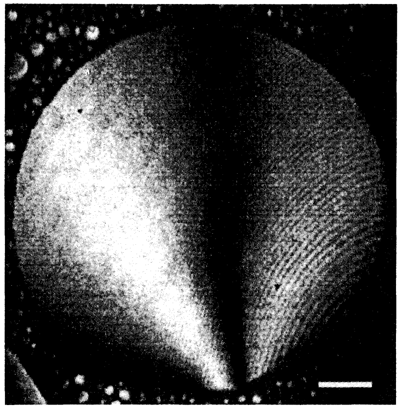
“The Hunting Of Snark”
Lewis Carroll

What is Boojum?

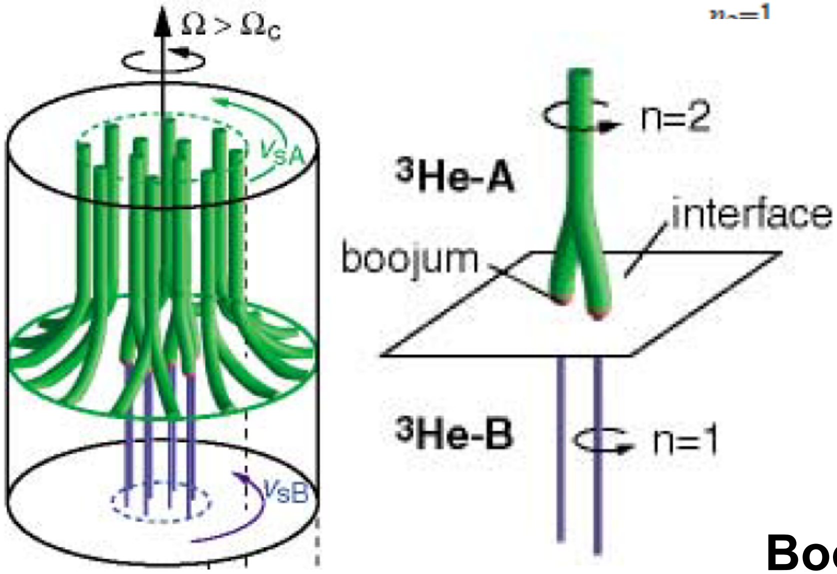
In physics, named by
D.Mermin (1976)

