Theory of Nuclear Matter and Neutron Stars

M. Baldo & G.F. Burgio & U. Lombardo & I. Vidaña & H.-J. S. : Catania
H. Chen & A. Li & Z.H. Li & H.Q. Song & X.R. Zhou & W. Zuo : China
T. Maruyama & S. Chiba & T. Tatsumi & **N. Yasutake** : Japan
T. Rijken : Nijmegen

PRC 61, 055801 (2000)
PRC 69, 018801 (2004)
PRD 70, 043010 (2004)
A&A 451, 213 (2006)
PRC 73, 058801 (2006)
PRC 74, 047304 (2006)
PRD 74, 123001 (2006)
PRD 76, 123015 (2007)
PRC 78, 028801 (2008)
PRC 83, 025804 (2011)
PRC 84, 035801 (2011)
PRD 84, 105023 (2011)
A&A 551, A13 (2013)
PRD 91, 105002 (2015)
EPJA 52, 21 (2016)
PRC 94, 024322 (2016)
PRC 96, 044309 (2017)

- BHF approach of hypernuclear matter
- Hypernuclei
- Neutron star properties
- Quark matter and hybrid stars

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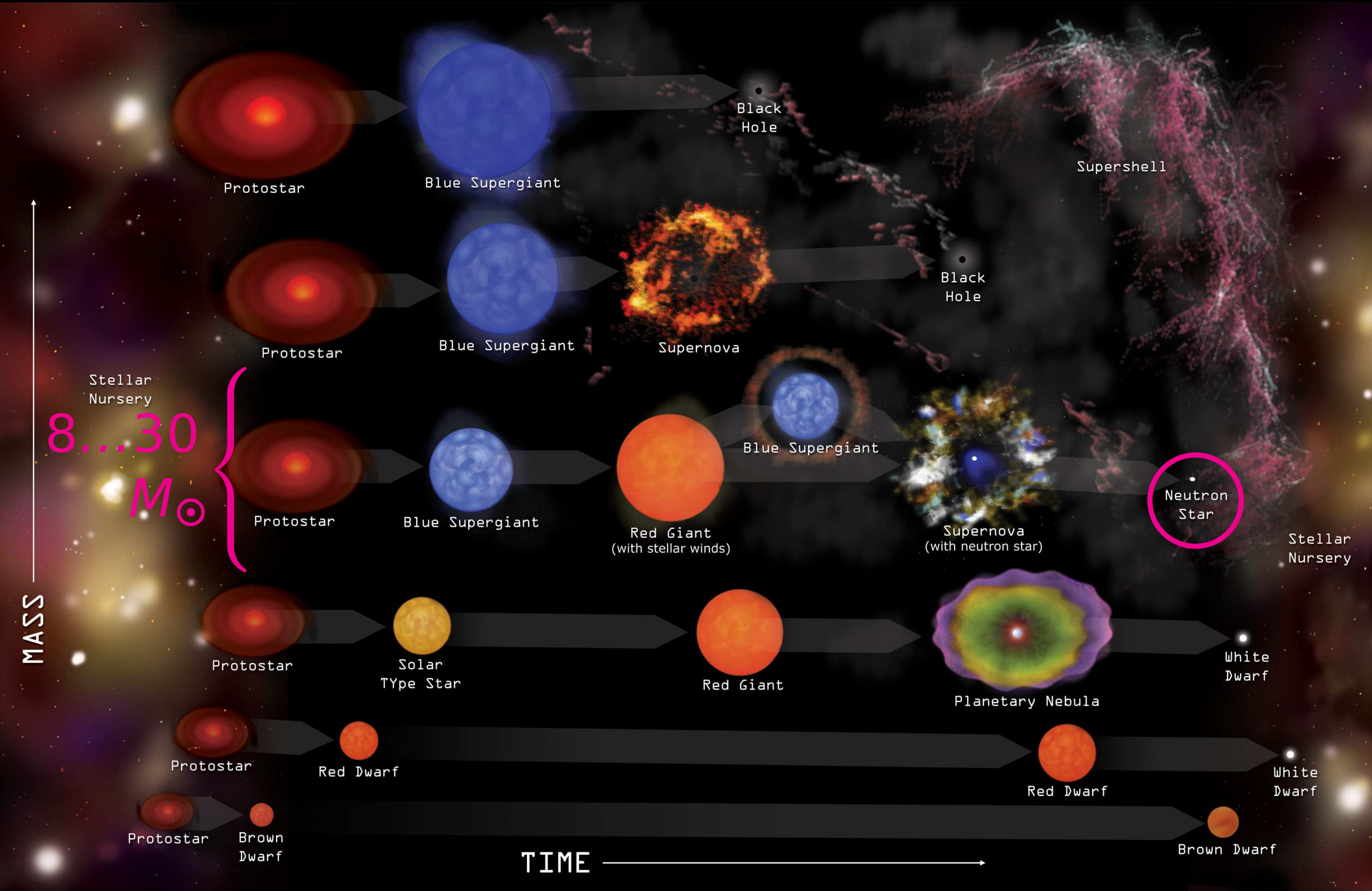

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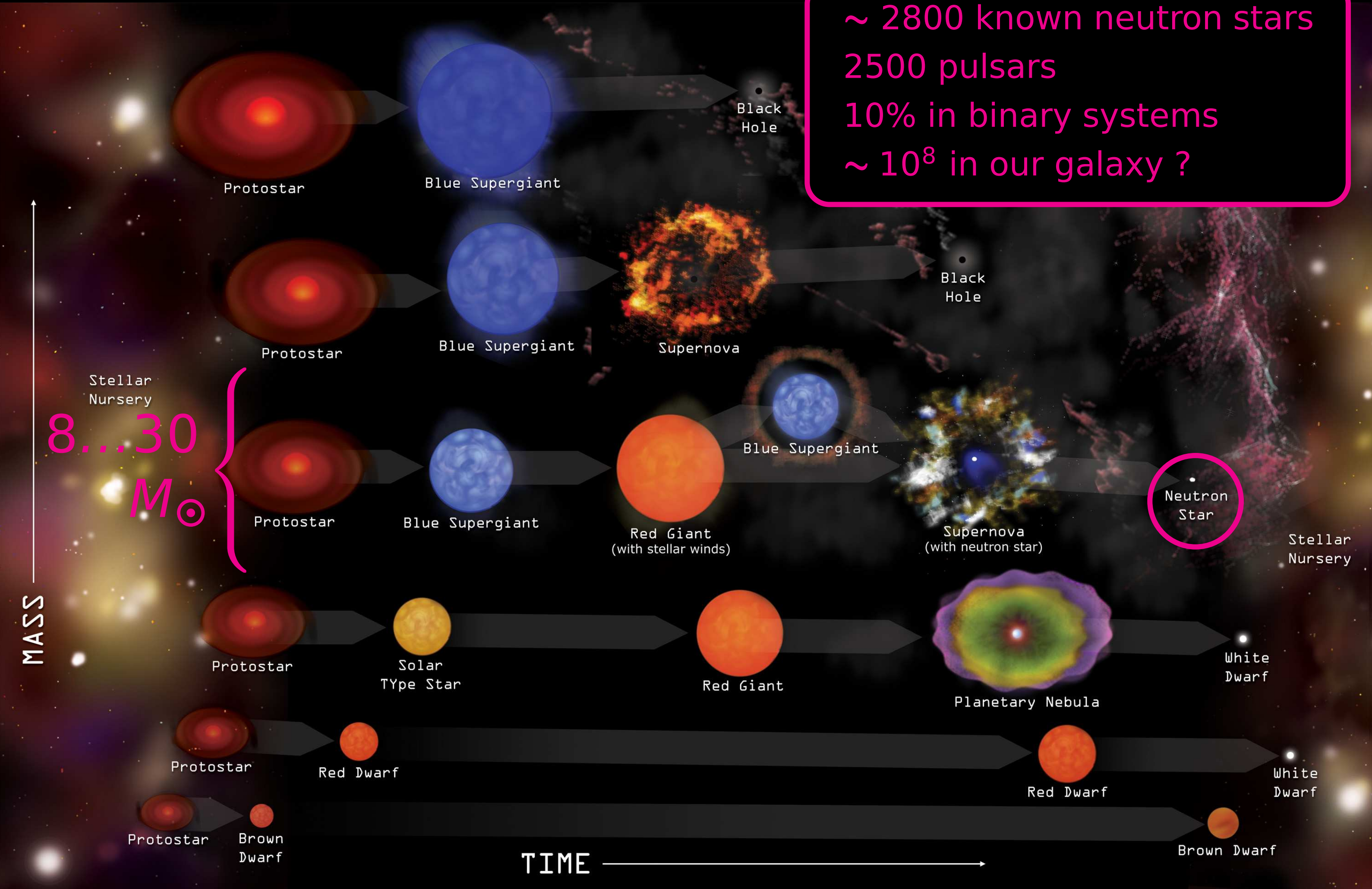
~ 2800 known neutron stars

2500 pulsars

10% in binary systems

~ 10^8 in our galaxy ?

