CFT approach to multi-channel SU(N) Kondo effect

Sho Ozaki (Keio Univ.)

In collaboration with

Taro Kimura (Keio Univ.)

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T. Kimura and S.O., arXiv: 1611.07284 to be published in JPSJ

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T. Kimura and S. O., in preparation

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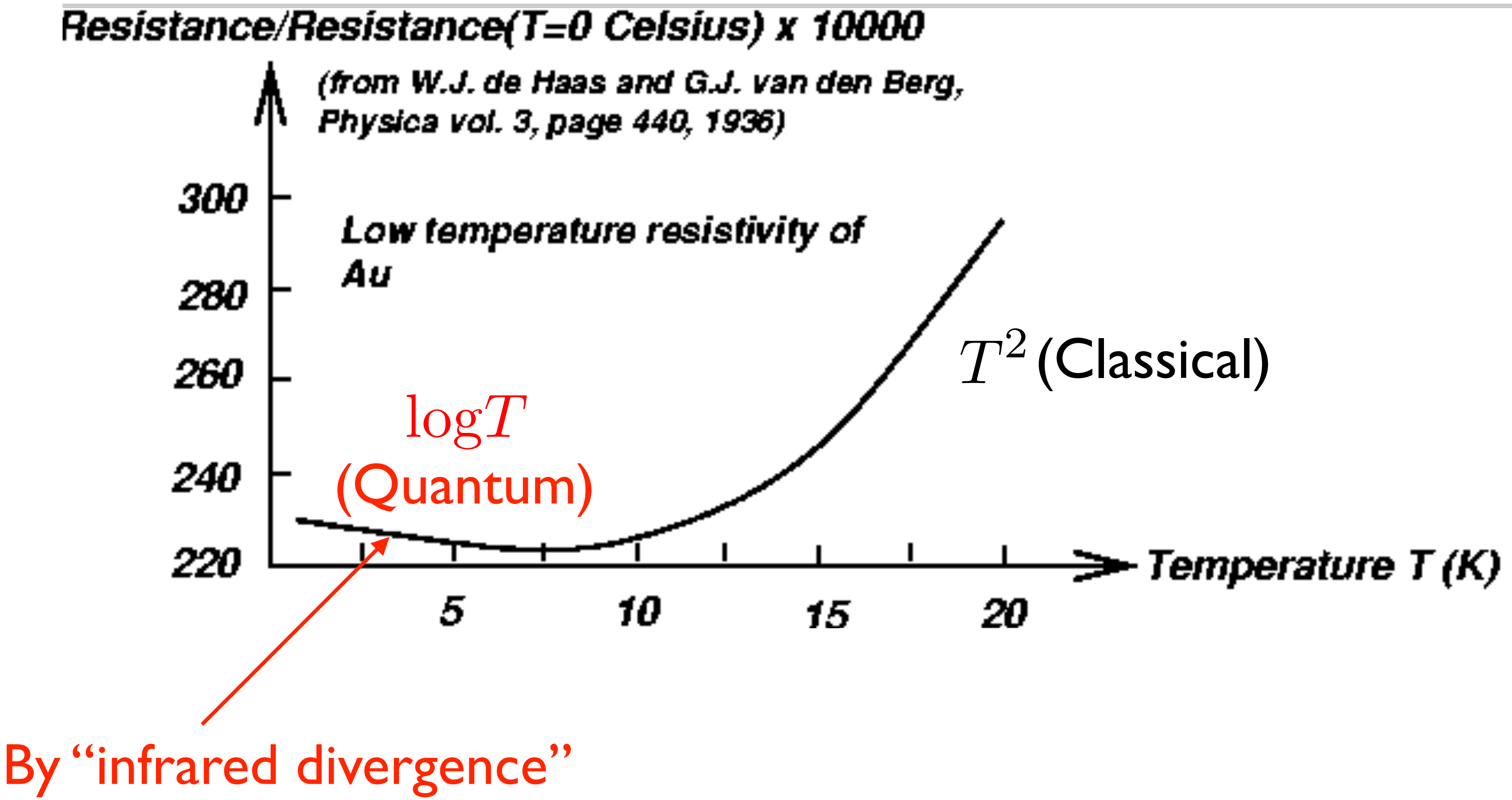
Kondo effect



Kondo effect is firstly observed in experiment as an enhancement of electrical resistivity of impure metals.

Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

Resistance Minimum in Dilute Magnetic Alloys

Jun KONDO

Jun Kondo
(1930-)

J. Kondo has explained the phenomenon based on the second order perturbation of interaction between conduction electron and impurity.

Conditions for the appearance of Kondo effect

0) Localized (Heavy) impurity

i) Fermi surface

ii) Quantum fluctuation (loop effect)

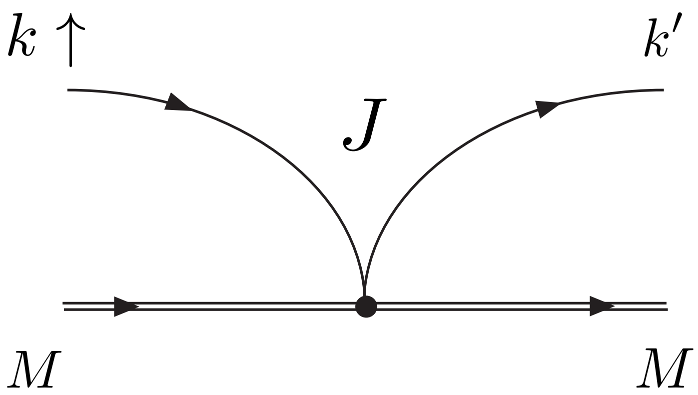
iii) Non-Abelian property of interaction
(spin-flip int.)

- s-d model (Kondo model)

$$H_{sd} = \sum_{k,\sigma} \epsilon_k c^\dagger_{k,\sigma} c_{k,\sigma} - J \sum_{k,k'} \vec{s}_{k'k} \cdot \vec{S} \qquad (J < 0)$$

- Scattering amplitude $T(k \uparrow \to k' \uparrow)$

<u>Born term</u>



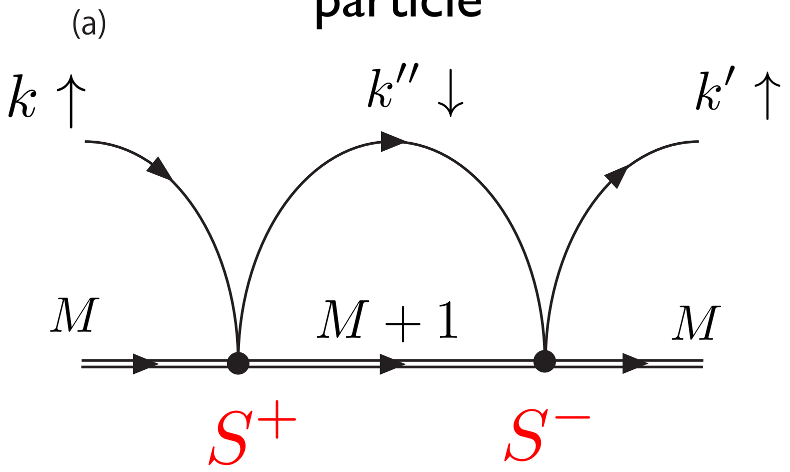
$$T^{(1)} = \langle k' \uparrow | - J \vec{s}_{k'k} \cdot \vec{S} | k \uparrow \rangle = -J S_z$$

$$|k \uparrow\rangle = c^\dagger_{k\uparrow} |FS\rangle$$

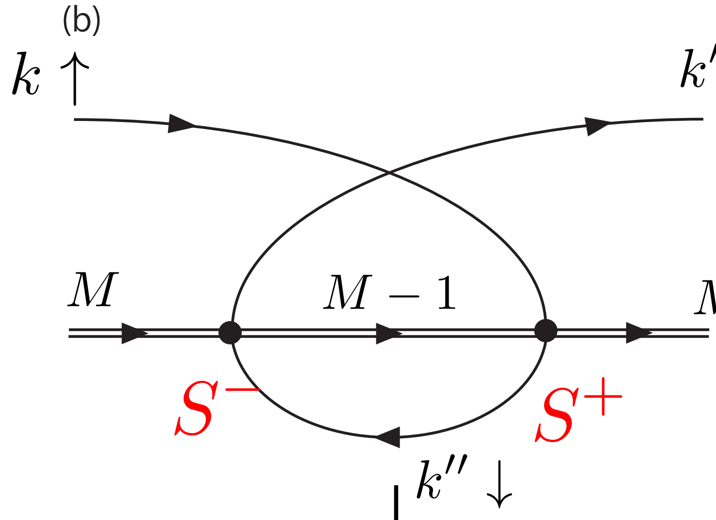
Second order perturbation theory

- only spin flip processes -

particle



hole



$$T_a^{(2)} = \langle k' \uparrow | \sum_{k''} \frac{\left(\frac{J}{2} S^- c^\dagger_{k'\uparrow} c_{k''\downarrow}\right)\left(\frac{J}{2} S^+ c^\dagger_{k''\downarrow} c_{k\uparrow}\right)}{\epsilon - \epsilon_{k''} + i\eta} | k \uparrow \rangle$$

$$T_b^{(2)} = \langle k' \uparrow | \sum_{k''} \frac{\left(\frac{J}{2} S^+ c^\dagger_{k''\downarrow} c_{k\uparrow}\right)\left(\frac{J}{2} S^- c^\dagger_{k'\uparrow} c_{k''\downarrow}\right)}{\epsilon_{k''} - \epsilon - i\eta} | k \uparrow \rangle$$

$$\epsilon = \epsilon_{k'} = \epsilon_k$$

$$\longrightarrow \quad T^{(2)} = \frac{J^2}{4} \left[S^+, S^- \right] \rho_F \log \left(\frac{D}{T} \right)$$

ρ_F : density of state on the Fermi surface, D : Bandwidth

Total amplitude

$$T = T^{(1)} + T^{(2)} + \cdots$$

$$\simeq T^{(1)} \left(1 + \frac{J}{2} \rho_F \log \left(\frac{T}{D} \right) \right)$$

At low temperature (IR) regions: $T \ll D \quad (J < 0)$

$$1 \simeq \frac{J}{2} \rho_F \log \left(\frac{T}{D} \right) \qquad \text{@ } T \simeq T_{\rm K}$$

The perturbative expansion breaks down.

→ We need a some non-perturbative method to analyze the Kondo effect at IR regions.

Non-perturbative approach for Kondo effect

- Numerical renormalization group [Wilson]

- Bethe ansatz [Andrei] [Wiegmann] ....

