Theory of Nuclear Matter and Neutron Stars

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- BHF approach of hypernuclear matter
- Hypernuclei
- Neutron star properties
- Quark matter and hybrid stars

PRC 61, 055801 (2000)
PRC 69, 018801 (2004)
PRD 70, 043010 (2004)
PRC 73, 058801 (2006)
PRC 74, 047304 (2006)
PRD 74, 123001 (2006)
PRD 76, 123015 (2007)
PRC 78, 028801 (2008)
PRC 83, 025804 (2011)
PRC 84, 035801 (2011)
PRD 84, 105023 (2011)
PRD 91, 105002 (2015)
EPJA 52, 21 (2016)
PRC 94, 024322 (2016)
PRC 96, 044309 (2017)
∼ 2800 known neutron stars
2500 pulsars
10% in binary systems
∼ 10^{8} in our galaxy?
∼ 2800 known neutron stars
2500 pulsars
10% in binary systems
∼ 10⁸ in our galaxy?
A Theorist’s View of a Neutron Star:

A huge nucleus: $\sim 10^{57}$ nucleons:

- $n p e \mu$ mesons?
- quarks?

$M \sim 1 \ldots 2 M_{\odot}$

$T \ll 1 \text{ MeV} \approx 10^{10} \text{K}$

$\rho \sim \rho_0$ 

$\sim 10 \rho_0$, $\sim 10 \text{km}$

The only “laboratory” for $\rho_B \sim 10 \rho_0$ in the Universe!

Need EOS of nuclear matter including hyperons and quarks
Hypermatter in the Neutron Star: \( \rho = \rho_N + \rho_Y \)

- \( N = qqq: \) \( n\bar{p} \) (939 MeV)
- \( Y = qqs: \) \( \Lambda^0 \) (1116 MeV)
- \( qss: \) \( \Sigma^{-0+} \) (1193 MeV)
- \( \Xi^{-0} \) (1318 MeV)

- \( V_{NN} \): Argonne, Bonn, Paris, ... potential
- \( V_{NY} \): Nijmegen (NSC89,NSC97,ESC08...)
- \( V_{YY} \): ? (no scattering data)

In free space weak decay: \( Y \to N + \pi \) etc. \((c\tau \approx 8 \text{ cm})\)
In dense nucleonic medium the decay is Pauli-blocked!

We need to compute the energy density of this system ...
Brueckner Theory of (Hyper)Nuclear Matter:

- Effective in-medium interaction $G$ from potential $V$:

$$G = V + V_G$$

parameter-free!

$$e_k = m + \frac{k^2}{2m} + U(k)$$

Results: binding energy $\epsilon(\rho_n, \rho_p, \rho_\Lambda, \rho_\Sigma) = \sum_{i} \sum_{k < k_F^{(i)}} \left[ e_k^{(i)} - \frac{U_i(k)}{2} \right]$ s.p. properties, cross sections, ...

K.A. Brueckner and J.L. Gammel; PR 109, 1023 (1958) for nuclear matter

Extension to hypernuclear matter ...
• Framework: Brueckner-Bethe-Goldstone hole-line expansion

\[
\frac{E}{A} = \frac{3 k_F^2}{52m} + \left( \sum_{k<k_F} \right) + \cdots + \mathcal{O}(\kappa^4)
\]

\[
\approx \left[ 22 - 40 + 2 + ? \right] \text{MeV}
\]

\[\rho = \rho_0, \text{ symmetric matter, } V_{18} \text{ potential} \]

• Expansion parameter \( \kappa \sim \rho V_{\text{core}} \approx 0.2 \):

\[
\kappa \equiv \frac{\sum_k n(k > k_F)}{\sum_k < k_F} = \rho \int d^3r \left\langle |\eta(r)|^2 \right\rangle_{S,T} = N \frac{V_{\text{core}}}{V} = \left( \frac{c}{d} \right)^3
\]

- Hierarchy of n-body correlations/clusters within hard-core range \( c \), avg. distance \( d \):
- Justified for hard-core potentials
- Correlation parameter for different NN potentials:

\[ \kappa \approx \rho V_{\text{core}}(\rho) \]

Small up to large density

Hard vs. soft potentials

- Binding energy up to three hole lines:

\[ B/A = T + E_2 + E_3 \]

Hole-line expansion appears well converged, but misses slightly for AV18 the empirical saturation point of nuclear matter
Diagrams up to 3HL:


 Fadeev calculation:

\[ T^{(3)} = \cdots + \cdots + \cdots + \cdots + \cdots \]
Three-Nucleon Forces:

1. \( N^* = \Delta, R, \ldots \)
2. \( \mu = \pi, \rho, \sigma, \omega \)
3. Only small effect required \([\delta(B/A) \approx 1 \text{ MeV at } \rho_0]\)
4. Model dependent, no final theory yet
5. Use and compare microscopic and phenomenological TBF...
   - Microscopic TBF of P. Grangé et al., PRC 40, 1040 (1989): Exchange of \( \pi, \rho, \sigma, \omega \) via \( \Delta(1232), R(1440), \bar{N}N \) Parameters compatible with two-nucleon potential (Paris, \( V_{18}, \ldots \))
   - Urbana IX phenomenological TBF: Only 2\( \pi \)-TBF + phenomenological repulsion Fit saturation point
- BHF binding energy and saturation point of nuclear matter:

- Nuclear mass formula

- Coester band:

- Dependence on NN potential
- TBF needed to improve saturation properties
Include Hyperons:

- Technical difficulty: coupled channels:

\[
\Lambda \Lambda = \Lambda \Lambda + \Lambda \{\Lambda \Sigma^- \Sigma^0\} + \ldots
\]
Include Hyperons:

- Technical difficulty: coupled channels:

\[
G = n\Lambda \Lambda + \{\Lambda^{-} \Sigma^{0}\} + \ldots
\]

\[
G = \Lambda \Lambda \Lambda + \{n p n\} + \ldots
\]

\[
G = \Lambda \Lambda \Lambda + \{\Lambda \Lambda \Sigma^{0} \Sigma^{+}\} + \ldots
\]

\[
G = \Lambda \Lambda \Lambda + \{\Lambda \Sigma^{0} \Sigma^{0} \Sigma^{-}\} + \ldots
\]

\[
G = \Lambda \Lambda \Lambda + \{\Xi^{0} \Xi^{-}\} + \ldots
\]

\[
G = \Lambda \Lambda \Lambda + \{n p\} + \ldots
\]
Example BHF Results: Input:

- Hyperon-nucleon potentials (NSC89) vs. Paris NN:

![Graph showing various interactions between nucleons and hyperons](image)

- "Soft" cores, Strong coupling $N\Lambda \leftrightarrow N\Sigma$
Example BHF Results: Output:

s.p. potentials

A18+TBF NN & ESC08 NY+YY Potentials
\( \rho_N = 0.17 \text{ fm}^{-3} \), \( \rho_{\Lambda}/\rho_N = 0.0 \) \( \rho_N = 0.17 \text{ fm}^{-3} \), \( \rho_{\Lambda}/\rho_N = 0.5 \)

\[ k \text{ [fm}^{-1} \]

Re \( U \) [MeV]

\( N \) \( \Sigma \) \( \Xi \) \( \Lambda \)

\[ r_N = 0.17 \text{ fm}^{-3}, \frac{r_{\Lambda}}{r_N} = 0.5 \]

Hyperons are weaker bound than nucleons
Neutron Stars

Vela pulsar
«Recipe» for Neutron Star Structure Calculation:

Brueckner results: $\epsilon(\{\rho_i\})$; $i = n, p, e, \mu, \Lambda, \Sigma, u, d, s, \ldots$

Chemical potentials: $\mu_i = \frac{\partial \epsilon}{\partial \rho_i}$

Beta-equilibrium: $\mu_i = b_i \mu_n - q_i \mu_e$

Charge neutrality: $\sum_i x_i q_i = 0$

Composition: $x_i(\rho)$

Equation of state: $p(\rho) = \rho^2 \frac{d(\epsilon/\rho)}{d\rho}(\rho, x_i(\rho))$

TOV equations:

$$\frac{dp}{dr} = -\frac{Gm\epsilon}{r^2} \frac{(1 + p/\epsilon)(1 + 4\pi r^3 p/m)}{1 - 2Gm/r}$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon$$

Structure of the star: $\rho(r), M(R)$ etc.
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Generic implications for EOS and stellar structure:

- Hyperon onset occurs at $\rho \sim 2\ldots3 \rho_0$
- Softer EOS
- NS structure including hyperons
  . . . and including quark matter
Data ?