8...30 $M_\odot$



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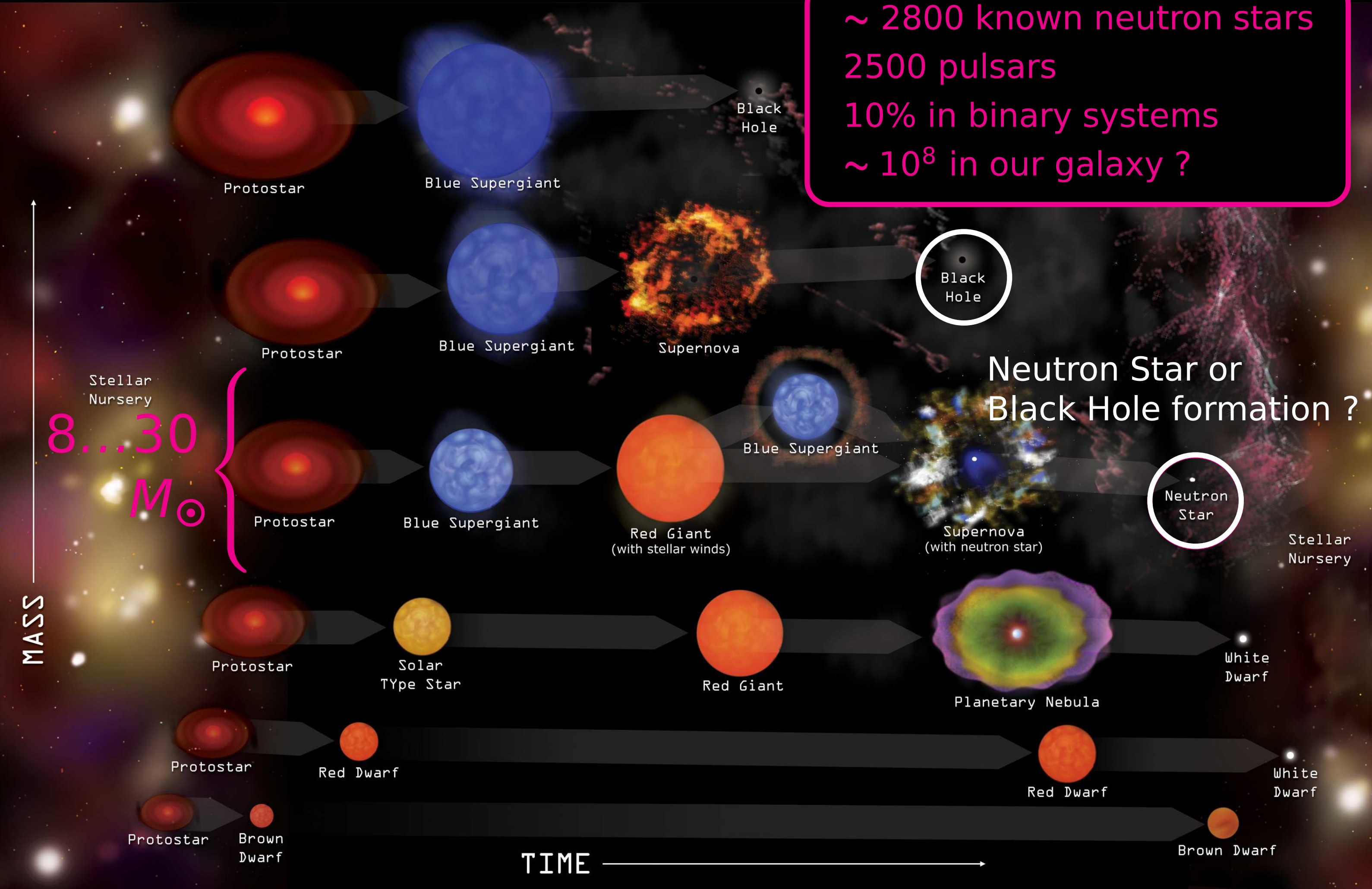
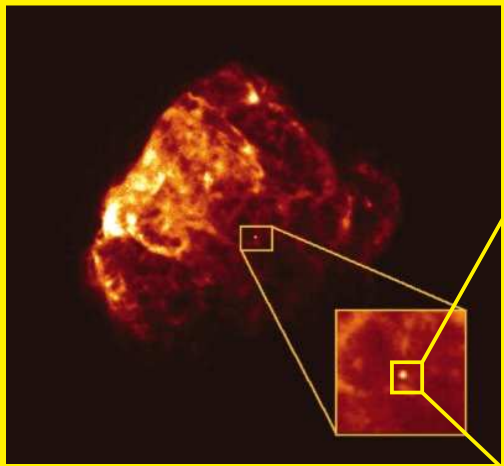
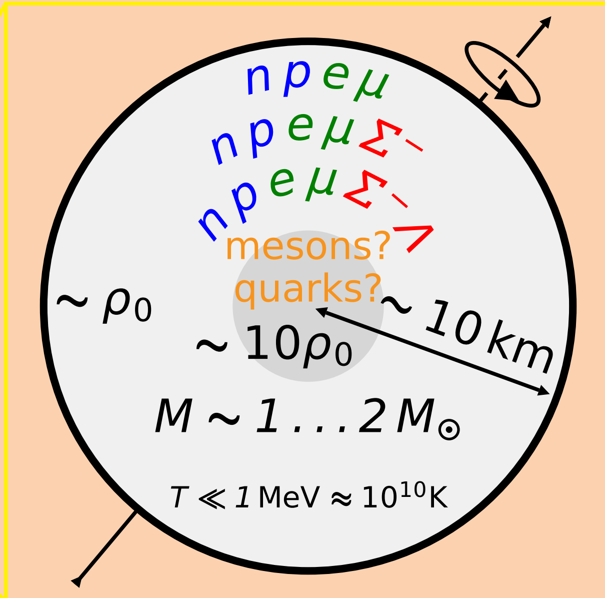


A Theorist's View of a Neutron Star:

A huge nucleus: $\sim 10^{57}$ nucleons :



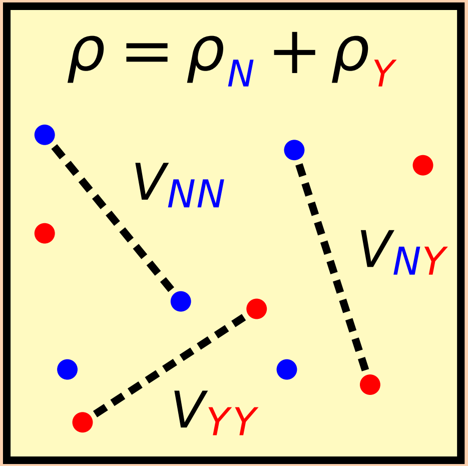
ROSAT image of *Puppis A*



↪ The only "laboratory" for $\rho_B \sim 10\rho_0$ in the Universe !

Need EOS of nuclear matter including hyperons and quarks

Hypernuclear Matter in the Neutron Star:



$N = qqq$: n, p (939 MeV)

$Y = qqs$: Λ^0 (1116 MeV)

Σ^{-0+} (1193 MeV)

qss: Ξ^{-0} (1318 MeV)

V_{NN}: Argonne, Bonn, Paris, ... potential

V_{NY}: Nijmegen (NSC89,NSC97,ESC08...)