- 1+1 dim. conformal field theory (CFT) approach

  [Affleck-Ludwig]

  k(multi)-channel SU(2) Kondo

Non-perturbative approach for Kondo effect

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  [Affleck-Ludwig]

  k(multi)-channel SU(2) Kondo

  k-channel SU(N) Kondo with k >= N

Non-perturbative approach for Kondo effect

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  [Affleck-Ludwig]

  k(multi)-channel SU(2) Kondo

  k-channel SU(N) Kondo with k >= N

  k-channel SU(N) Kondo including N > k >1

  T. Kimura and S. O, arXiv:1611.07284

Boundary CFT

Assuming that the impurity is sufficiently dilute, and the interaction is a contact-type one, the s-wave approx. is valid, which leads to the following one-dim. effective theory:

$$\mathcal{H} = i\psi^\dagger(x)\frac{\partial\psi(x)}{\partial x} + \lambda_{\mathrm{K}}\psi^\dagger(x)t^a\psi(x)S^a\delta(x)$$

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Currents

$J^a(x) =: \psi^\dagger(x)t^a\psi(x):$ color

$J^A(x) =: \psi^\dagger(x)T^A\psi(x):$ flavor

$J(x) =: \psi^\dagger(x)\psi(x):$ charge

Normal order product

$$:OO(x):= \lim_{\epsilon\to 0}\{O(x)O(x+\epsilon) - \langle O(x)O(x+\epsilon)\rangle\}$$

Boundary CFT

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Normal order product

$$:OO(x):= \lim_{\epsilon\to 0}\{O(x)O(x+\epsilon) - \langle O(x)O(x+\epsilon)\rangle\}$$

The Sugawara form of the Hamiltonian density

$$\mathcal{H} = \frac{1}{N+k}J^aJ^a + \frac{1}{k+N}J^AJ^A + \frac{1}{2kN}JJ + \lambda_{\mathrm{K}}J^aS^a\delta(x)$$

Boundary CFT

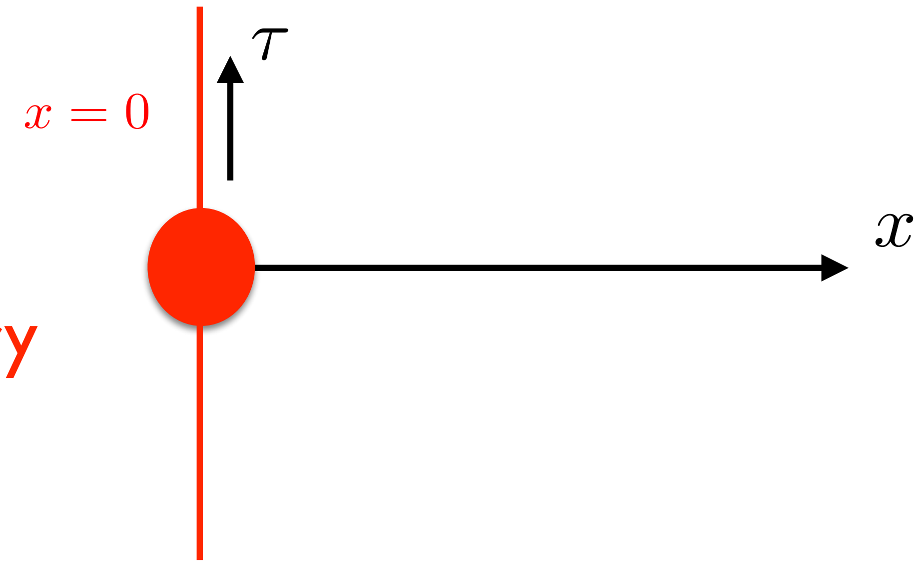
$$\mathcal{H} = \frac{1}{N+k} J^a J^a + \frac{1}{k+N} J^A J^A + \frac{1}{2kN} JJ + \lambda_{\mathrm{K}} J^a S^a \delta(x)$$

$$\mathcal{J}^a = J^a + \frac{\lambda_{\mathrm{K}}}{2}(N+k) S^a \delta(x)$$

$$\mathcal{H} = \frac{1}{N+k} \mathcal{J}^a \mathcal{J}^a + \frac{1}{k+N} J^A J^A + \frac{1}{2kN} JJ$$

Impurity effect

→ Boundary of the theory



g-factor and Impurity entropy

Partition function

$$Z(L,\beta) \xrightarrow{L\to\infty} \underbrace{g_{R_{\mathrm{imp}}}}_{\text{boundary (impurity)}} \times \underbrace{\mathrm{e}^{\frac{\pi c L}{6\beta}}}_{\text{bulk}},$$

universal quantities

$c = Nk$: central charge

$g_{R_{\mathrm{imp}}} = \frac{S_{R_{\mathrm{imp}}0}}{S_{00}}$: g-factor

(S_{mn} : modular S-matrix)

Free energy

$$F = -\frac{1}{\beta}\log Z$$

Entropy

$$S(T) = -\frac{\partial F}{\partial T}$$

Impurity entropy at IR fixed point (T=0)

$$S_{\mathrm{imp}} = S(T) - S_{\mathrm{bulk}}(T)|_{T=0} = \log\left(g_{R_{\mathrm{imp}}}\right)$$

g-factor and Impurity entropy

g-factor provides the impurity entropy:

$$S_{\rm imp} = \log(g_{R_{\rm imp}}) \,,$$

$R_{\rm imp}$: fundamental representation

ex) SU(2), k=1 (standard Kondo effect)

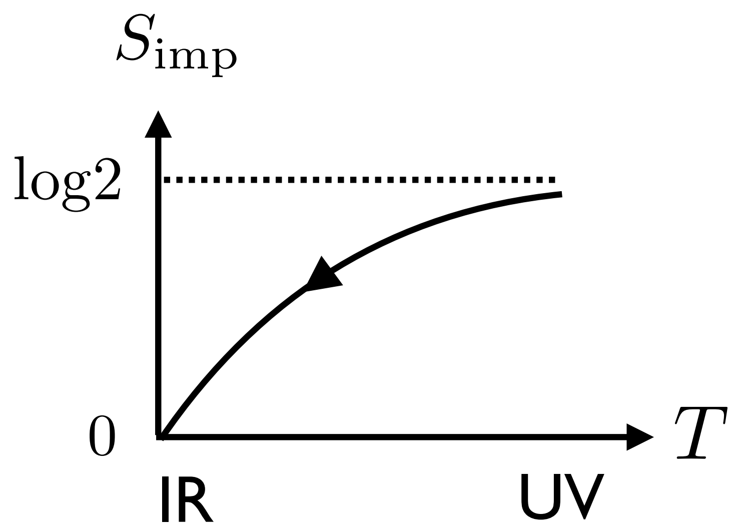
$$S_{\rm imp} = \log(2s+1)$$

In UV, $s = 1/2$

$$S_{\rm imp} \to \log 2$$

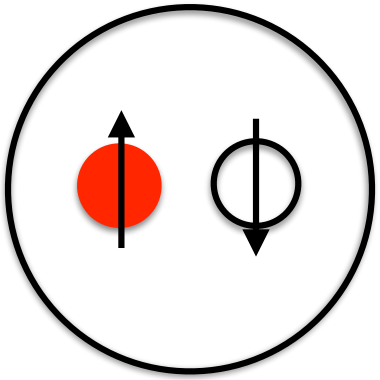
In IR, $s \to 0$ (Kondo singlet)

$$S_{\rm imp} \to 0$$



Overscreening Kondo effect in multi-channel SU(2) Kondo model

Standard Kondo effect

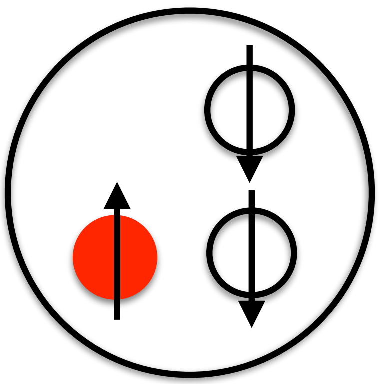


$k = 1$ (single channel)

→ Fermi liquid at IR fixed point

→ g : integer

Overscreening Kondo effect



$k = 2$ (two channel)

→ non-Fermi liquid at IR fixed point

→ g : non-integer

g-factor in SU(N) Kondo effect @ IR fixed point (zero temperature)

N	$k=1$	$k=2$	$k=3$	$k=4$	$k=\infty$
2	1	1.4142...	1.6180...	1.7320...	2
3	1	1.6180...	2	2.2469...	3
4	1	1.7320...	2.2469...	2.6131...	4
∞	1	2	3	4	∞

- For k=1, g-factor is always unity, and thus the system becomes the Fermi liquid.
- In general with k > 1, the g-factor becomes non-integer, which indicates that the system is described by non-Fermi liquid.

1+1 dim. (boundary) CFT approach

Correlation functions are exactly determined by

- Conformal symmetry in 1+1 dim.
- Kac-Moody algebra

$$[\mathcal{J}^a(x), \mathcal{J}^b(y)] = if^{abc}\mathcal{J}^c(x)\delta(x-y) + \frac{i}{2\pi}\left(\frac{k}{2}\right)\delta^{ab}\frac{\partial}{\partial x}\delta(x-y)$$

$$\langle O_1(x)O_2(y)\rangle = \frac{C_{1,2}}{|x-y|^{\Delta}}$$

Δ determined by conformal symmetry

$C_{1,2}$ determined by KM algebra

From the correlation functions, one can evaluate T-dep. of several observables of k-channel SU(N) Kondo effect in IR regions.

Specific heat, susceptibility and the Wilson ratio

- Bulk contributions to C & χ

$$C_{\text{bulk}} = \frac{\pi}{3} N k T$$

$$\chi_{\text{bulk}} = \frac{k}{2\pi}$$

These are well known properties of free Nk (bulk) fermions in 1+1 dim.

▶ Impurity contributions to C & χ

i) k=1 & arbitrary N [Affleck 1990]

Leading irrelevant operator

$$\delta\mathcal{H}_1 = \lambda_1 \mathcal{J}^a \mathcal{J}^a(x)\delta(x)$$

$$\lambda_1 \sim 1/T_{\mathrm{K}}$$

From the perturbation w.r.t. $\delta\mathcal{H}_1$

$$C_{\mathrm{imp}} = -\lambda_1 \frac{k(N^2-1)}{3}\pi^2 T$$

$$\chi_{\mathrm{imp}} = -\lambda_1 \frac{k(N+k)}{2}$$

with $k=1$

⟶ Typical Fermi liquid behaviors

ii) k > 1, Overscreening case

Leading irrelevant operator

$$\delta\mathcal{H} = \lambda\mathcal{J}^a_{-1}\phi^a(x)\delta(x)$$

ϕ^a :adjoint operator, appearing when k >1
scaling dimension is $\Delta = \dfrac{N}{N+k}$.

$\mathcal{J}^a_n$:Fourier mode of $\mathcal{J}^a(x)$

$\lambda \sim 1/T_{\mathrm{K}}^{\Delta}$ $\quad\quad \Delta < 1$

From the perturbation with respect to the leading irrelevant operator, we can evaluate $C_{\mathrm{imp}}, \chi_{\mathrm{imp}}$.