Boojums on the container wall
in superfluid ^{3}He-A

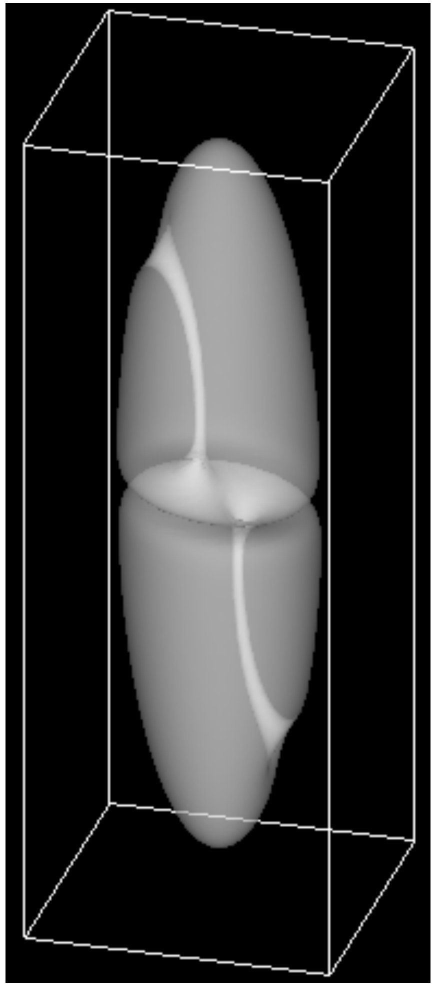




Boojum in
liquid crystal



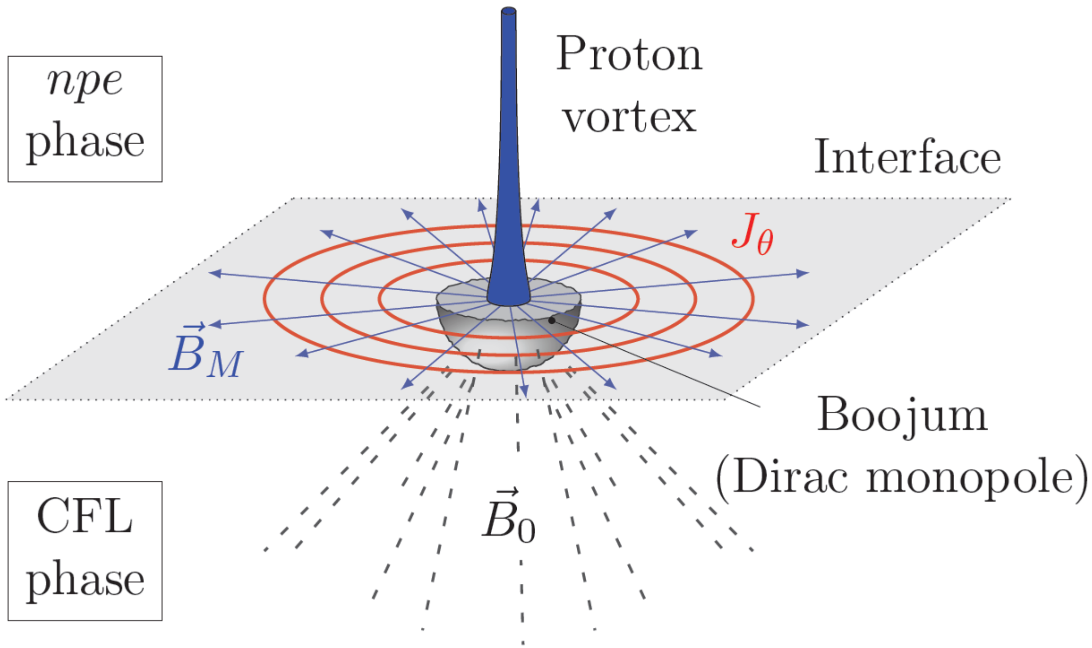
Boojums in ^{3}He A-B phase
boundary



Boojum in two component BECs
Kasamatsu-Takeuchi-
MN-Tsubota, JHEP1011:068,2010
[arXiv:1002.4265]