V_{YY}: ? (no scattering data)

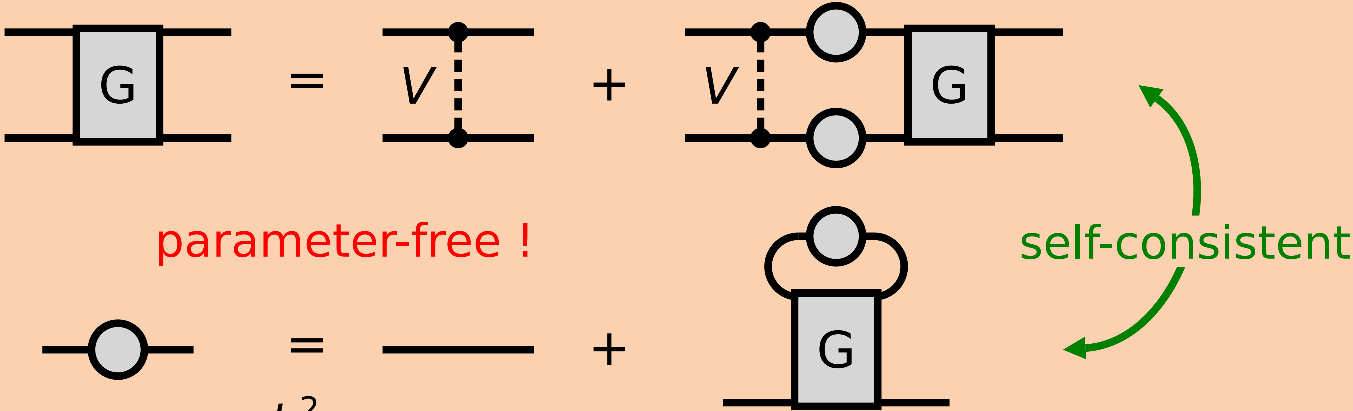
In free space weak decay: $Y \rightarrow N + \pi$ etc. ($c\tau \approx 8$ cm)

In dense nucleonic medium the decay is Pauli-blocked !

We need to compute the energy density of this system ...

Brueckner Theory of (Hyper)Nuclear Matter:

- Effective in-medium interaction G from potential V:



$$e_k = m + \frac{k^2}{2m} + U(k)$$

Results: binding energy $\epsilon(\rho_n, \rho_p, \rho_\Lambda, \rho_\Sigma) = \sum_i \sum_{k<k_F^{(i)}} \left[e_k^{(i)} - \frac{U_i(k)}{2} \right]$

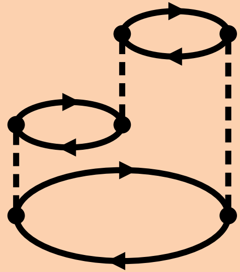
s.p. properties, cross sections, ...

K.A. Brueckner and J.L. Gammel; PR 109, 1023 (1958) for nuclear matter

Extension to hypernuclear matter ...

- Framework: Brueckner-Bethe-Goldstone hole-line expansion

$$\frac{E}{A} = \frac{3}{5}\frac{k_F^2}{2m} +$$ 

$$+$$ 
$$+ \cdots + \mathcal{O}(\kappa^4)$$

$$\approx \Big[\ 22 \quad - \quad 40 \quad + \quad 2 \quad + \quad ? \ \Big] \mathrm{MeV}$$

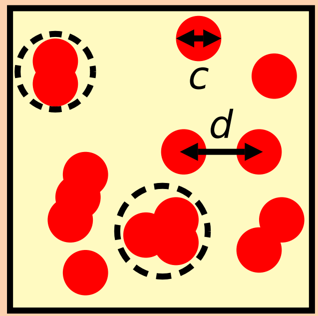
$\rho = \rho_0$, symmetric matter, V_{18} potential

- Expansion parameter $\kappa \sim \rho V_{\rm core} \approx 0.2$:

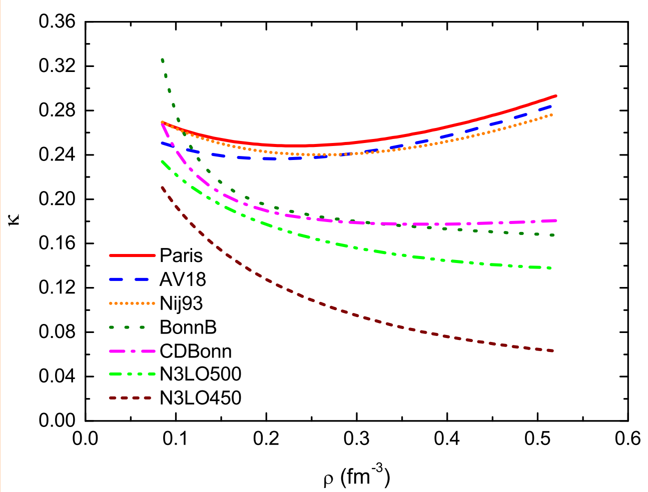
$$\kappa \equiv \frac{\sum_k n(k > k_F)}{\sum_{k<k_F}} = \rho \int d^3\boldsymbol{r} \left\langle |\eta(\boldsymbol{r})|^2 \right\rangle_{S,T} = N\frac{V_{\rm core}}{V} = \left(\frac{c}{d}\right)^3$$

η: $u - \phi$: defect function

- Hierarchy of n-body correlations/clusters within hard-core range c, avg. distance d:
- Justified for hard-core potentials



- Correlation parameter for different NN potentials:

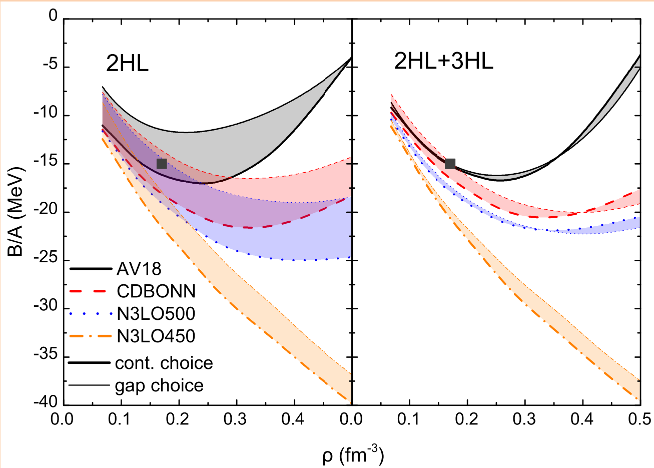


$$\kappa \approx \rho V_{\text{core}}(\rho)$$

Small up to large density

Hard vs. soft potentials

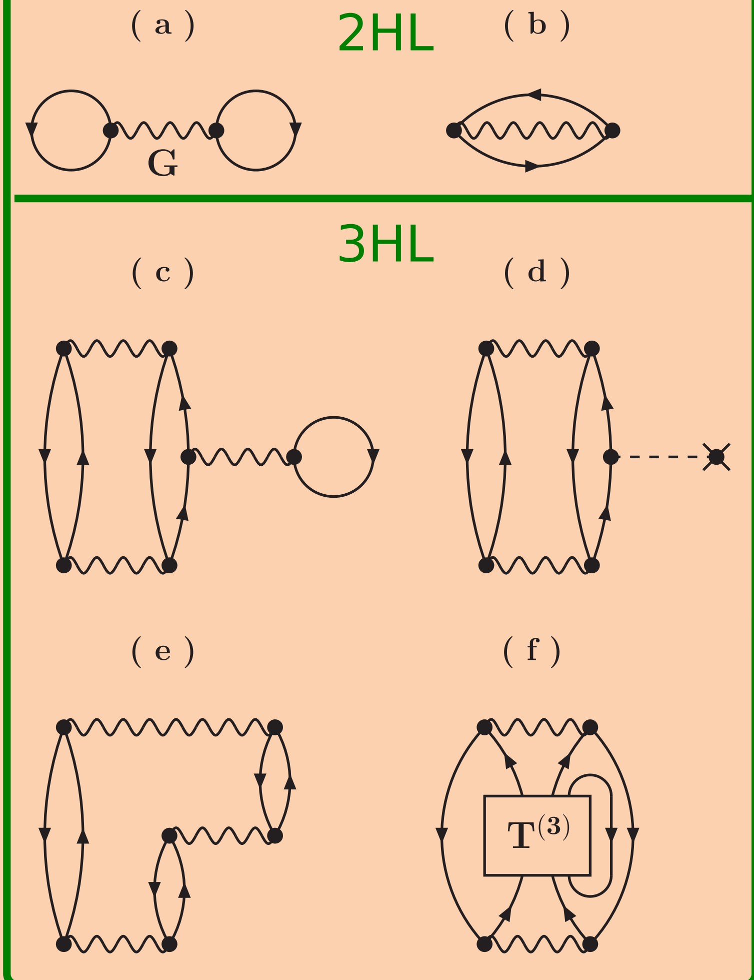
- Binding energy up to three hole lines:



$$B/A = T + E_2 + E_3$$

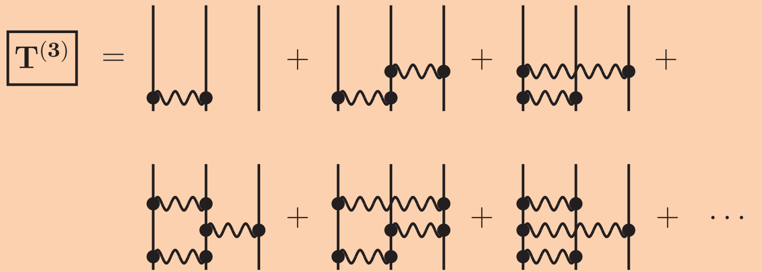
Hole-line expansion appears well converged, but misses slightly for AV18 the empirical saturation point of nuclear matter