Observables

$$Z = \mathrm{e}^{-\beta F(T,\lambda,h)} \qquad\qquad L \to \infty$$

$$= \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \, \mathrm{e}^{-\int d^2x\, \mathcal{H}} \exp\left\{ + \int_{-\beta/2}^{\beta/2} d\tau \left[\lambda \mathcal{J}^a_{-1} \phi^a(\tau, 0) + \frac{h}{2\pi} \int_{-L}^{L} dx \; \mathcal{J}^3(\tau, x) \right] \right\}$$

$$= Z_0 \left\langle \exp\left\{ + \int_{-\beta/2}^{\beta/2} d\tau \left[\lambda \mathcal{J}^a_{-1} \phi^a(\tau, 0) + \frac{h}{2\pi} \int_{-L}^{L} dx \; \mathcal{J}^3(\tau, x) \right] \right\} \right\rangle$$

Free energy can be divided in to bulk and impurity parts

$$F = L f_{\mathrm{bulk}} + f_{\mathrm{imp}}$$

which is expressed in terms of the correlation functions of the leading irrelevant operators.

$$\longrightarrow \quad C = -T \frac{\partial^2 F}{\partial T^2} \, , \;\; \chi = - \left. \frac{\partial^2 F}{\partial h^2} \right|_{h=0}$$

Specific heat of k-channel SU(N) Kondo effect

$$C_{\rm imp} = \begin{cases} \dfrac{\lambda^2}{2}\pi^{1+2\Delta}(2\Delta)^2(N^2-1)(N+k/2)\left(\dfrac{1-2\Delta}{2}\right)\dfrac{\Gamma(1/2-\Delta)\Gamma(1/2)}{\Gamma(1-\Delta)}T^{2\Delta} & (k>N) \\ \lambda^2\pi^{1+2\Delta}(N^2-1)(N+k/2)(2\Delta)^2\, T\log\left(\dfrac{T_{\rm K}}{T}\right) & (k=N) \\ -\lambda_1\dfrac{k}{3}(N^2-1)\pi^2 T + 2\lambda^2\pi^2(N^2-1)(N+k/2)\dfrac{2\Delta}{1+2\Delta}\left(\dfrac{\beta_{\rm K}^{-2\Delta+1}}{2\Delta-1}\right)T & (N>k>1) \end{cases}$$

$$\delta\mathcal{H}_1 = \lambda_1 \mathcal{J}^a\mathcal{J}^a(x)\delta(x)$$

$$\delta\mathcal{H} = \lambda \mathcal{J}^a_{-1}\phi^a(x)\delta(x)$$

Specific heat of k-channel SU(N) Kondo effect

$$C_{\mathrm{imp}} = \begin{cases} \frac{\lambda^2}{2}\pi^{1+2\Delta}(2\Delta)^2(N^2-1)(N+k/2)\left(\frac{1-2\Delta}{2}\right)\frac{\Gamma(1/2-\Delta)\Gamma(1/2)}{\Gamma(1-\Delta)}\underline{T^{2\Delta}} & (k>N) \\ \lambda^2\pi^{1+2\Delta}(N^2-1)(N+k/2)(2\Delta)^2\,\underline{T\log\left(\frac{T_{\mathrm{K}}}{T}\right)} & (k=N) \\ -\lambda_1\frac{k}{3}(N^2-1)\pi^2\underline{T}+2\lambda^2\pi^2(N^2-1)(N+k/2)\frac{2\Delta}{1+2\Delta}\left(\frac{\beta_{\mathrm{K}}^{-2\Delta+1}}{2\Delta-1}\right)\underline{T} & (N>k>1) \end{cases}$$

Low T scaling

$$C_{\mathrm{imp}} \propto \begin{cases} T^{2\Delta} & (k>N) & \text{Non-Fermi} \\ T\log(T_{\mathrm{K}}/T) & (k=N) & \text{Non-Fermi} \\ T & (N>k>1) & \text{Fermi} \end{cases}$$

For N > k >1, although the g-factor (at IR fixed point) exhibits non-Fermi liquid signature, T-dep. of C_{imp} shows Fermi liquid behavior. ⟶ Fermi/non-Fermi mixing [T. Kimura and S. O, arXiv:1611.07284]

Susceptibility of k-channel SU(N) Kondo effect

$$\chi_{\rm imp} = \begin{cases} \dfrac{\lambda^2}{2}\pi^{2\Delta-1}(N+k/2)^2(1-2\Delta)\dfrac{\Gamma(1/2-\Delta)\Gamma(1/2)}{\Gamma(1-\Delta)}\,T^{2\Delta-1} & (k>N) \\ 2\lambda^2(N+k/2)^2\,\log\left(\dfrac{T_{\rm K}}{T}\right) & (k=N) \\ -\lambda_1\dfrac{k(N+k)}{2}+2\lambda^2(N+k/2)^2\left(\dfrac{\beta_{\rm K}^{-2\Delta+1}}{2\Delta-1}\right) & (N>k>1) \end{cases}$$

Low T scaling

$$\chi_{\rm imp} = \begin{cases} T^{2\Delta-1} & (2k>N) \quad \text{Non-Fermi} \\ \log(T_{\rm K}/T) & (2k=N) \quad \text{Non-Fermi} \\ \text{const.} & (N>k>1) \quad \text{Fermi} \end{cases}$$

The Wilson ratio of QCD Kondo effect

$$R_{\mathrm{W}} = \left(\frac{\chi_{\mathrm{imp}}}{C_{\mathrm{imp}}}\right) \Bigg/ \left(\frac{\chi_{\mathrm{bulk}}}{C_{\mathrm{bulk}}}\right)$$

$$= \frac{(N+k/2)(N+k)^2}{3N(N^2-1)} \qquad (k \geq N)$$

Unknown parameters are canceled, and thus the Wilson ratio is universal.

$$R_{\mathrm{W}} = \frac{(N+k/2)(N+k/3)}{N^2-1} \frac{\gamma - \dfrac{k(N+k)}{(N+k/2)^2}}{\gamma - \dfrac{k(N+k/3)}{N(N+k/2)}} \quad (N > k > 1)$$

with $\gamma = 4\dfrac{\lambda^2}{\lambda_1} T_{\mathrm{K}}^{2\Delta-1}$

For N >= k, the Wilson ratio is no longer universal, which depends on the detail of the system, such as $\lambda,\ T_{\mathrm{K}}$

IR behaviors of k-channel SU(N) Kondo effect

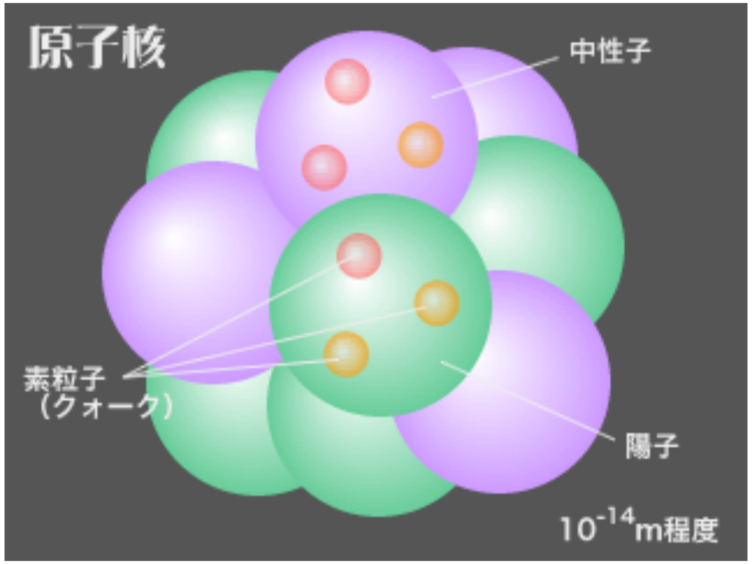
T. Kimura and S. O, arXiv:1611.07284

	k >= N	N > k > 1
g-factor (IR fixed point)	non-Fermi	non-Fermi
Low T scaling	non-Fermi	Fermi
Wilson ratio	universal	non-universal

Fermi/non-Fermi mixing

Application to high energy physics: QCD Kondo effect

Strong interaction



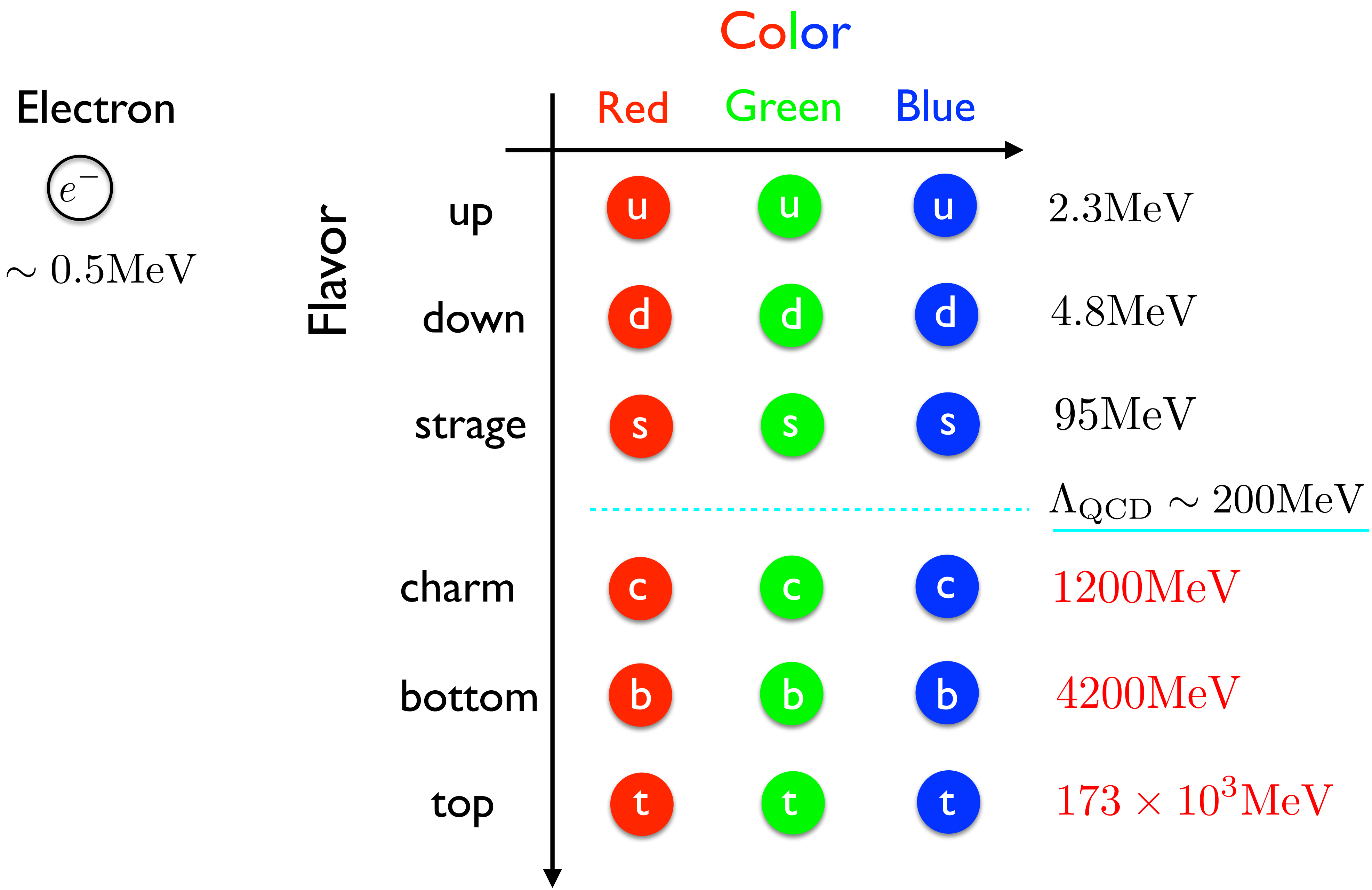
QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + \bar{q} \left(i\gamma_\mu D^\mu - M_q \right) q$$