Boojums at interface of nuclear matter/quark matter

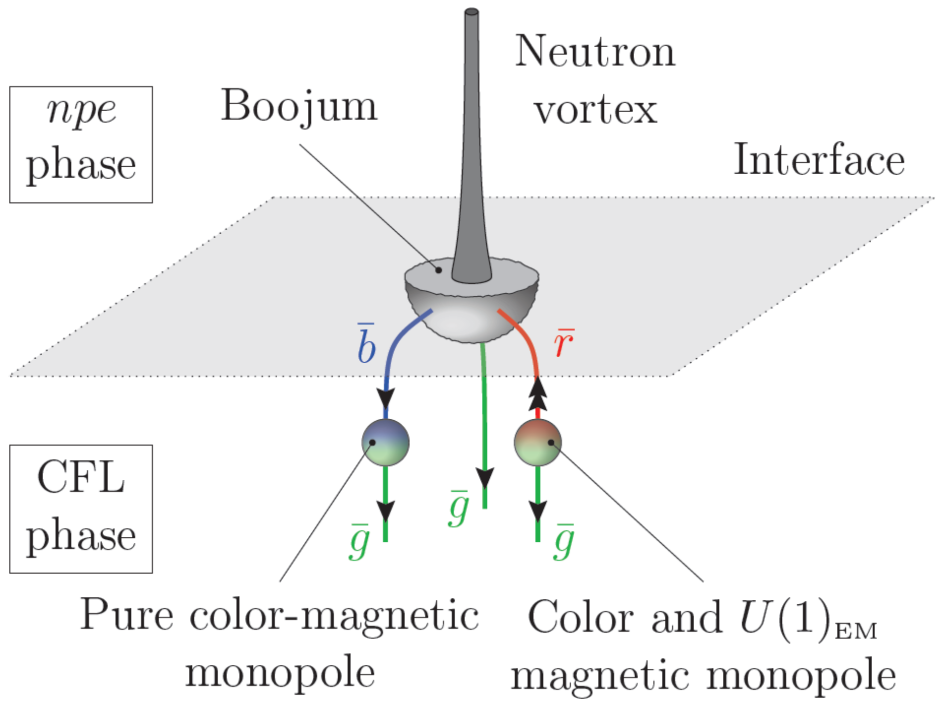
Proton boojum



Dirac monopole

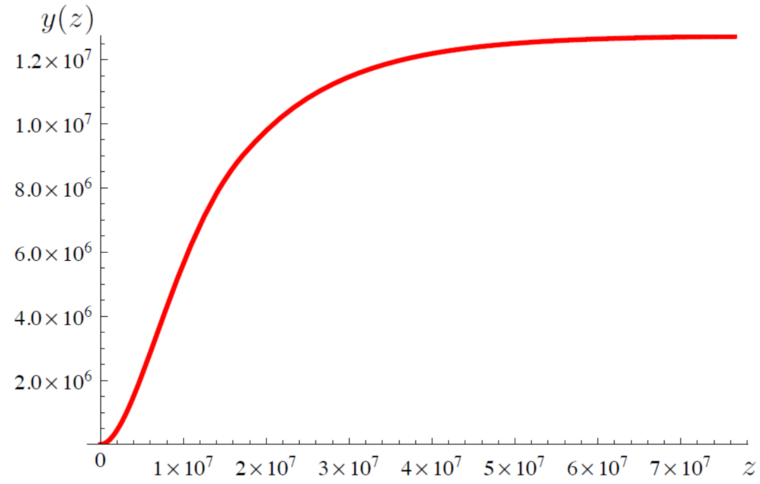
Cipriani,Vinci & MN, arXiv:1208.5704

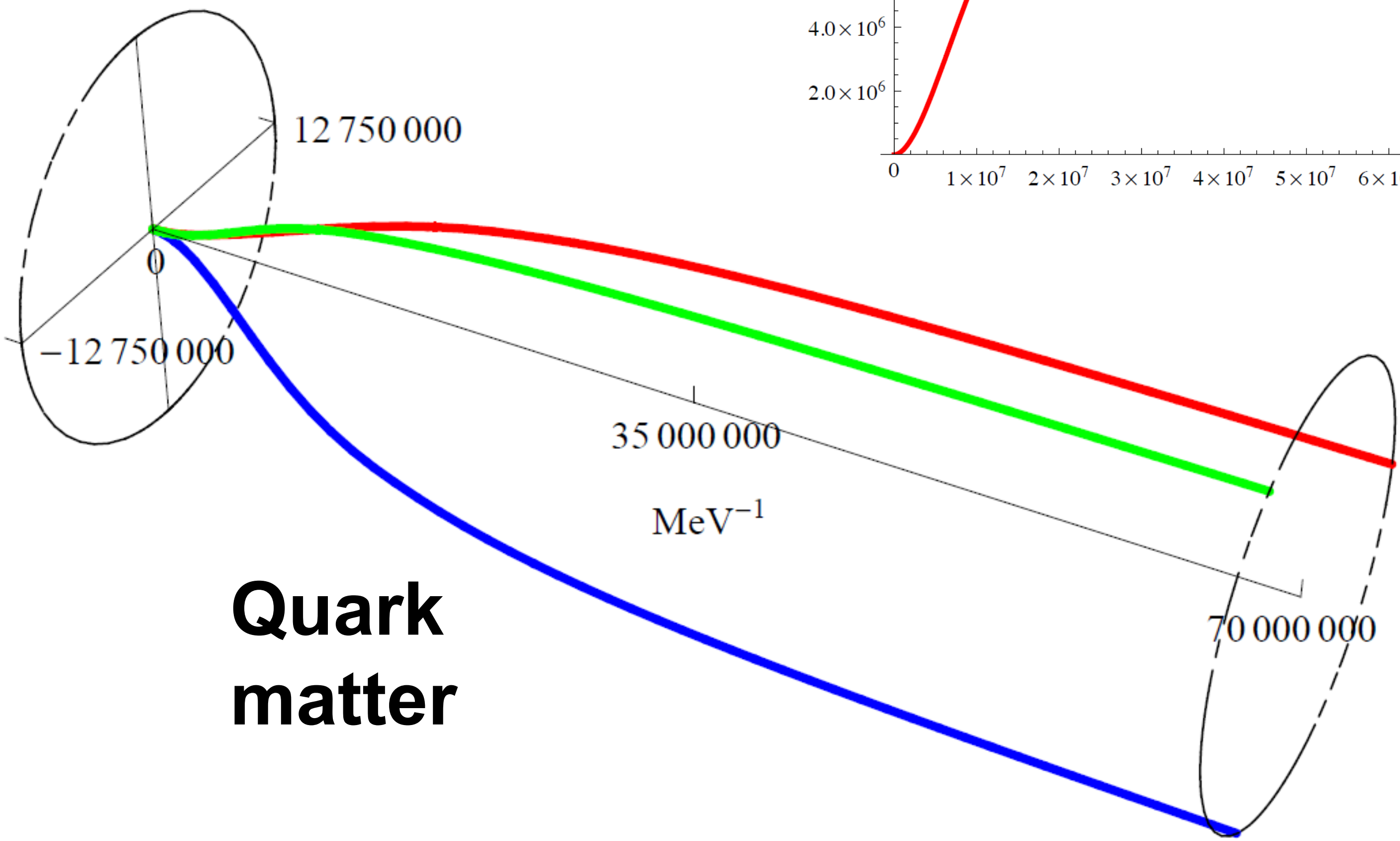
Neutron boojum
= Colorful boojum



Confined color monopoles

Neutron boojum
= Colorful boojum





Cipriani,Vinci & MN, arXiv:1208.5704

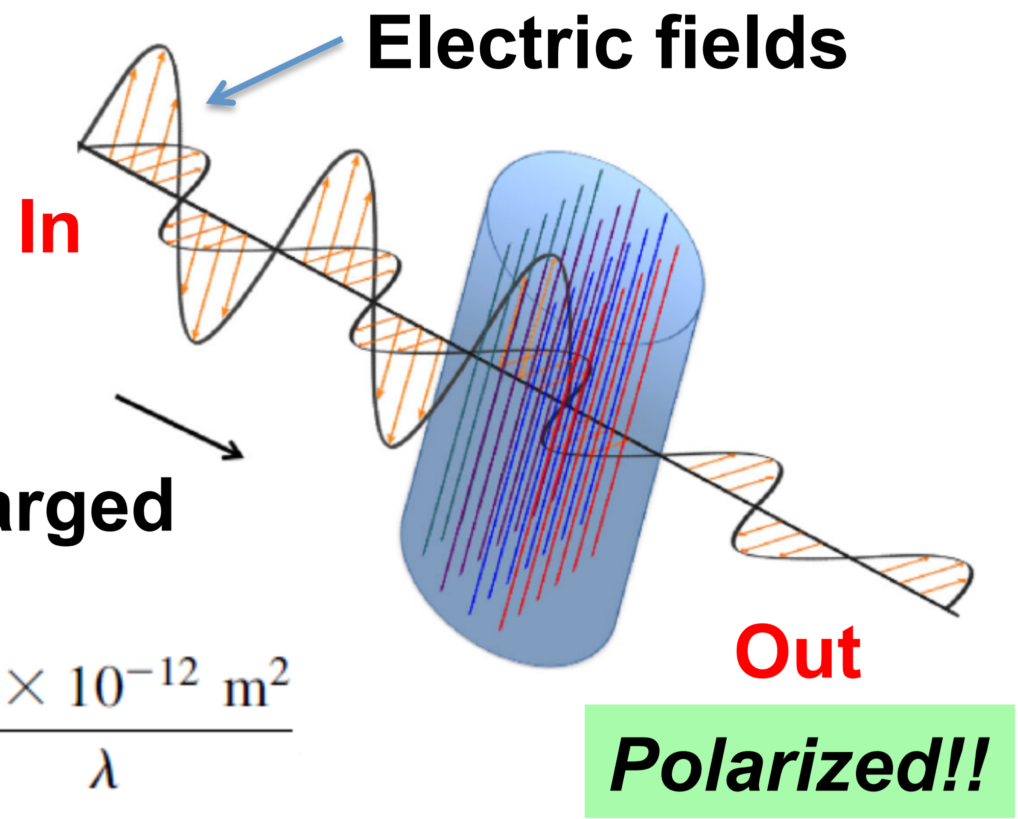
Cosmic polarizer

$$\mathcal{L}_{g\mathbb{C}P^2} = C \sum_{\alpha=0,3} K_\alpha [\mathcal{D}^\alpha \phi^\dagger \mathcal{D}_\alpha \phi + (\phi^\dagger \mathcal{D}^\alpha \phi)(\phi^\dagger \mathcal{D}_\alpha \phi)],$$

$$\mathcal{D}_\alpha \phi = (\partial_\alpha - ie\sqrt{6} A_\alpha T_8)\phi$$

$$\frac{SU(3)_{\mathrm{C+F}}}{SU(2)\times U(1)} = \mathbb{C}P^2$$

modes are electrically charged



$$L = \frac{\ell^2}{288\pi(CK_3\alpha\eta)^2\langle f(\phi)^2\rangle\lambda} \simeq \frac{6.5\times 10^{-12}\ \mathrm{m}^2}{\lambda}$$