Diagrams up to 3HL:

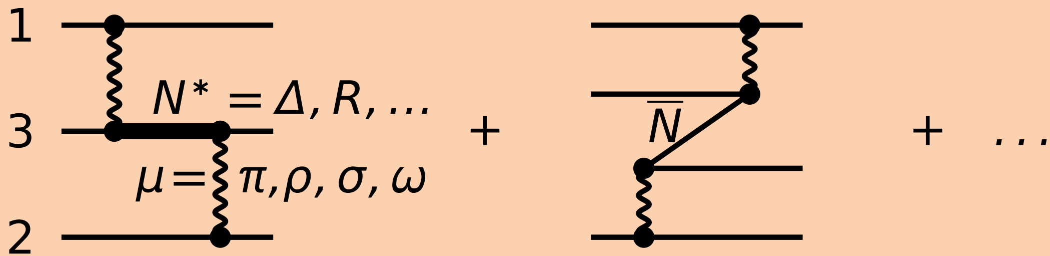


B.D. Day, PRC 24, 1203 (1981)

Fadeev calculation:

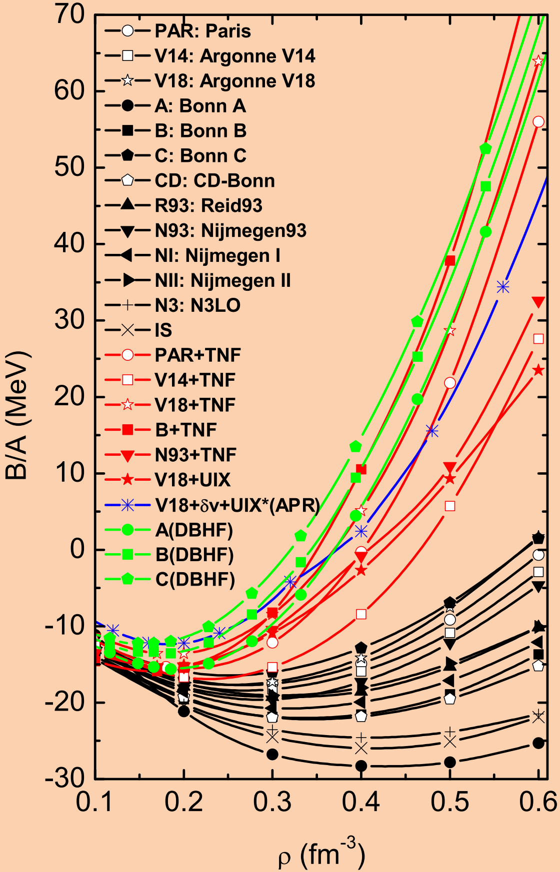


Three-Nucleon Forces:



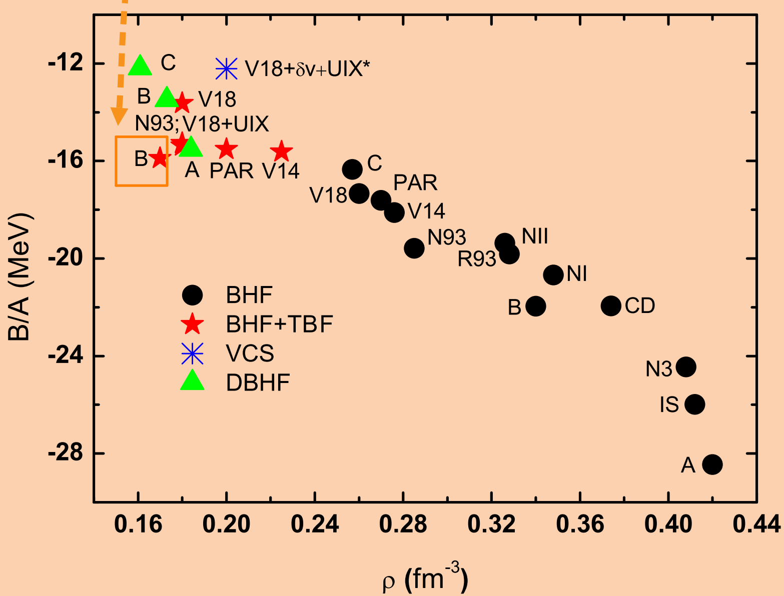
- Only small effect required [$\delta(B/A) \approx 1$ MeV at ρ_0]
- Model dependent, no final theory yet
- Use and compare microscopic and phenomenological TBF...
  - Microscopic TBF of P. Grangé et al., PRC 40, 1040 (1989):
    Exchange of $\pi, \rho, \sigma, \omega$ via $\Delta(1232)$, $R(1440)$, $N\bar{N}$
    Parameters compatible with two-nucleon potential (Paris,V_{18},...)
  - Urbana IX phenomenological TBF:
    Only 2π-TBF + phenomenological repulsion
    Fit saturation point

● BHF binding energy and saturation point of nuclear matter:



● Nuclear mass formula

Coester band:

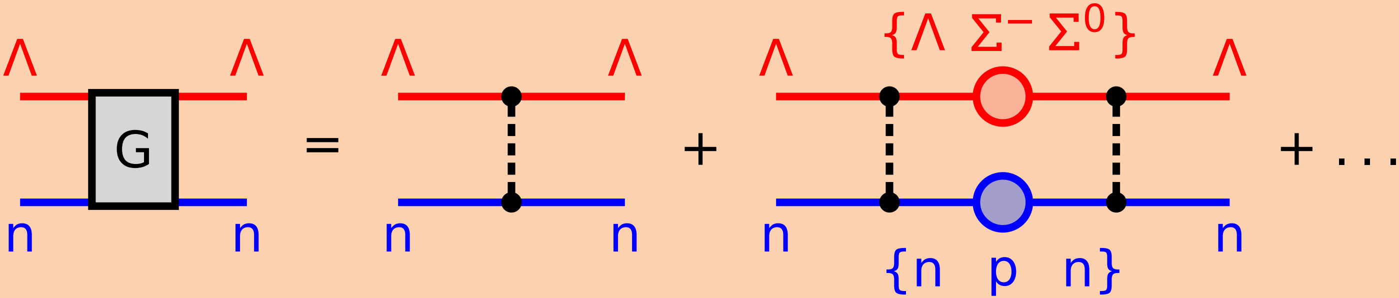


● Dependence on NN potential

● TBF needed to improve saturation properties

Include Hyperons:

- Technical difficulty: coupled channels:



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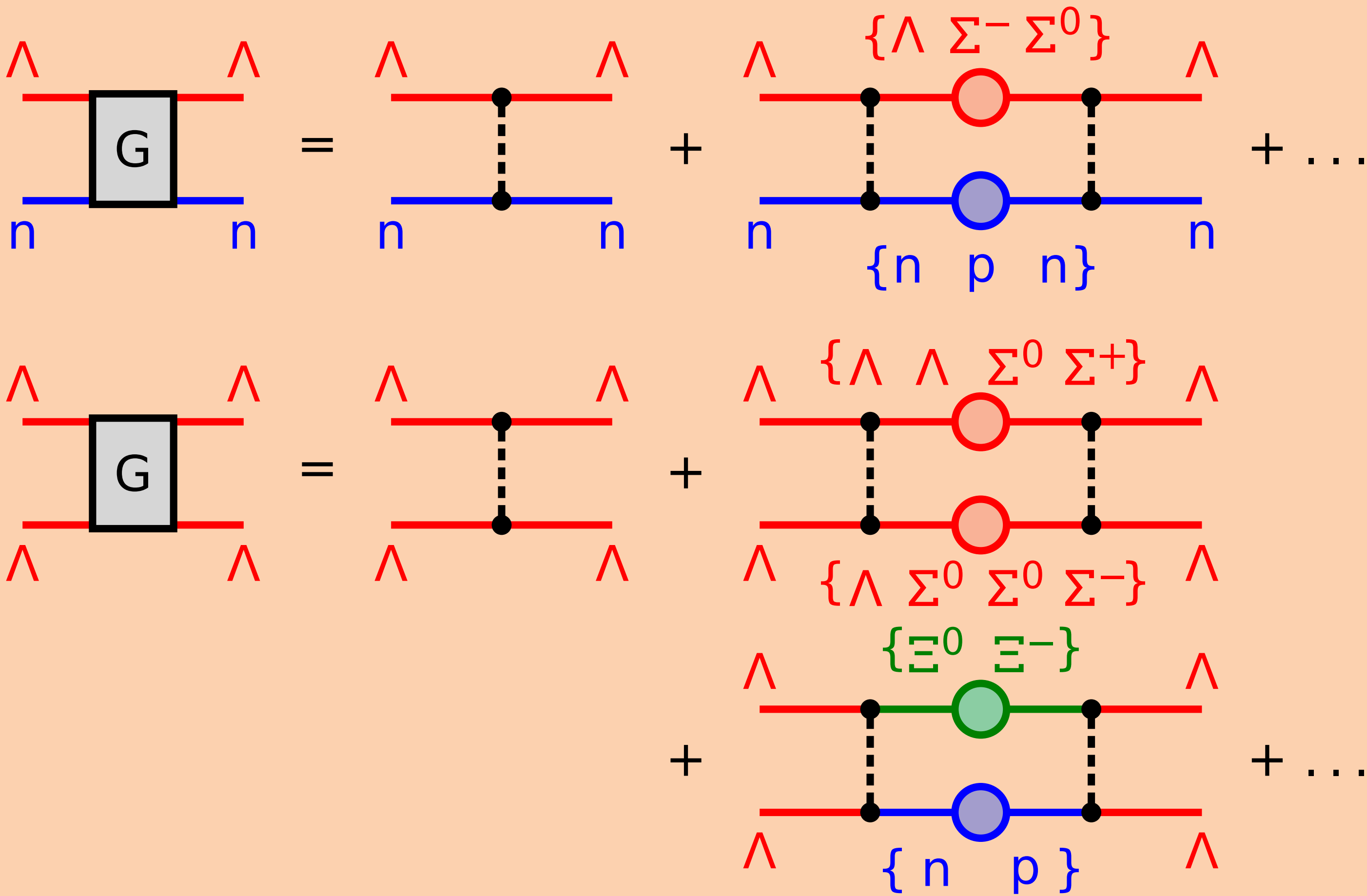
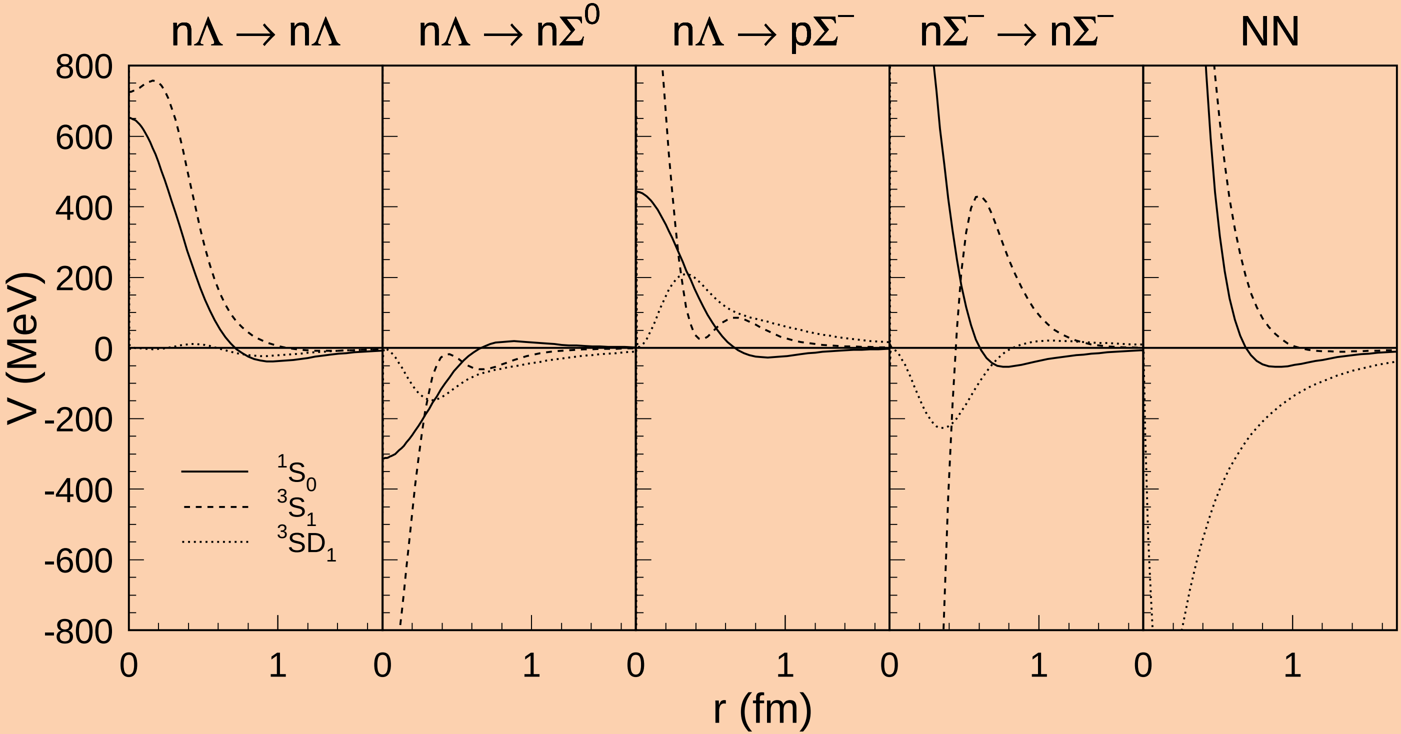


Example BHF Results: Input:

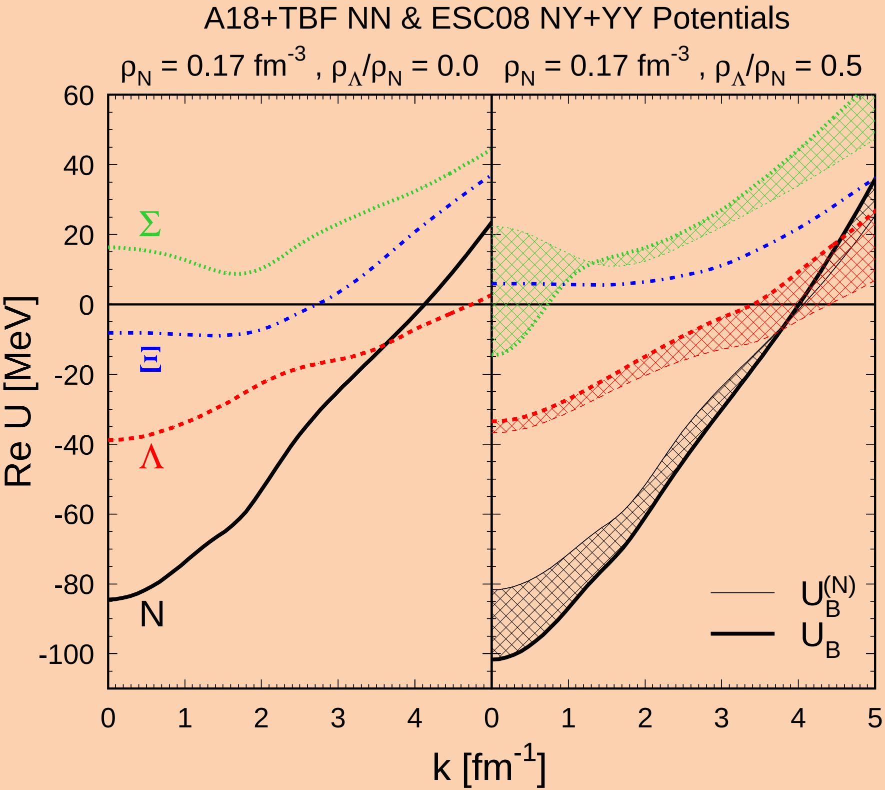
- Hyperon-nucleon potentials (NSC89) vs. Paris NN:



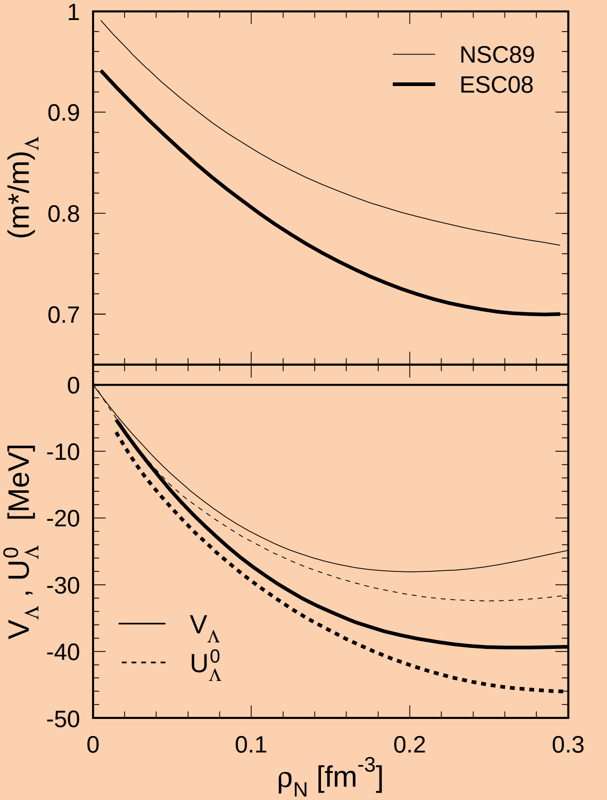
↪ "Soft" cores, Strong coupling $N\Lambda \longleftrightarrow N\Sigma$

Example BHF Results: Output:

s.p. potentials



Λ eff. mass & mean field



➥ Hyperons are weaker bound than nucleons

Neutron Stars

Vela pulsar

«Recipe» for Neutron Star Structure Calculation:

Brueckner results: $\boldsymbol{\epsilon}(\{\boldsymbol{\rho_i}\})\ ;\ i = n, p, e, \mu, \Lambda, \Sigma, u, d, s, \ldots$

Chemical potentials: $\mu_i = \frac{\partial \epsilon}{\partial \rho_i}$

Beta-equilibrium: $\mu_i = b_i \mu_n - q_i \mu_e$

Charge neutrality: $\sum_i x_i q_i = 0$

Composition: $x_i(\rho)$

Equation of state: $\boldsymbol{p}(\boldsymbol{\rho}) = \rho^2 \frac{d(\epsilon/\rho)}{d\rho}(\rho, x_i(\rho))$

TOV equations:

$$\frac{dp}{dr} = -\frac{Gm\epsilon}{r^2} \frac{(1 + p/\epsilon)(1 + 4\pi r^3 p/m)}{1 - 2Gm/r}$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon$$

Structure of the star: $\rho(r)$, $\boldsymbol{M}(\boldsymbol{R})$ etc.

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Chemical potentials: $\mu_i = \frac{\partial \epsilon}{\partial \rho_i}$

$\mu_e = \mu_\mu = \mu_n - \mu_p$

$\mu_{\Sigma^-} = 2\mu_n - \mu_p$

$\mu_{\Sigma^0} = \mu_\Lambda = \mu_n$

$\mu_{\Sigma^+} = \mu_p$

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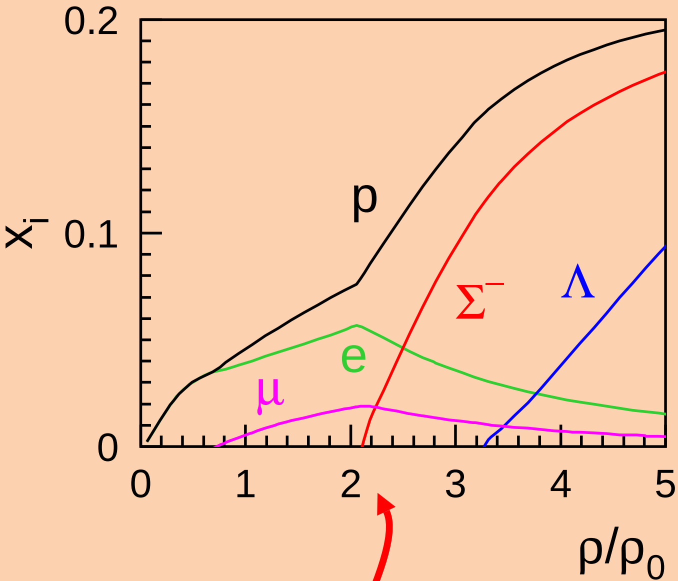
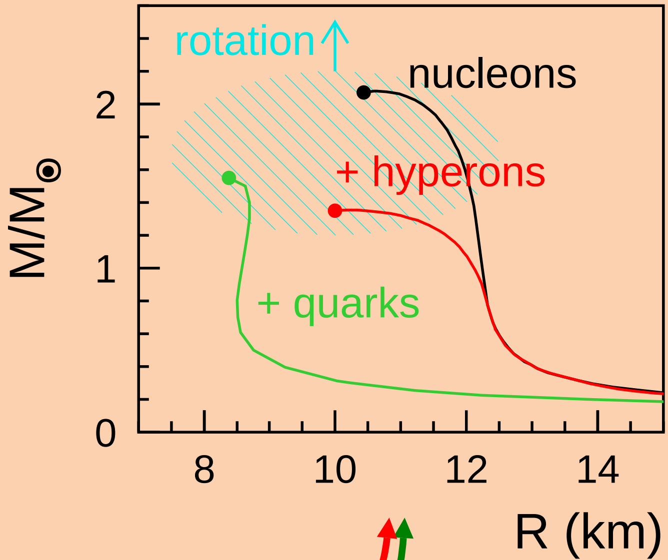
Structure of the star: $\rho(r)$, $\boldsymbol{M}(\boldsymbol{R})$ etc.

• Generic implications for EOS and stellar structure:



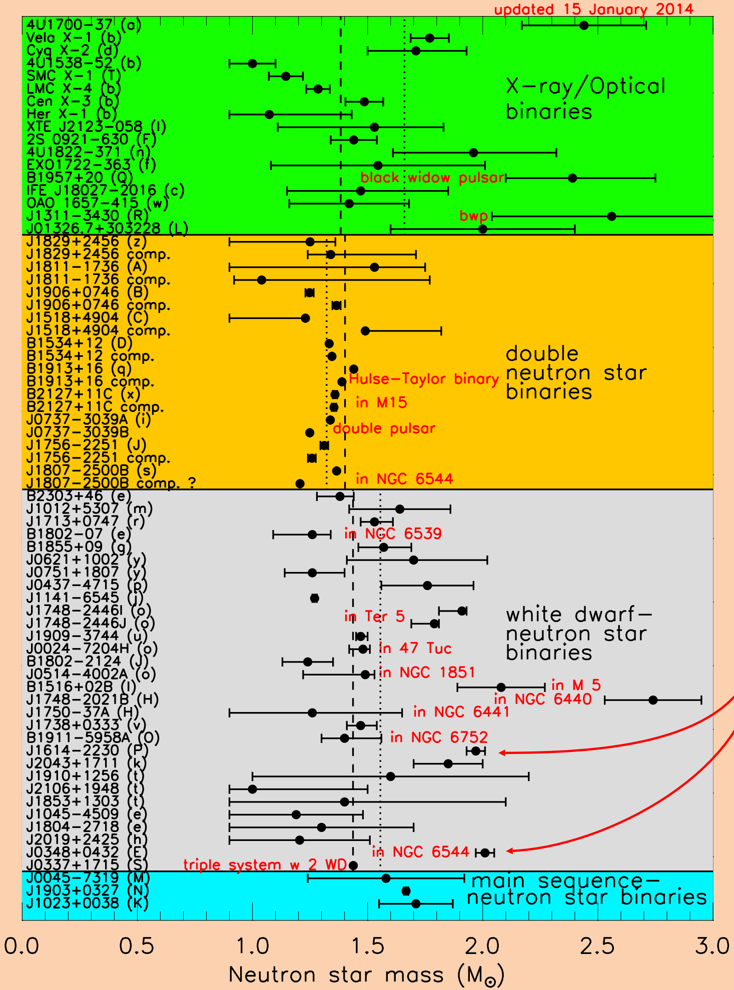


- Hyperon onset occurs at $\rho \sim 2 \ldots 3\, \rho_0$
- Softer EOS
- NS structure including hyperons

  . . . and including quark matter


Data ?
Vela pulsar

Observational Data: Masses



Courtesy of J. Lattimer

The heaviest neutron stars:

Recent: $\sim 1.97 M_\odot$ (Nature 09466)

$\sim 2.01 M_\odot$ (Science 340)

No combined (M, R) measurements!