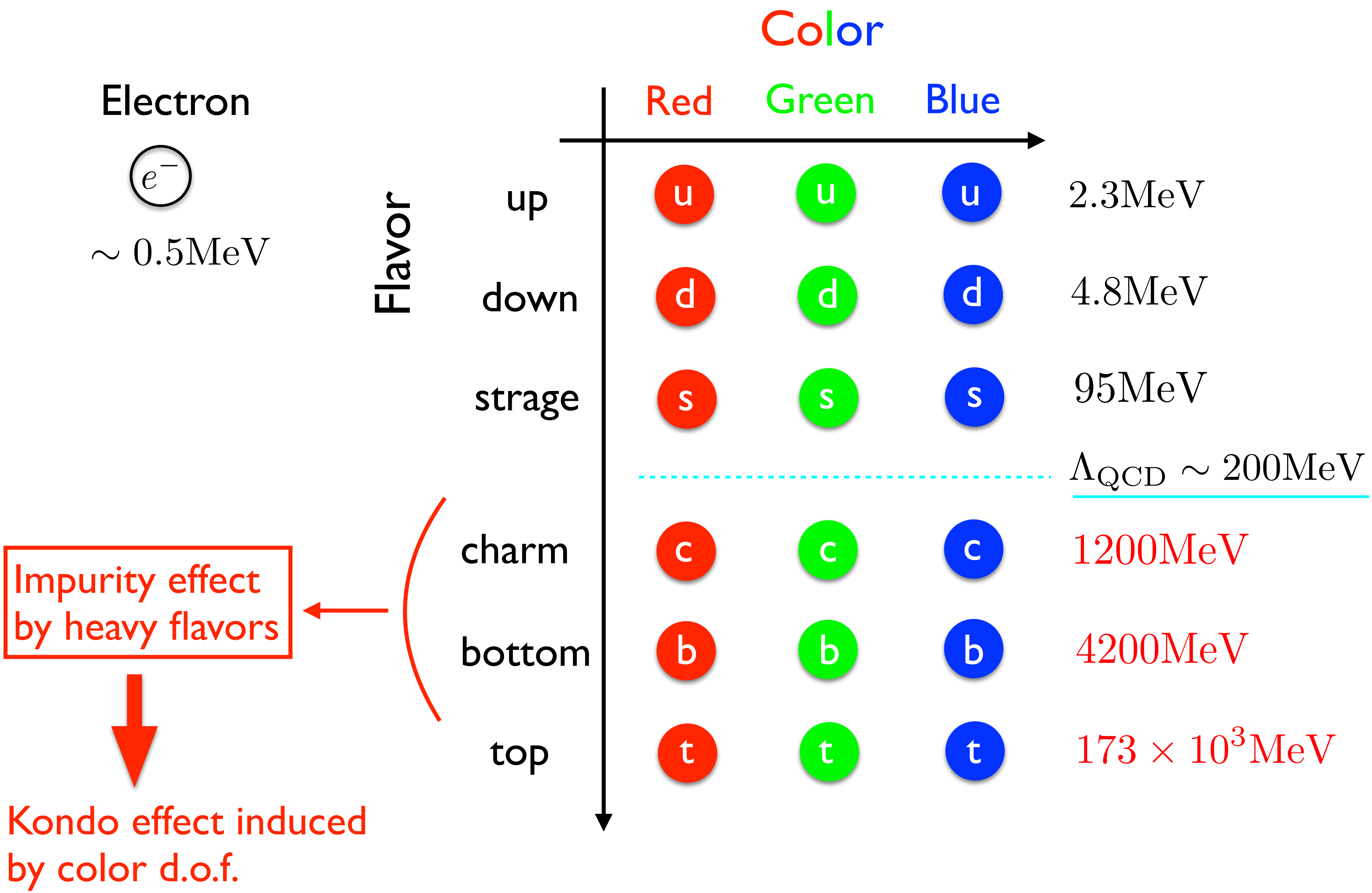
$$D_\mu = \partial_\mu - igA^a_\mu t^a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc} A^b_\mu A^c_\nu$$

Quarks


Color
Electron
e^-
$\sim 0.5\text{MeV}$
Red
Green
Blue
Flavor
up
u
u
u
2.3MeV
down
d
d
d
4.8MeV
strage
s
s
s
95MeV
$\Lambda_{\text{QCD}} \sim 200\text{MeV}$
charm
c
c
c
1200MeV
bottom
b
b
b
4200MeV
top
t
t
t
$173 \times 10^3\text{MeV}$

Quarks


Color
Red
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$\sim 0.5\mathrm{MeV}$
Flavor
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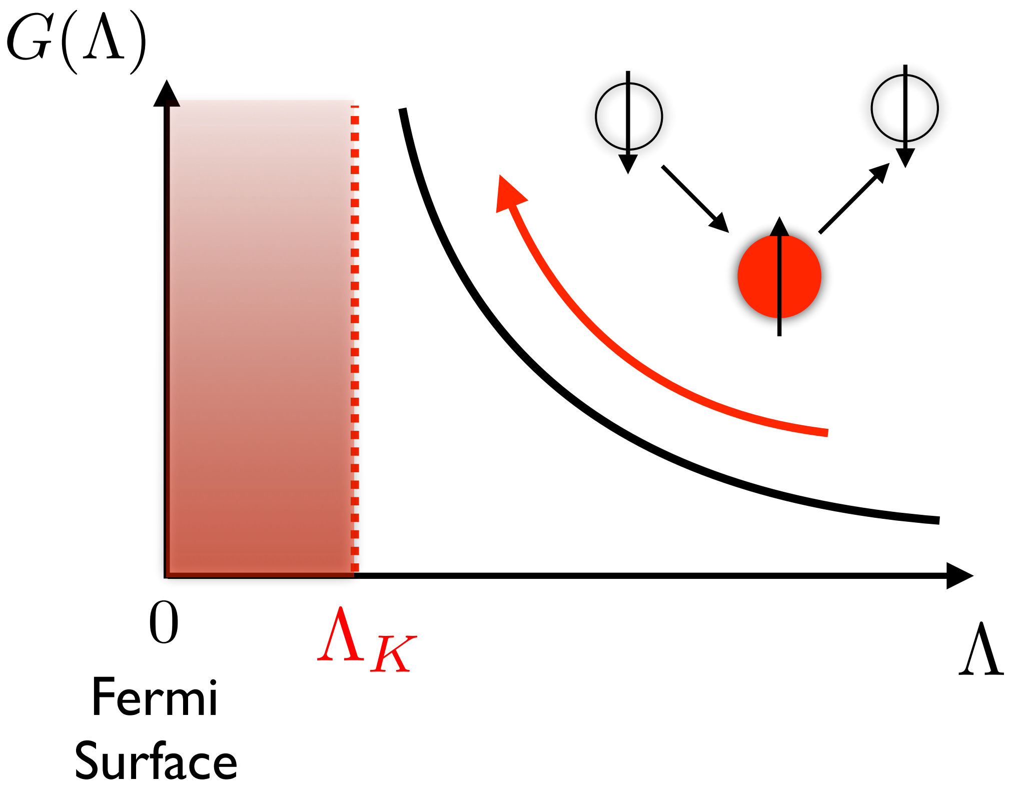
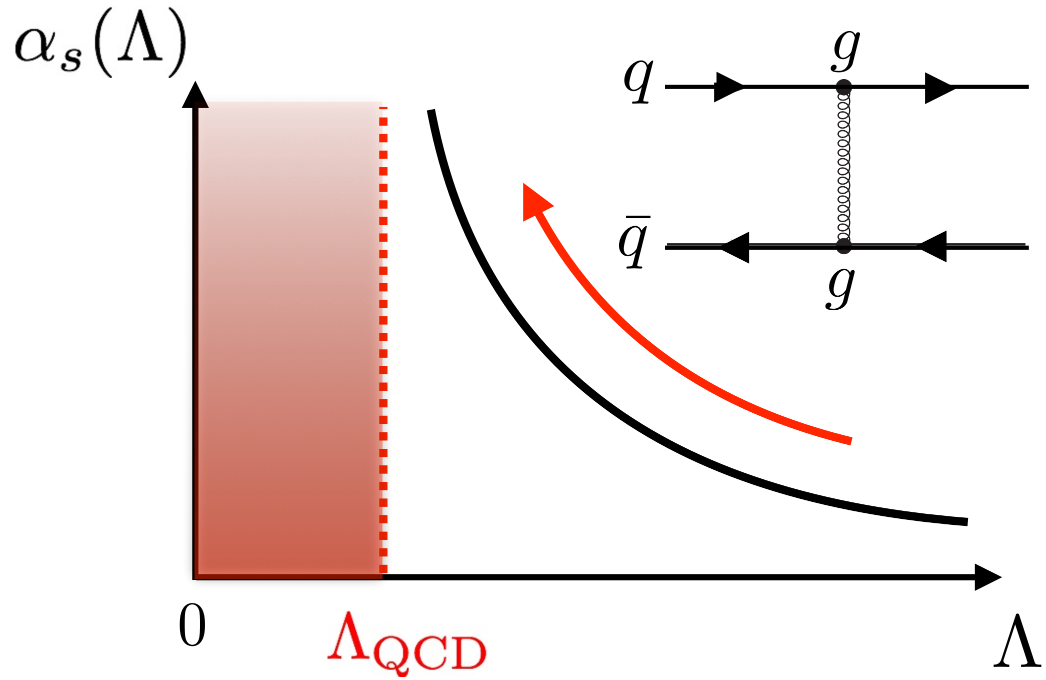
strage
charm
bottom
top
u u u
d d d
s s s
c c c
b b b
t t t
2.3MeV
4.8MeV
95MeV
$\Lambda_{\mathrm{QCD}} \sim 200\mathrm{MeV}$
1200MeV
4200MeV
$173 \times 10^3\mathrm{MeV}$
Impurity effect
by heavy flavors
Kondo effect induced
by color d.o.f.