$$\lambda \geq 6.5\times 10^{-15}\ \mathrm{m} \equiv \lambda_c$$

Fermions trapped inside a vortex core

$$\Phi_{\alpha i} = \left(\begin{array}{c|cc} \Delta_1(r)e^{i\theta} & 0 & 0 \\ \hline 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{array}\right)$$

$$H = SU(3)_{\mathrm{C+F}}$$

$$\downarrow$$

$$K = [SU(2) \times U(1)]_{\mathrm{C+F}}$$

@ vortex core

Triplet Majorana fermion

protected by SO(3)

$$q = \begin{pmatrix} u_r & d_r & s_r \\ u_g & d_g & s_g \\ u_b & d_b & s_b \end{pmatrix}$$

Yasui, Itakura & MN, Phys.Rev.D81,105003(2010) [arXiv:1001.3730]

Index theorem Fujiwara,Fukui,MN&Yasui ('11)

Non-Abelian statistics Yasui,Itakuta&MN('11), Hirono,Yasui,Itakura&MN('12)

These fermions are important for transportation coeff. of quasi-particles.

Conclusion

If quark matter (CFL phase) is realized in neutron star core, non-Abelian vortices (color flux tubes) *must* be created and constitute a colorful lattice.

Discussion

What is an observational signal?
Confirm/deny quark matter.

1. The existence of vortex lattice implies:
   * Lattice oscillation: *Tkachenko, Kelvin* modes
   * Transportation of particles, anisotropic neutrino emission?
   * EOS more stiff (?): anisotropic pressure?
2. Its rigid rotation yields gravitational wave radiation(?)

Questions, discussions, or seminar: nitta@phys-h.keio.ac.jp

Landau-Ginzburg model from QCD

Iida&Baym('01)
Giannakis&Ren('02)
Iida,Matsuura,
Tachibana&Hatsuda('04)

$$\mathcal{L} = \mathrm{Tr}(K_0 \mathcal{D}_0 \Phi^\dagger \mathcal{D}_0 \Phi - K_3 \mathcal{D}_i \Phi^\dagger \mathcal{D}_i \Phi),$$
$$+ \frac{\varepsilon}{2} F_{0i}^2 - \frac{1}{4\lambda} F_{ij}^2 - V,$$

$$V_{\mathrm{GL}} = \mathrm{Tr}\left[\Phi^\dagger \left\{ \left(\alpha + \frac{2\epsilon}{3} \right) \mathbf{1}_3 + \epsilon X_3 \right\} \Phi \right]$$
$$+ \beta_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 + \beta_2 \mathrm{Tr}[(\Phi^\dagger \Phi)^2],$$

$$\alpha = 4N(\mu) \log \frac{T}{T_c}, \quad \beta_{1,2} = \frac{7\zeta(3)}{8(\pi T_c)^2} N(\mu) \equiv \beta,$$

$$K_3 = \frac{1}{3} K_0 = \frac{7\zeta(3)}{12(\pi T_c)^2} N(\mu), \quad \epsilon = N(\mu) \frac{m_s^2}{\mu^2} \log \frac{\mu}{T_c},$$

Effective action on a vortex

Eto,Nakano&MN('09)

$$\frac{H}{K} = \frac{SU(3)_{\mathrm{C+F}}}{SU(2)\times U(1)} = \mathbf{C}P^2$$

1+1 dim. world-sheet theory

$$\mathcal{L}_{\mathbb{C}P^2} = C \sum_{\alpha=0,3} K_\alpha[\partial^\alpha \phi^\dagger \partial_\alpha \phi + (\phi^\dagger \partial^\alpha \phi)(\phi^\dagger \partial_\alpha \phi)],$$