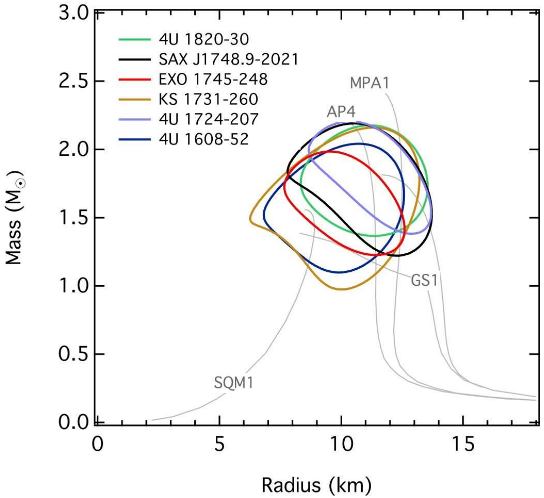
(Would practically fix the EOS)

Observational Data: Radii

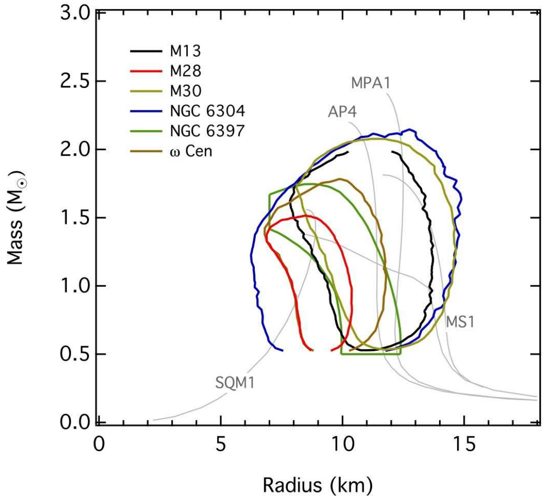
Neutron Star Radius Results

F. Özel et al., APJ820, 28 (2016)

Six Burst Sources



Six qLMXBs

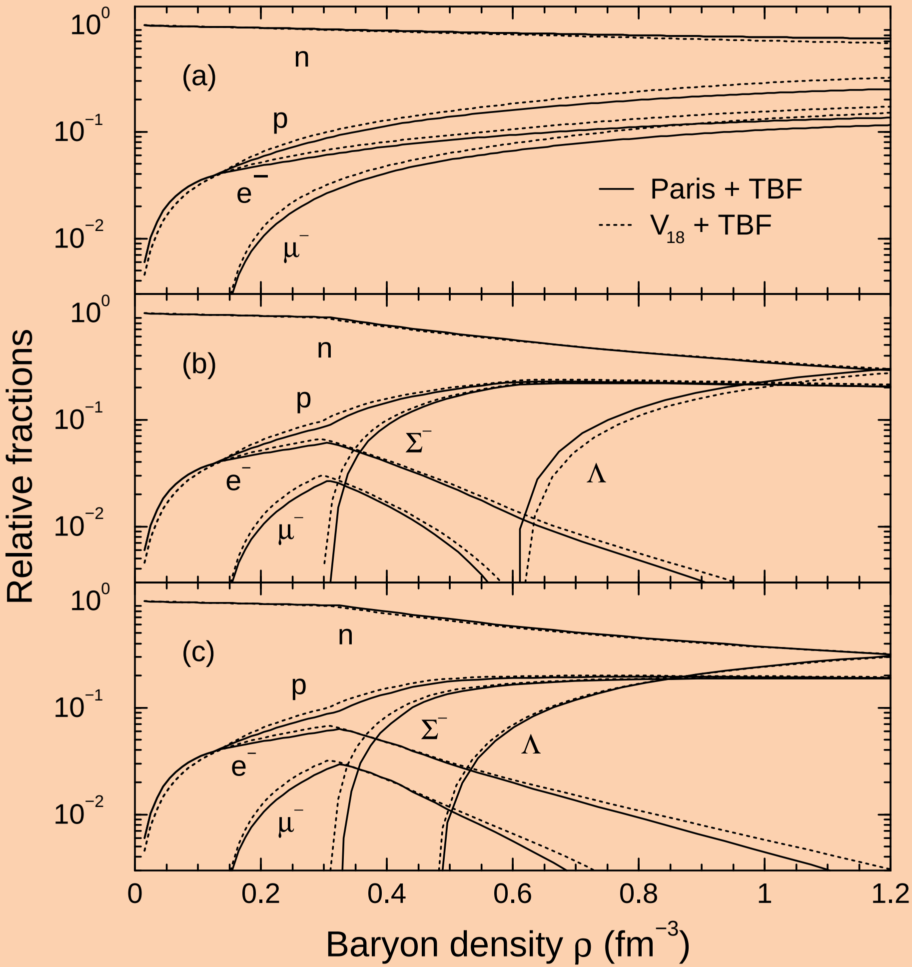


Ozel et al. 2015

➥ The measurement is difficult: currently no accurate results


J0617 in IC 443
BHF Results ...

Composition of neutron star matter:

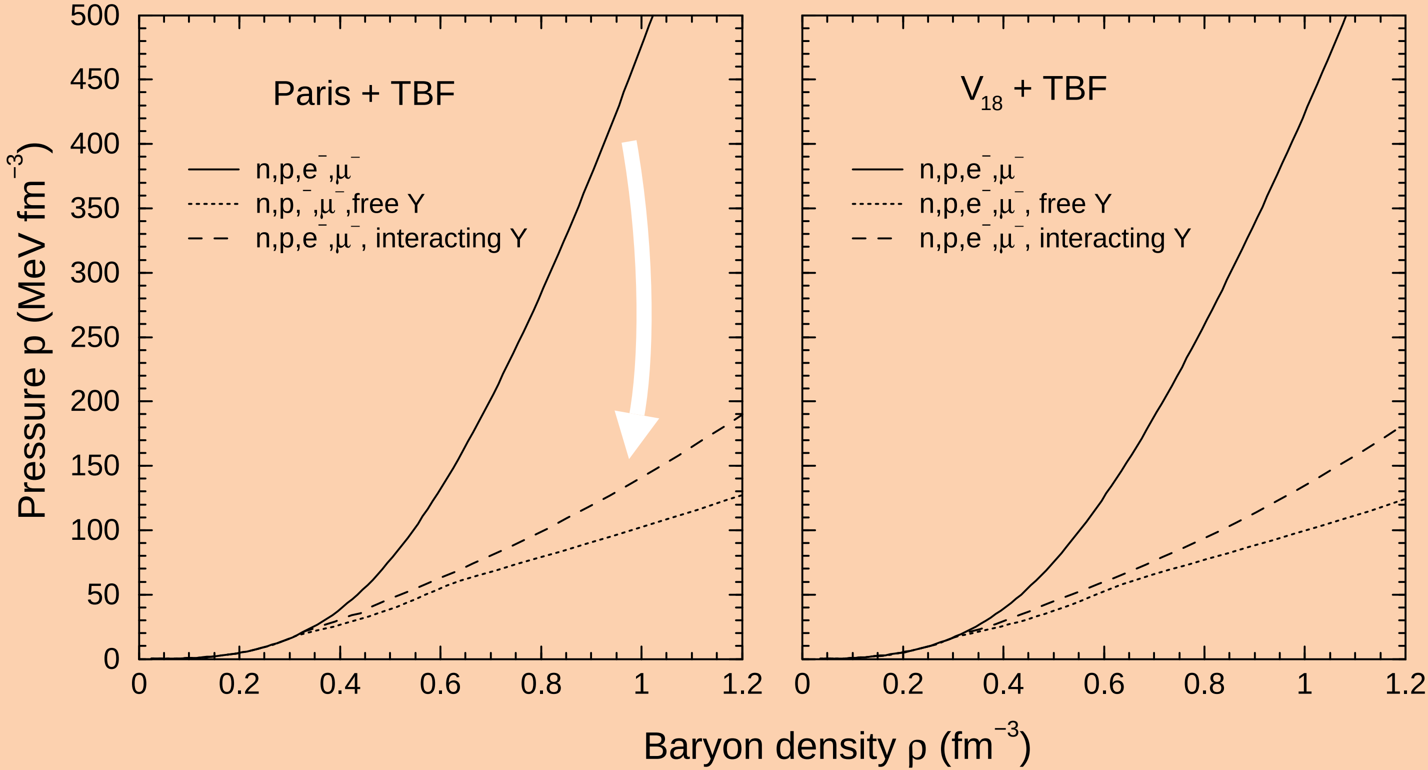


No hyperons

Free hyperons

Interacting hyperons
(Σ^- repulsive, Λ attractive)
NY interaction determines
Y onset