Asymptotic freedom in Kondo effect and QCD

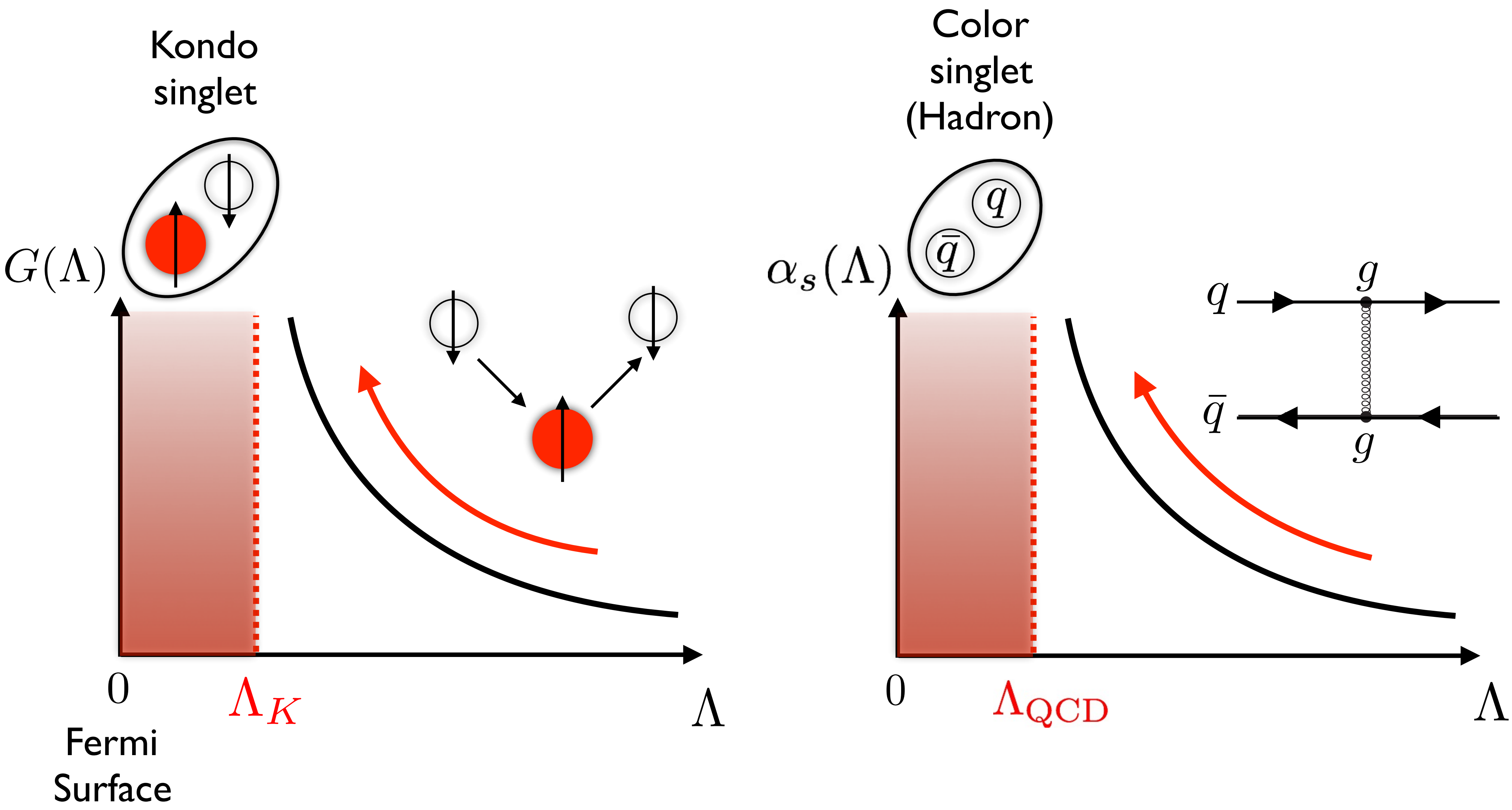
Kondo effect



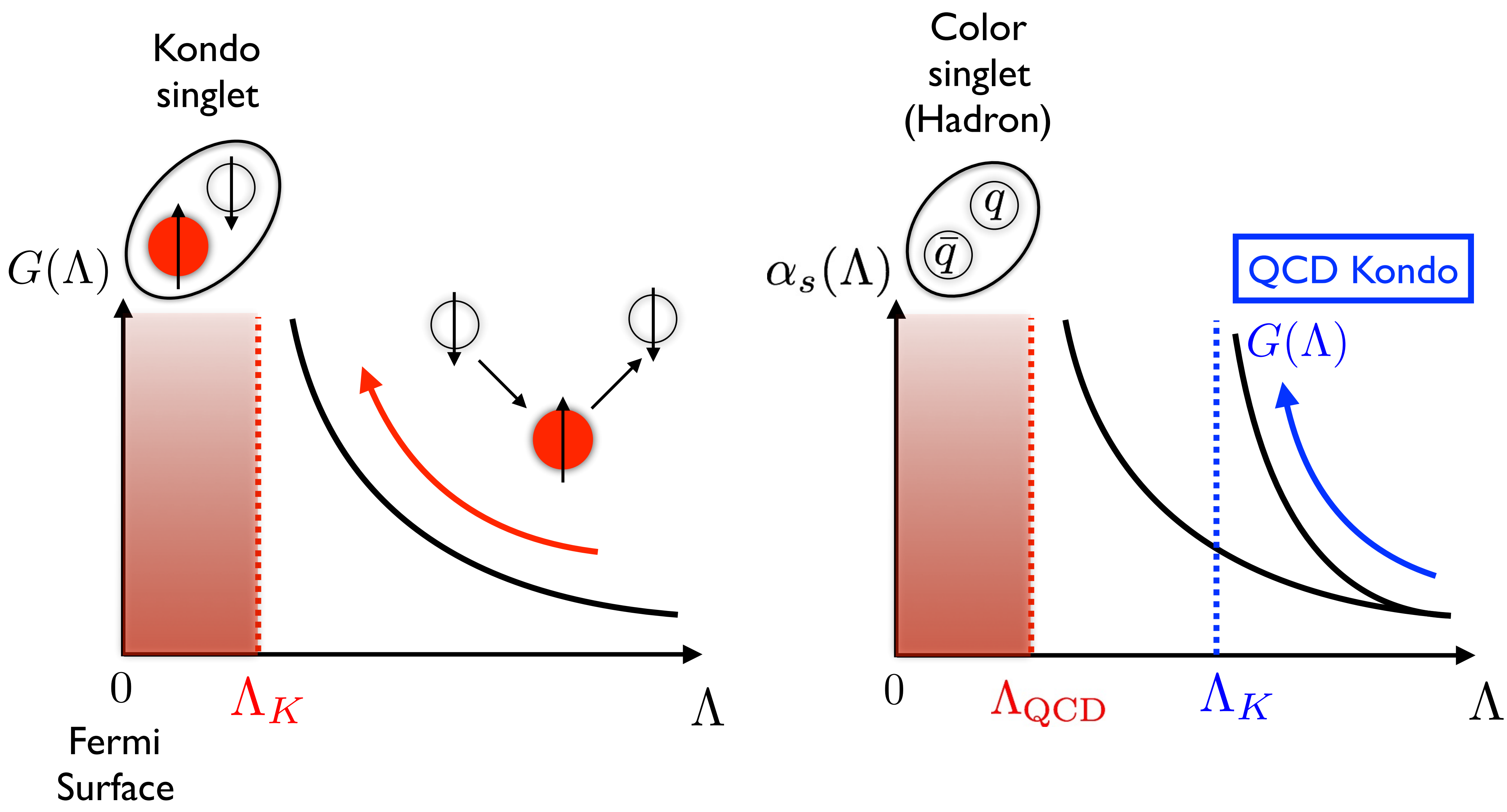
Running coupling of QCD



Asymptotic freedom in Kondo effect and QCD



Asymptotic freedom in Kondo effect and QCD



Conditions for the appearance of Kondo effect

0) Heavy impurity

i) Fermi surface

ii) Quantum fluctuation (loop effect)

iii) Non-Abelian property of interaction
(spin-flip int.)