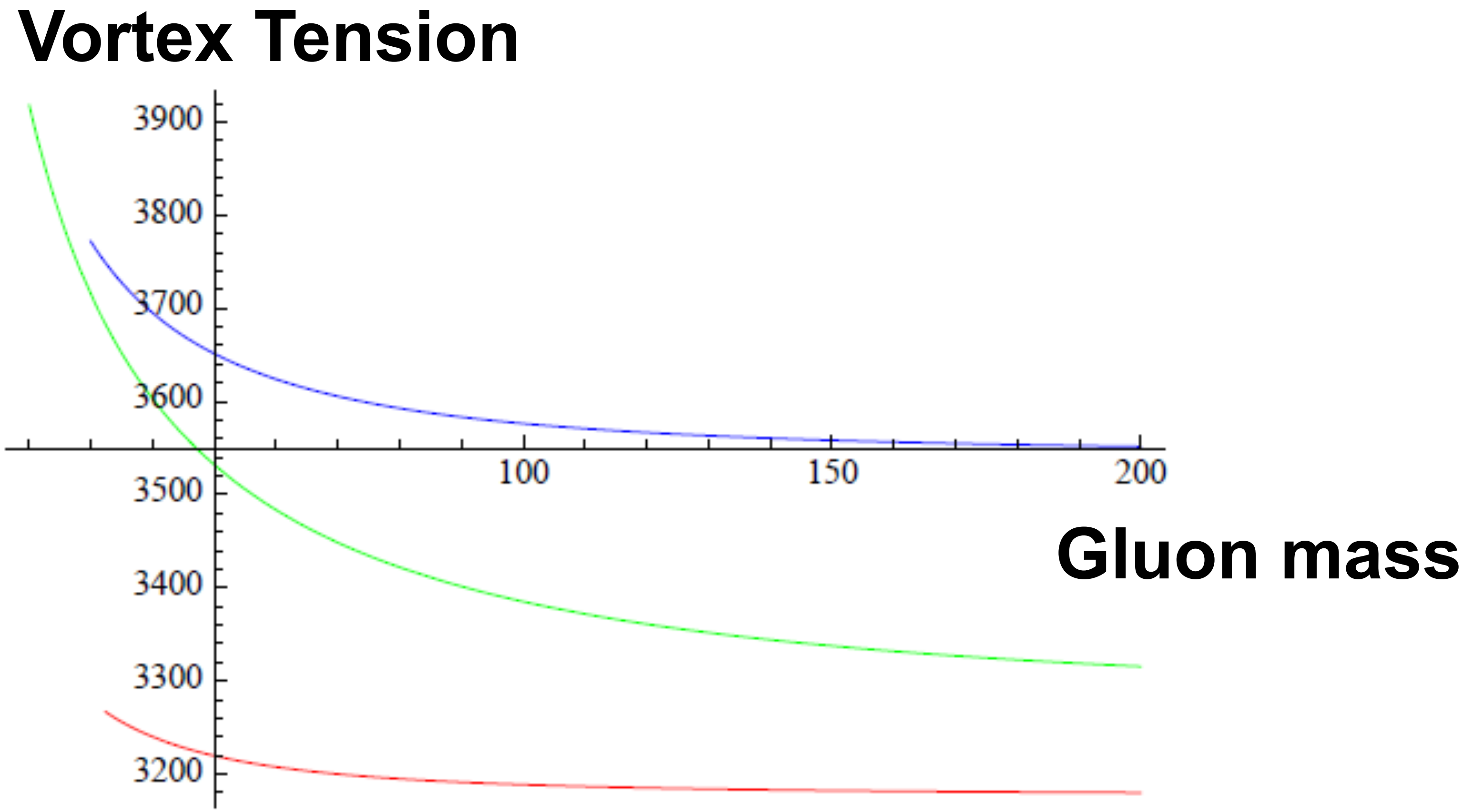
$$\mathcal{L}_{g\mathbb{C}P^2} = C \sum_{\alpha=0,3} K_\alpha[\mathcal{D}^\alpha \phi^\dagger \mathcal{D}_\alpha \phi + (\phi^\dagger \mathcal{D}^\alpha \phi)(\phi^\dagger \mathcal{D}_\alpha \phi)],$$