Vela pulsar
Observational Data: Masses

The heaviest neutron stars:
Recent: \( \sim 1.97 \, M_\odot \) (Nature 09466)
\( \sim 2.01 \, M_\odot \) (Science 340)

No combined \((M, R)\) measurements!
(Would practically fix the EOS)
Observational Data: Radii

Neutron Star Radius Results
F. Özel et al., APJ820, 28 (2016)

Six Burst Sources
Six qLMXBs

The measurement is difficult: currently no accurate results
BHF Results ...
• Composition of neutron star matter:

No hyperons

Free hyperons

Interacting hyperons (\(\Sigma^-\) repulsive, \(\Lambda\) attractive)

NY interaction determines \(Y\) onset
- EOS of neutron star matter:

Strong softening due to hyperons!
(More Fermi seas available)
Mass-radius relations with different nucleonic TBF:

- Large variation of $M_{\text{max}}$ with nucleonic TBF
- Self-regulating softening due to hyperon appearance (stiffer nucleonic EOS $\rightarrow$ earlier hyperon onset)
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NSC89 NY potential
No YY
No hyperon TBF

11.5-13.0 km
Mass-radius relations using different \( NY,YY \) potentials:

- Maximum mass independent of potentials!
- Maximum mass too low (< 1.4 \( M_\odot \))!
- Proof for “quark” matter inside neutron stars?
Mass-radius relations using different \( NY, YY \) potentials:

- **Maximum mass independent of potentials**
- **Maximum mass too low \((< 1.4M_\odot)\)**
- **Proof for “quark” matter inside neutron stars?**
Quark Matter EOS of Dense Matter:

- Problem: No “exact” results from QCD:
  Large theoretical uncertainties, limited predictive power

- Current strategy:
  Use available eff. quark models (MIT, NJL, CDM, DSM, ...) in combination with the hadronic EOS

- An important constraint (from heavy ion collisions):
  In symmetric matter phase transition not below $\approx 3\rho_0$

  E.g., the simplest (MIT) quark model requires a density-dependent bag “constant”:
  \[
  \epsilon_Q = B + \epsilon_{\text{kin}} + \alpha_s \times \ldots
  \]

  \[
  B(\rho) = B_\infty + (B_0 - B_\infty) \exp \left[-\beta \left(\frac{\rho}{\rho_0}\right)^2\right]
  \]
A more sophisticated approach: Dyson-Schwinger model:

\[ \Sigma(p) = \int \frac{d^4 p}{(2\pi)^4} \frac{\lambda^a}{2} \gamma_\rho D_{\rho \sigma}(k) \Gamma^a_\sigma(q, p) \]

Compute the quark propagator \( S(q) \) from QCD

Allows to calculate

the bag constant:
Different quark EOS’s: bag models, color dielectric model:

- Maximum masses: 1.5...1.9 $M_\odot$, Radii are different!
Details of the phase transition: neutron star profiles:

- **Bulk Gibbs**
- **Screened Gibbs**
- **Maxwell**

\[ \rho \quad \varepsilon \quad p \quad n \quad p \quad n \]

- **BHF[V18+UIX+NSC89] & MIT[B=100, a=0, s=40], M/M_s=1.40**

- Hyperons replaced by strange quark matter
- Very different possible internal structures
- Surface tension + screening enforce ‘quasi’ Maxwell construction (exact for \( \sigma \geq 70 \text{ MeV/fm}^2 \))
Mass-radius relations with different hadron-quark phase transition constructions:

- Screened Gibbs constr. very close to Maxwell construction
- Maximum mass independent of phase transition

$$e.m. \text{ interaction vs. surface tension : }$$

$$\sim 50 \text{ fm}$$

$$M/M_\odot$$ vs. $$R [\text{km}]$$

- nucleon
- hyperon
- hyp+quark $$B=100$$
- hyp+quark $$B=B_{\text{eff}}(\rho_B)$$

**Maxwell**

**Bulk Gibbs**

$$\sigma=40$$
Summary:

- Neutron star physics probes the 4 fundamental interactions:
  - Gravitation: Densest object in the Universe
  - Strong: Nuclear EOS
  - Weak: Beta-equilibrium of matter, Neutrino physics
  - EM: Charge-neutrality, Mixed-phase structures, Crust

Conclusions:

- Hyperons cannot be ignored!
- BHF EOS with hyperons predicts $M_{\text{max}}$ not above $\sim 1.7 M_\odot$
- Need "quark matter" to reach higher masses
- Currently $M_{\text{max}} \approx 1.9 M_\odot$ for hybrid stars in this approach
However:
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We do not know dark matter.
However:

We do not know dark matter.

We do not know dark energy.
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Do we know GR at $10\rho_0$?