Conditions for the appearance of QCD Kondo effect

0) Heavy quark impurity

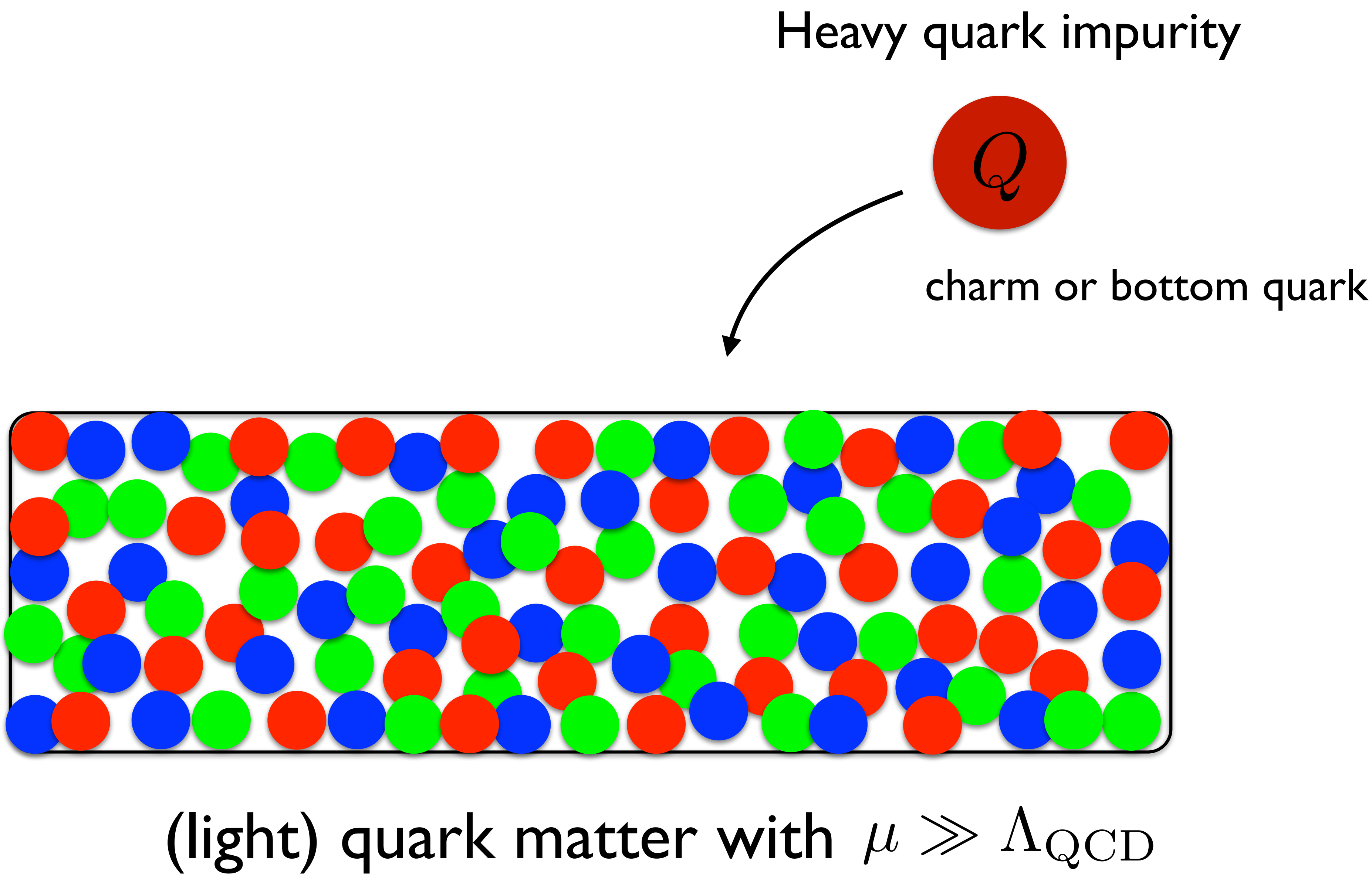
i) Fermi surface of light quarks

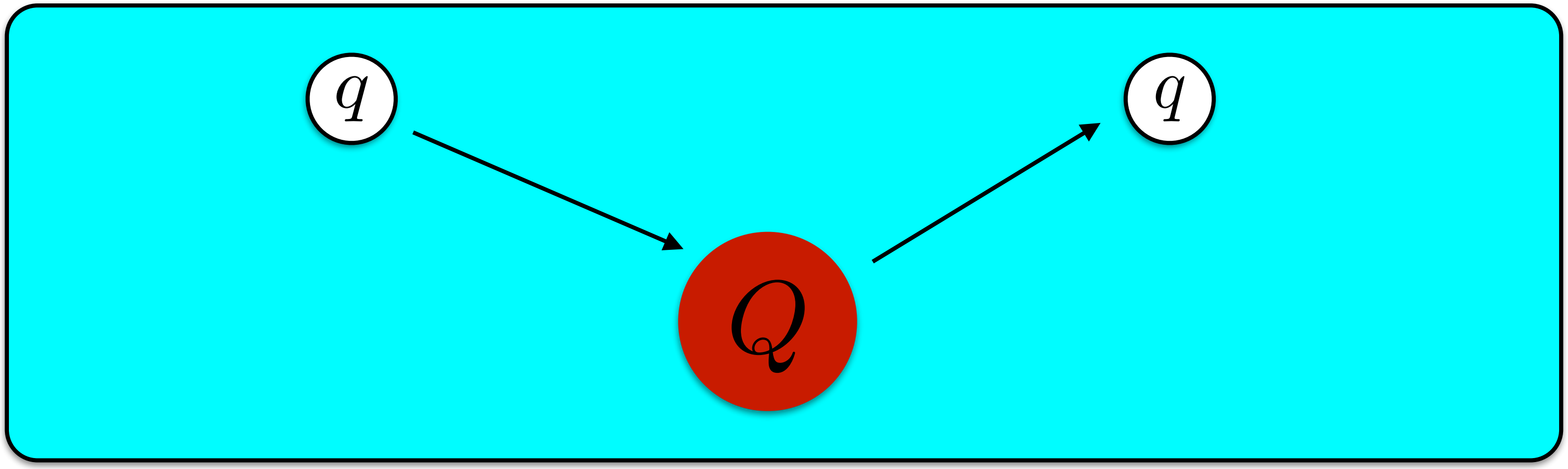
ii) Quantum fluctuation (loop effect)

iii) Color exchange interaction in QCD

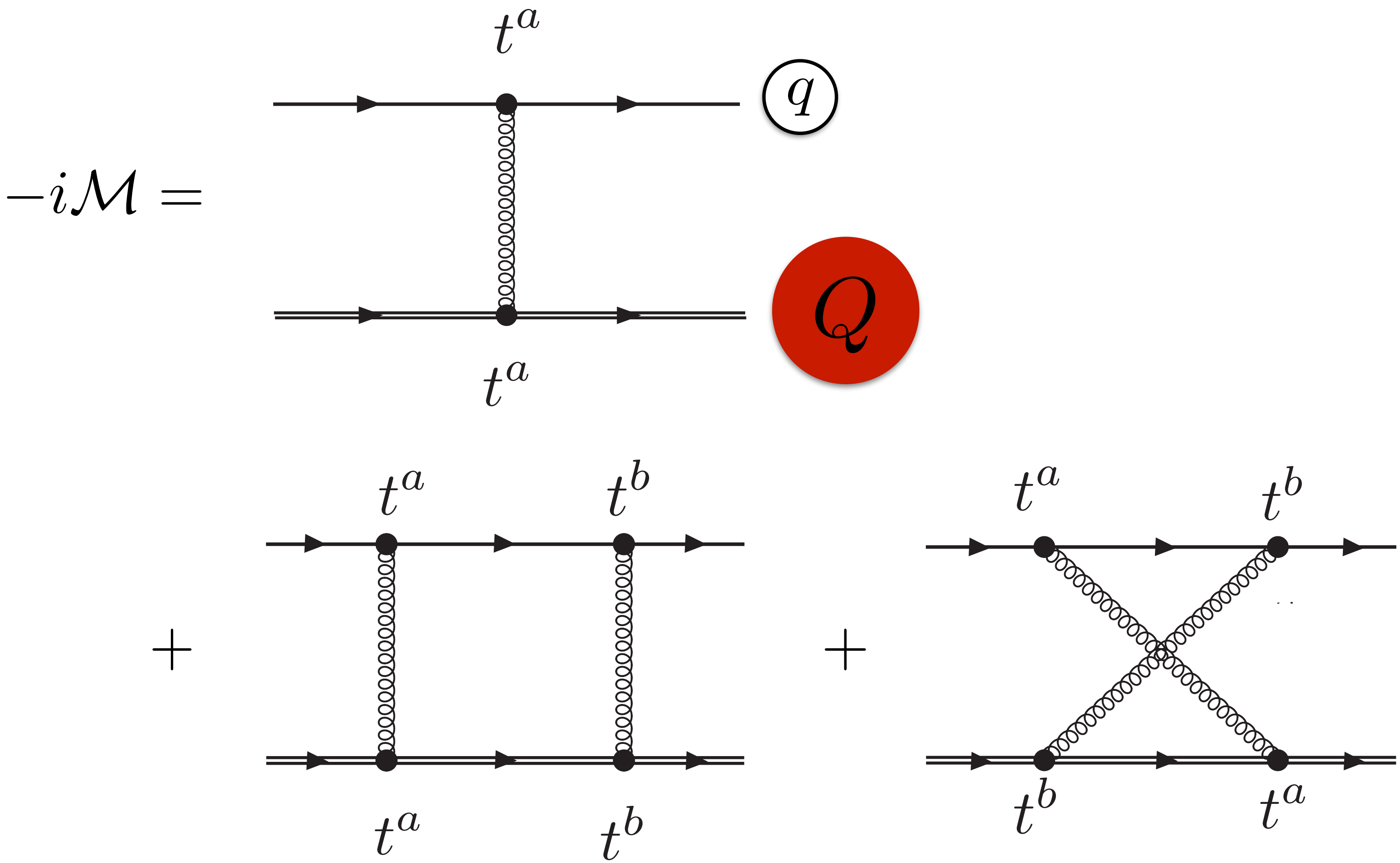
QCD Kondo effect

K. Hattori, K. Itakura, S. O. and S. Yasui, PRD92 (2015) 065003


Heavy quark impurity
Q
charm or bottom quark
(light) quark matter with $\mu \gg \Lambda_{\rm QCD}$



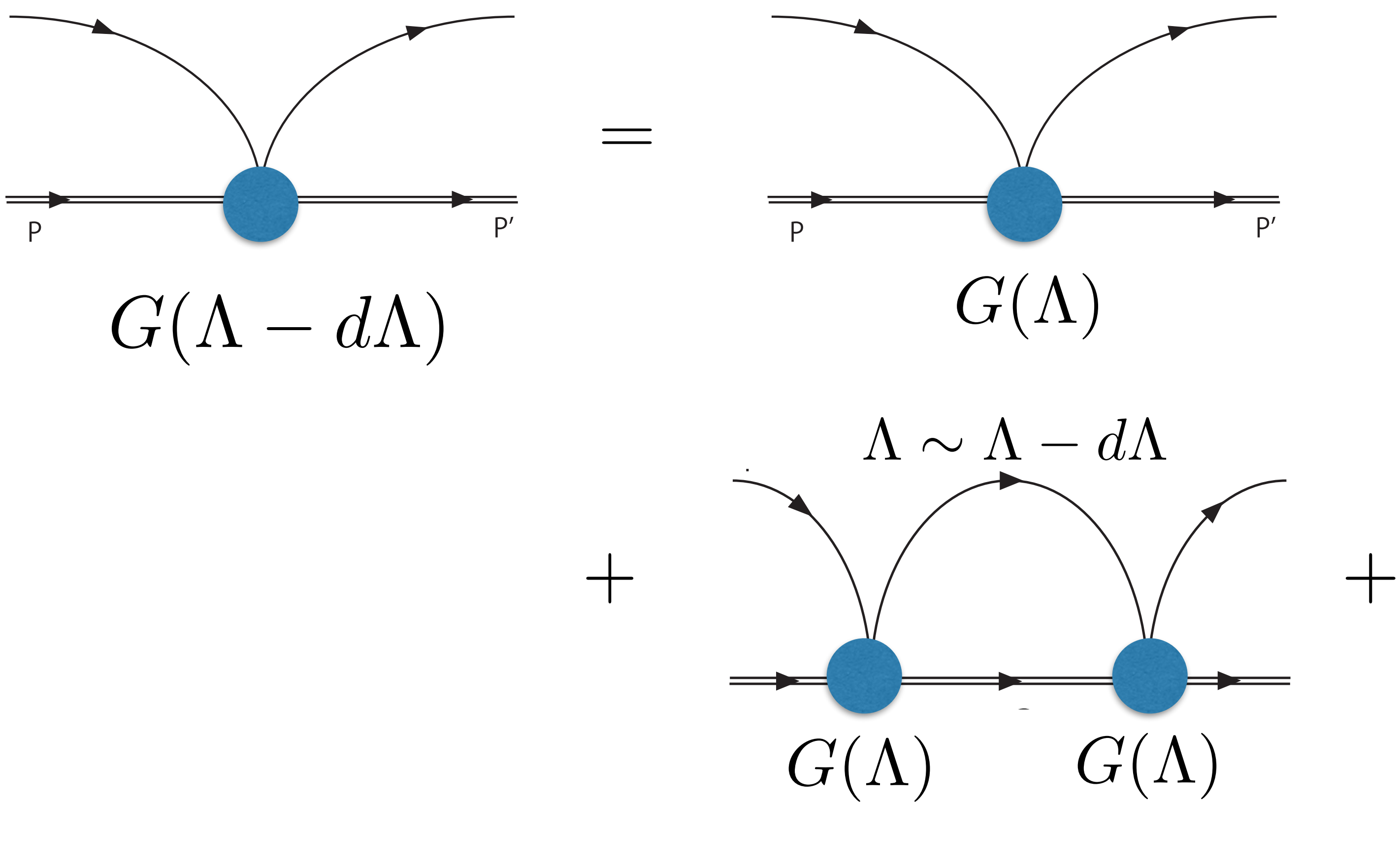
(light) quark matter with $\mu \gg \Lambda_{\mathrm{QCD}}$