$$\mathcal{D}_\alpha \phi = (\partial_\alpha - ie\sqrt{6}A_\alpha T_8)\phi$$

Nambu-Goldstone modes localized around the vortex

= Gapless modes propagating along the vortex line

Dependence of vortex tension of gluon mass

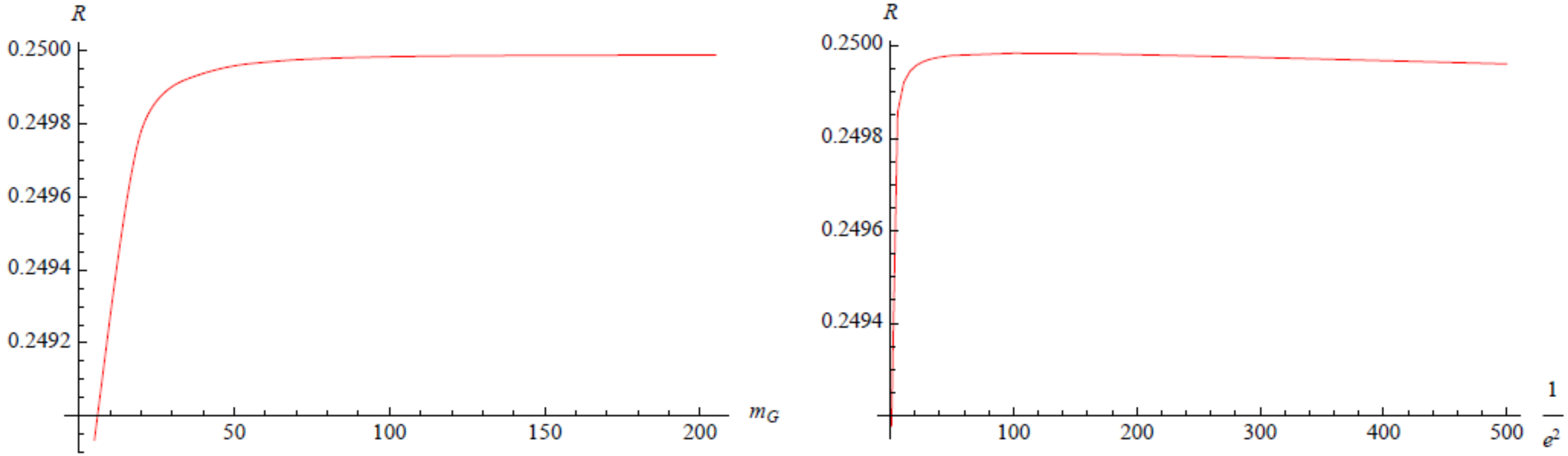


Decay probability of metastable CP^1 vortices

Vinci,Cipriani&MN,
Phys.Rev. D86:085018,2012[arXiv:1206.3535]

$$P \sim e^{-\beta R}, \quad R \equiv \frac{\mathcal{T}_{\text{pure}} - \mathcal{T}_{\mathbb{C}P^1}}{\mathcal{T}_{\text{pure}} - \mathcal{T}_{\text{BDM}}} \qquad \beta \sim K_1^2/\lambda_1 \sim \mu^2/T_c^2 \sim 2500$$