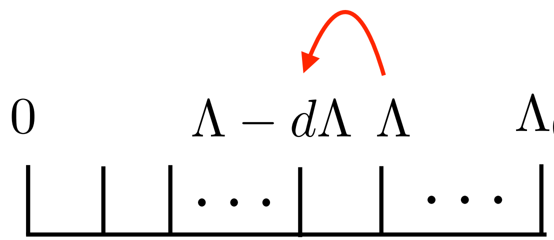


Heavy quark: $M_Q \to \text{large}$

Renormalization group equation of scattering amplitude

~poor man's scaling~





Initial scale

$$\Lambda_0 = \Lambda_{UV} \simeq k_F$$

Renormalization group equation of scattering amplitude

$$\Lambda \frac{dG(\Lambda)}{d\Lambda} = -\frac{N_c}{2}\rho_F G^2(\Lambda)$$

Solution $\longrightarrow$

$$G(\Lambda) = \frac{G(\Lambda_0)}{1 + \frac{N_c}{2}\rho_F G(\Lambda_0)\log(\Lambda/\Lambda_0)}$$

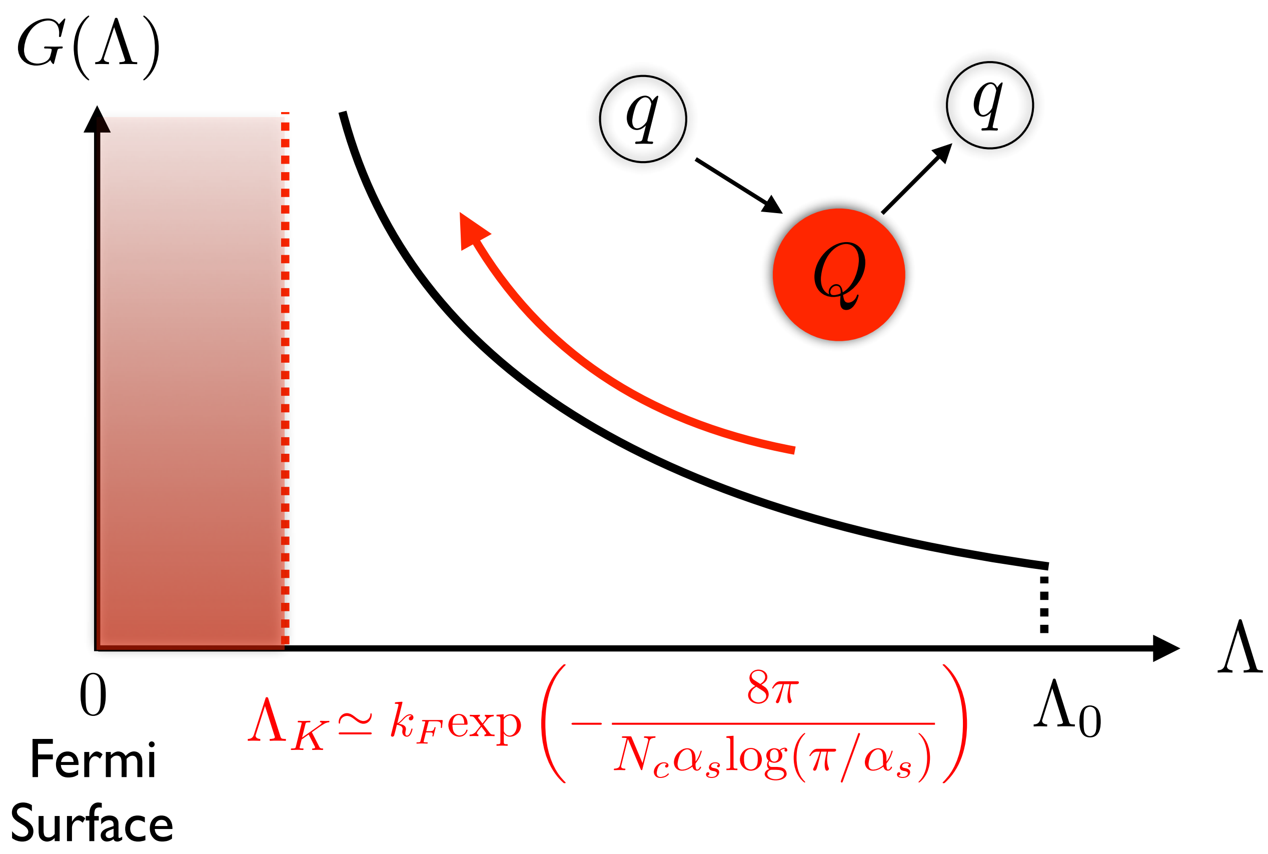
Initial scale

$$\Lambda_0 = \Lambda_{UV} \simeq k_F$$

Kondo scale (from the Landau pole)

$$\Lambda_K \simeq k_F \exp\left(-\frac{8\pi}{N_c \alpha_s \log(\pi/\alpha_s)}\right)$$

QCD Kondo effect

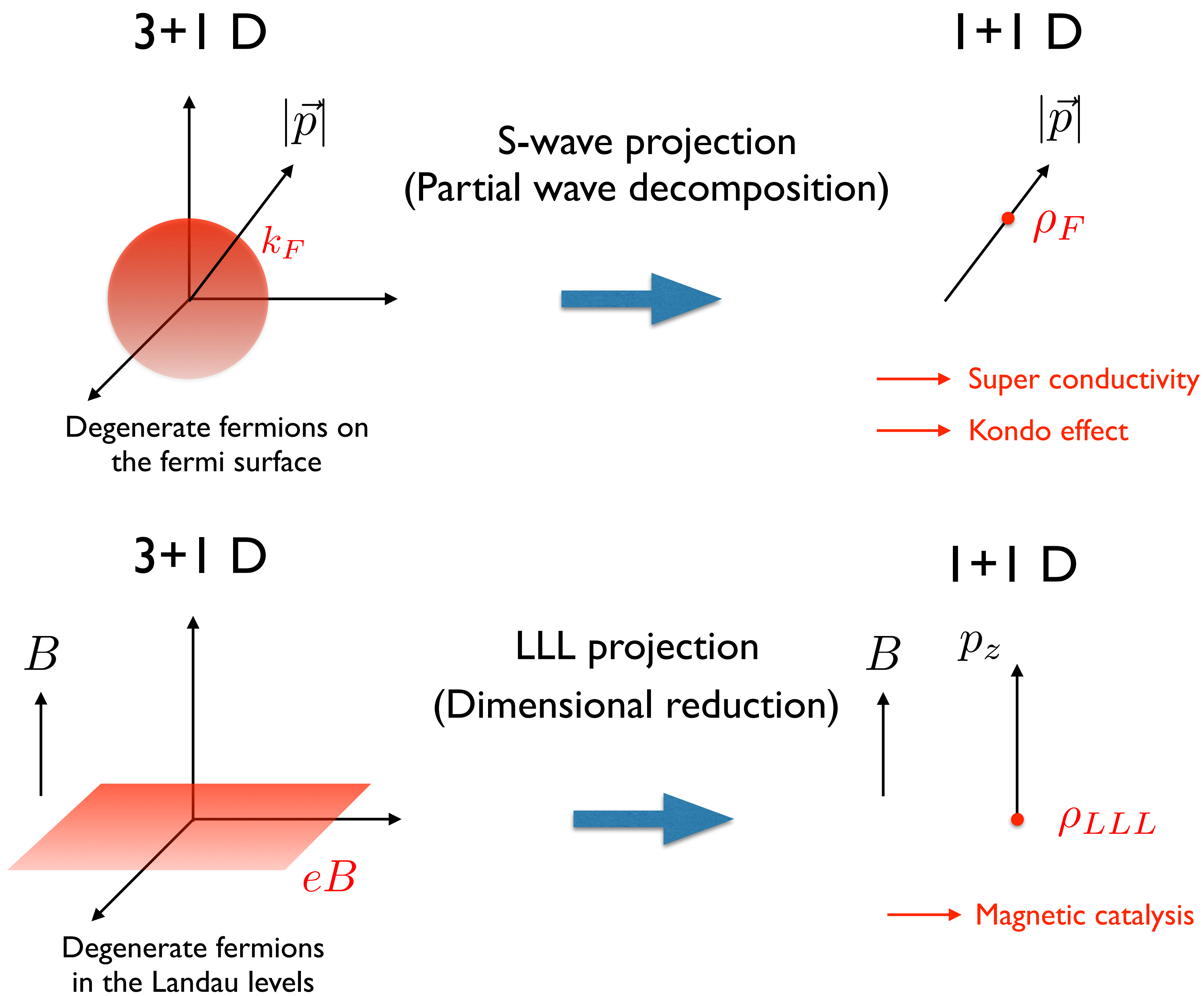


- The strength of the q-Q interaction increases as the energy scale decreases, and the system becomes non-perturbative one below the Kondo scale.
- This indicates a change of mobility of light quarks.

⟶ Several transport coefficients will be largely affected by QCD Konde effect.

Magnetically induced QCD Kondo effect

S. O., K. Itakura and Y. Kuramoto, PRD94 (2016) 074013


3+1 D
1+1 D
|p|
S-wave projection
(Partial wave decomposition)
|p|
k_F
ρ_F
Super conductivity
Kondo effect
Degenerate fermions on
the fermi surface
3+1 D
1+1 D
B
LLL projection
(Dimensional reduction)
B
p_z
ρ_LLL
eB
Magnetic catalysis
Degenerate fermions
in the Landau levels

Conditions for the appearance of QCD Kondo effect

0) Heavy quark impurity

i) Fermi surface of light quarks

ii) Quantum fluctuation (loop effect)

iii) Color exchange interaction in QCD

Conditions for the appearance of

"Magnetically induced QCD Kondo effect"

0) Heavy quark impurity

i) Strong magnetic field

ii) Quantum fluctuation (loop effect)

iii) Color exchange interaction in QCD

The magnetic field does not affect color degrees of freedom.

Renormalization group equation

$$\Lambda \frac{dG(\Lambda)}{d\Lambda} = -\frac{N_c}{2}\rho_{LLL} G^2(\Lambda)$$

$\xrightarrow{\text{solution}}$

$$G(\Lambda) = \frac{G(\Lambda_0)}{1 + \frac{N_c}{2}\rho_{LLL} G(\Lambda_0) \log(\Lambda/\Lambda_0)}$$

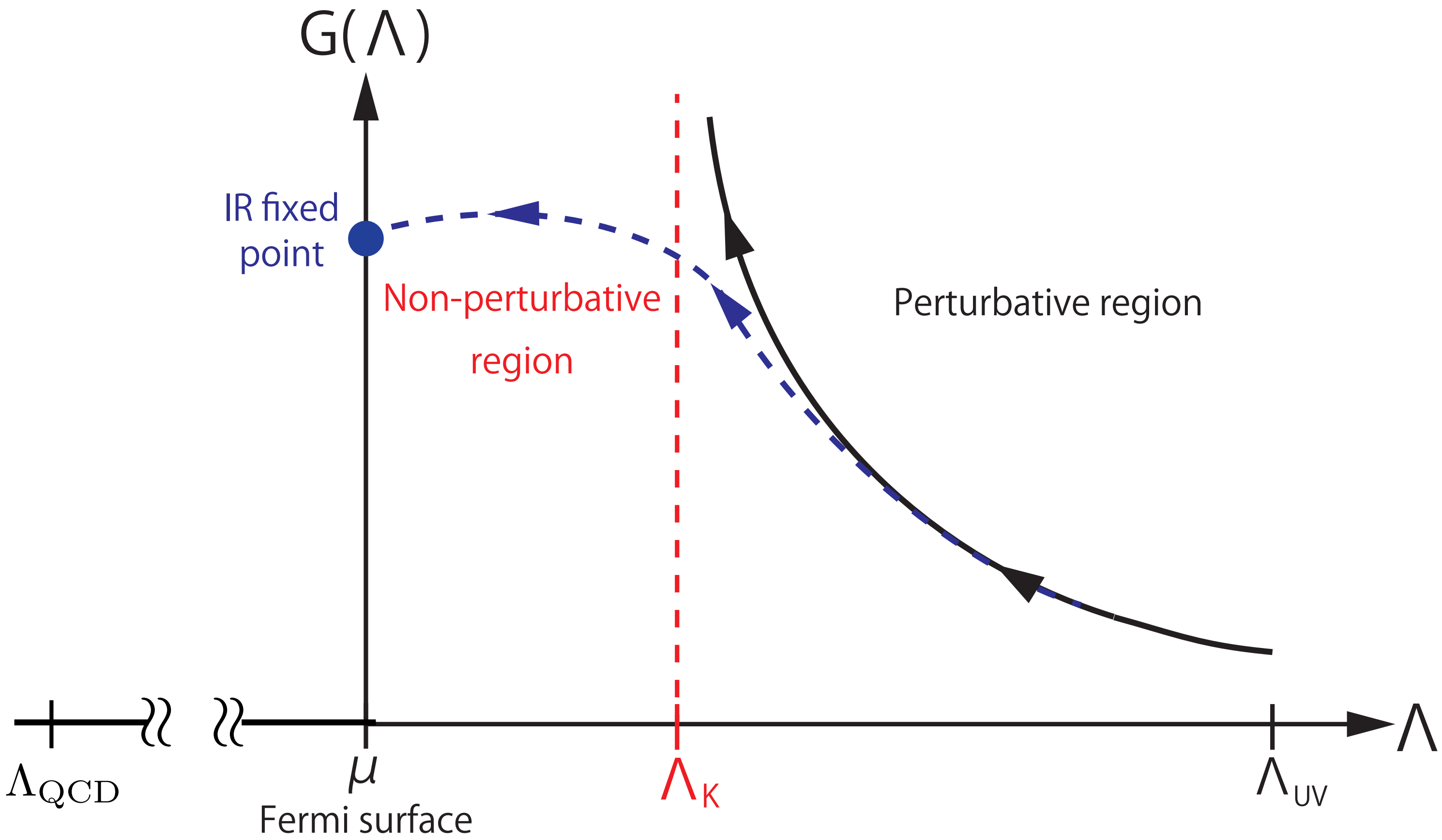
Kondo scale (from the Landau pole)

$$\Lambda_K \simeq \sqrt{e_q B}\,\alpha_s^{1/3} \exp\left\{ -\frac{2\pi}{N_c \alpha_s \log(\pi/\alpha_s)} \right\}$$

QCD Kondo effect from CFT

T. Kimura and S. O, in preparation

QCD Kondo effect



In order to investigate QCD Kondo effect in IR region below Kondo scale, we have to rely on non-perturbative method.

Effective 1+1 dim. theory at high density

High density QCD in the presence of the heavy quark

$\xrightarrow{\text{s-wave}}$ 1+1 dim. (Dimensional reduction)

[E. Shuster & D.T. Son, and T. Kojo et al.]

$$S_{eff}^{1+1} = \int d^2x\, \bar{\Psi}\left[i\Gamma^\mu \partial_\mu\right]\Psi - G\Psi^\dagger t^a \Psi Q^\dagger t^a Q$$

with $G = \alpha_s \log\frac{4\mu^2}{m_g^2} = \alpha_s \log\frac{4\pi}{\alpha_s} \ll 1$

- Ψ is light quark fields with 2Nf components of flavor and Nc colors. The 2 comes from spin d.o.f. in 4 dim.
- This is nothing but k-channel SU(N) Kondo model in 1+1 dim., where $k = 2N_f$, $N = N_c$.

g-factor in QCD Kondo effect @ IR fixed point
(zero temperature)

- $N_c = 3$ $\qquad k = 2N_f$

$$g = \frac{1+\sqrt{5}}{2} \qquad (N_f = 1)$$

$$g = 2.24598... \quad (N_f = 2)$$

$$g = 2.53209... \quad (N_f = 3)$$

- In general Nc and Nf, the g-factor is non-integer, and thus QCD Kondo effect has non-Fermi liquid IR fixed point.

- In large Nc limit: $N_c \to \infty, \quad N_f$: fixed

  $g \to k = 2N_f$ Fermi liquid at IR fixed point

Specific heat of QCD Kondo effect

$$C_{\rm imp} = \begin{cases} \frac{\lambda^2}{2}\pi^{1+2\Delta}(2\Delta)^2(N_c^2-1)(N_c+N_f)\left(\frac{1-2\Delta}{2}\right)\frac{\Gamma(1/2-\Delta)\Gamma(1/2)}{\Gamma(1-\Delta)}\underline{T^{2\Delta}} & (2N_f > N_c) \\ \lambda^2\pi^{1+2\Delta}(N^2-1)(N_c+N_f)(2\Delta)^2\,\underline{T\log\left(\frac{T_{\rm K}}{T}\right)} & (2N_f = N_c) \\ -\lambda_1\frac{2}{3}(N_c^2-1)\pi^2\underline{T}+2\lambda^2\pi^2(N_c^2-1)(N_c+N_f)\frac{2\Delta}{1+2\Delta}\left(\frac{\beta_{\rm K}^{-2\Delta+1}}{2\Delta-1}\right)\underline{T} & (N_c > 2N_f) \end{cases}$$

Low T scaling

$$C_{\rm imp} \propto \begin{cases} T^{2\Delta} & (2N_f > N_c) & \text{Non-Fermi} \\ T\log(T_{\rm K}/T) & (2N_f = N_c) & \text{Non-Fermi} \\ T & (N_c > 2N_f) & \text{Fermi} \end{cases}$$

<u>For Nc > 2Nf, QCD Kondo effect shows Fermi/non-Fermi mixing.</u>

Susceptibility of QCD Kondo effect

$$\chi_{\mathrm{imp}} = \begin{cases} \dfrac{\lambda^2}{2}\pi^{2\Delta-1}(N_c+N_f)^2(1-2\Delta)\dfrac{\Gamma(1/2-\Delta)\Gamma(1/2)}{\Gamma(1-\Delta)}T^{2\Delta-1} & (2N_f > N_c) \\ 2\lambda^2(N_c+N_f)^2 \log\left(\dfrac{T_{\mathrm{K}}}{T}\right) & (2N_f = N_c) \\ -\lambda_1 N_f(N_c+2N_f) + 2\lambda^2(N_c+N_f)^2\left(\dfrac{\beta_{\mathrm{K}}^{-2\Delta+1}}{2\Delta-1}\right) & (N_c > 2N_f) \end{cases}$$

Low T scaling

$$\chi_{\mathrm{imp}} = \begin{cases} T^{2\Delta-1} & (2N_f > N_c) \quad \text{Non-Fermi} \\ \log(T_{\mathrm{K}}/T) & (2N_f = N_c) \quad \text{Non-Fermi} \\ \text{const.} & (N_c > 2N_f) \quad \text{Fermi} \end{cases}$$

The Wilson ratio of QCD Kondo effect

$$R_{\rm W} = \left(\frac{\chi_{\rm imp}}{C_{\rm imp}}\right) \Big/ \left(\frac{\chi_{\rm bulk}}{C_{\rm bulk}}\right)$$

$$= \frac{(N_c + N_F)(N_c + 2N_f)^2}{3N_c(N_c^2 - 1)} \qquad (2N_f \geq N_c)$$

Unknown parameters are canceled, and thus the Wilson ratio of QCD Kondo effect is universal for 2Nf >= Nc.

$$R_{\rm W} = \frac{(N_c + N_f)(N_c + 2N_f/3)}{N_c^2 - 1} \frac{\gamma - \dfrac{2N_f(N_c + 2N_f)}{(N_c + N_f)^2}}{\gamma - \dfrac{2N_f(N_c + 2N_f/3)}{N_c(N_c + N_f)}} \qquad (N_c > 2N_f)$$

with $\gamma = 4\dfrac{\lambda^2}{\lambda_1} T_{\rm K}^{2\Delta - 1}$

For Nc >= 2Nf, the Wilson ratio is no longer universal, which depends on the detail of the system, such as $\lambda,\ T_{\rm K}$

IR behaviors of QCD Kondo effect

	(k >= N)	(N > k > 1)
	2Nf >= Nc	Nc > 2Nf
g-factor (IR fixed point)	non-Fermi	non-Fermi
Low T scaling	non-Fermi	Fermi
Wilson ratio	universal	non-universal

Fermi/non-Fermi mixing

Summary

- We develop the CFT approach to general k-channel SU(N) Kondo effect and investigate its IR behaviors.
- In the vicinity of IR fixed point, the Kondo system shows Fermi/non-Fermi mixing for N > k > 1, while it shoes non-Fermi liquid behaviors for k >= N.
- We apply CFT approach to QCD Kondo effect and determine its IR behaviors below the Kondo scale.
- Our CFT analysis for k-channel SU(N) Kondo effect can be also applied to SU(3) Kondo effect in cold atom and SU(4) Kondo effect in Quantum dot systems with multi-channels.