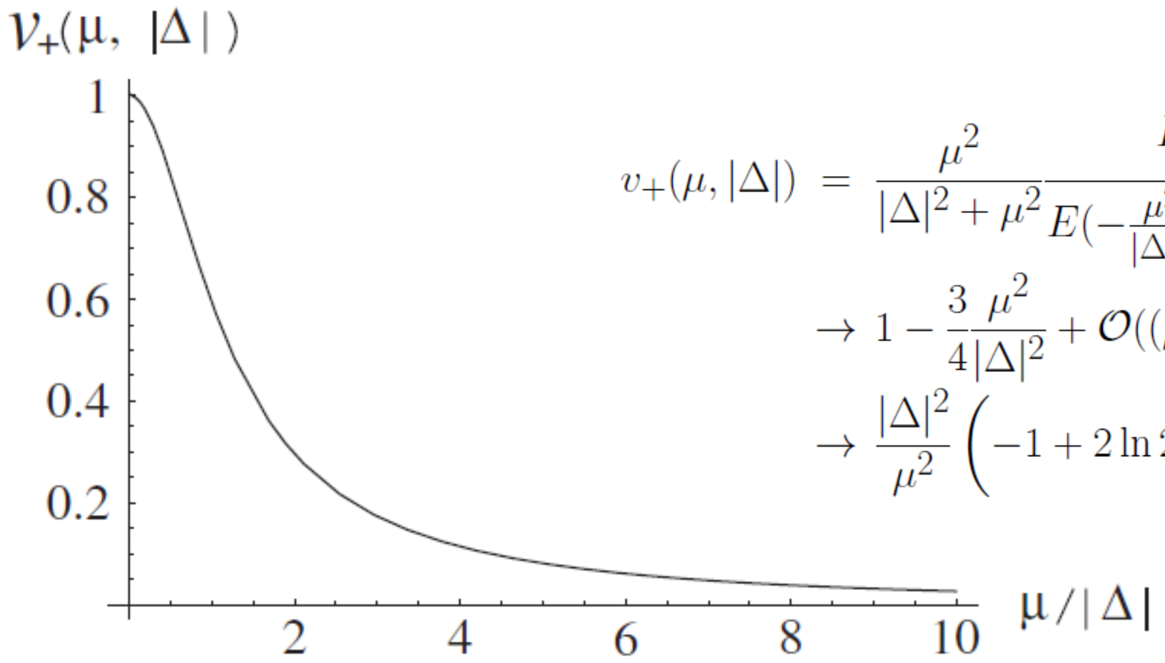
very small probability



Fermions trapped inside a vortex core

$$\Psi^{(1)} = C_1 \begin{pmatrix} \varphi(r,\theta) \\ \eta(r,\theta) \end{pmatrix}, \quad \Psi^{(2)} = C_2 \begin{pmatrix} \varphi(r,\theta) \\ -\eta(r,\theta) \end{pmatrix}, \quad \Psi^{(3)} = C_3 \begin{pmatrix} \varphi(r,\theta) \\ \eta(r,\theta) \end{pmatrix}$$

$$\varphi = \mathrm{e}^{-\int_0^r |\Delta_1(r')| dr'} \begin{pmatrix} J_0(\mu r) \\ i J_1(\mu r)\, \mathrm{e}^{i\theta} \end{pmatrix} \quad \eta = \mathrm{e}^{-\int_0^r |\Delta_1(r')| dr'} \begin{pmatrix} -J_1(\mu r)\, \mathrm{e}^{-i\theta} \\ i J_0(\mu r) \end{pmatrix}$$



$$\begin{aligned} v_+(\mu, |\Delta|) &= \frac{\mu^2}{|\Delta|^2 + \mu^2} \frac{E(-\frac{\mu^2}{|\Delta|^2})}{E(-\frac{\mu^2}{|\Delta|^2}) - K(-\frac{\mu^2}{|\Delta|^2})} - 1\,, \\ &\to 1 - \frac{3}{4}\frac{\mu^2}{|\Delta|^2} + \mathcal{O}((\mu/|\Delta|)^4), \ \textbf{for small } \mu/\Delta \\ &\to \frac{|\Delta|^2}{\mu^2}\left(-1 + 2\ln 2 + \ln\frac{\mu}{|\Delta|}\right) + \mathcal{O}((|\Delta|/\mu)^4) \ \textbf{for large } \mu/\Delta \end{aligned}$$

Index therem for fermion zero modes

T.Fujiwara, T.Fukui, MN, S.Yasui,
PRD84,076002 (2011) [arXiv:1105.2115]

$$q = \begin{pmatrix} u_r & d_r & s_r \\ u_g & d_g & s_g \\ u_b & d_b & s_b \end{pmatrix} \longleftarrow \Delta = \left(\begin{array}{cc|c} \Delta_Q & & \\ & \Delta_Q & \\ \hline & & \Delta_q \end{array}\right)$$

index

q	**triplet**
Q	**doublet**
$2Q$	**singlet**

Majorana zero modes

$\boxed{Q=0,\ q=1}$

1	**triplet**
0	**doublet**
0	**singlet**

Exchange statistics of NA Majorana fermion

$$\tau_k^{\mathbf{S},\mathcal{P}} = \sigma_k^{\mathbf{S}} \otimes h_k^{\mathcal{P}} \qquad \mathbf{S} = \mathbf{1},\, \mathbf{3},\, \mathbf{5},\, \mathcal{P} = \mathcal{E},\, \mathcal{O}$$

Ivanov's matrices

$$h_2^{\mathcal{E}} = h_2^{\mathcal{O}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \quad h_1^{\mathcal{E}} = h_1^{\mathcal{O}} = \begin{pmatrix} e^{i\frac{\pi}{4}} & 0 \\ 0 & e^{-i\frac{\pi}{4}} \end{pmatrix}$$

$$h_3^{\mathcal{E}} = h_3^{\mathcal{O}\dagger} = h_1^{\mathcal{E}}$$

NEW

$$\sigma_1^{\mathbf{1}} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_2^{\mathbf{1}} = \frac{1}{2} \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}, \quad \sigma_3^{\mathbf{1}} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma_1^{\mathbf{3}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \sigma_2^{\mathbf{3}} = \frac{1}{2} \begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix}, \quad \sigma_3^{\mathbf{3}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\sigma_1^{\mathbf{5}} = \sigma_2^{\mathbf{5}} = \sigma_3^{\mathbf{5}} = 1$$

Coxeter group

(n=4,Type A_3)

***N*-simplex and Coxeter group are hidden in the Hilbert space !!**