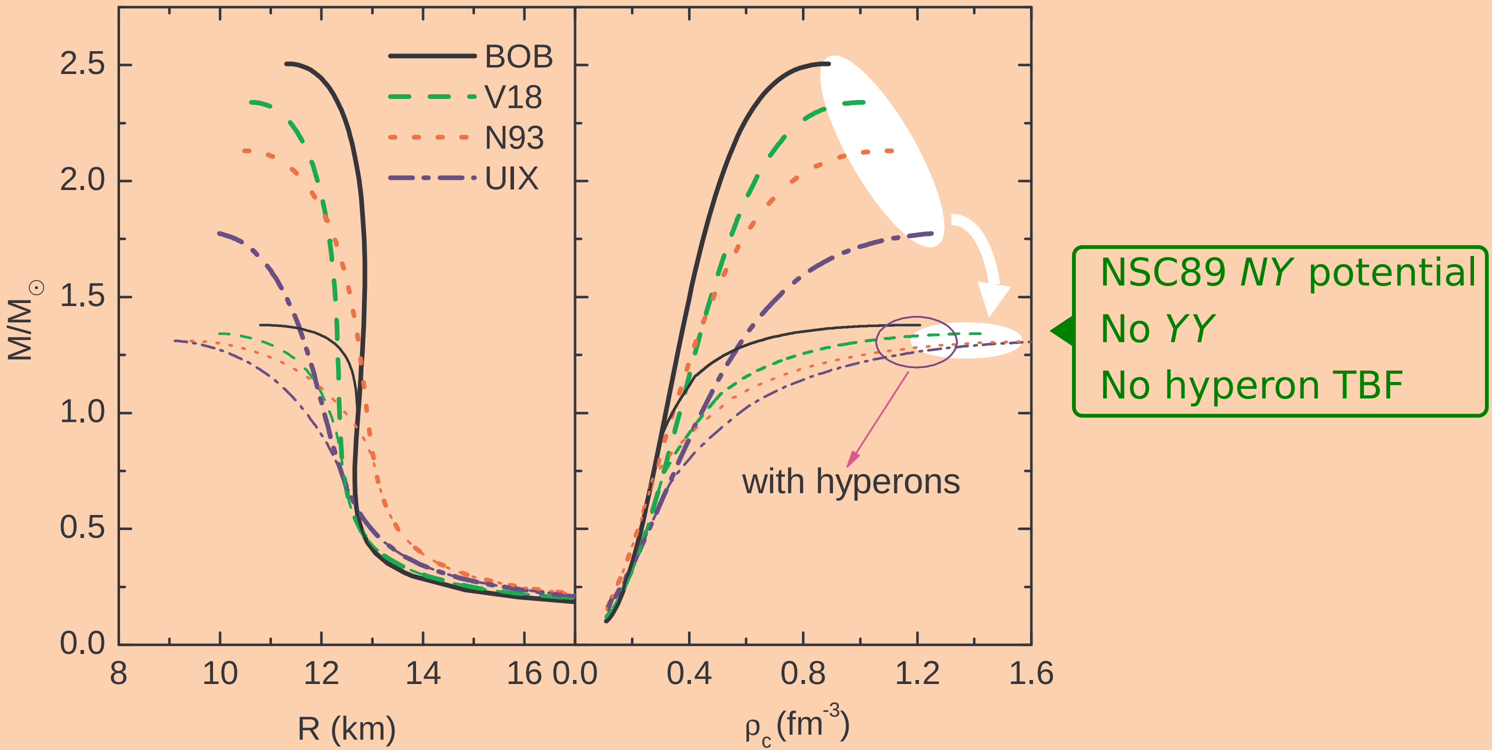
• EOS of neutron star matter:



↪ Strong softening due to hyperons !

(More Fermi seas available)

- Mass-radius relations with different nucleonic TBF:



↪ Large variation of M_{max} with nucleonic TBF

Self-regulating softening due to hyperon appearance

(stiffer nucleonic EOS → earlier hyperon onset)

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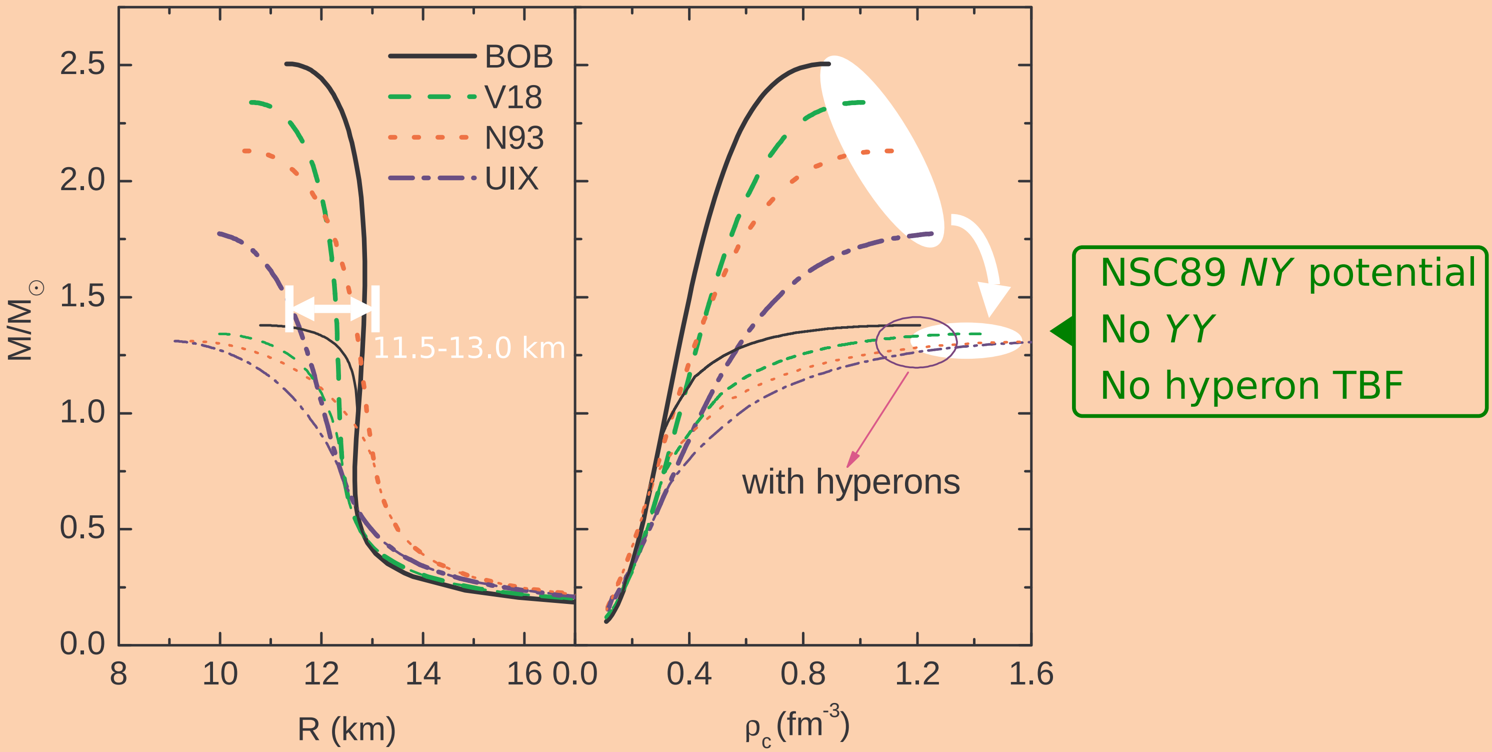

↪ Large variation of M_{max} with nucleonic TBF
Self-regulating softening due to hyperon appearance
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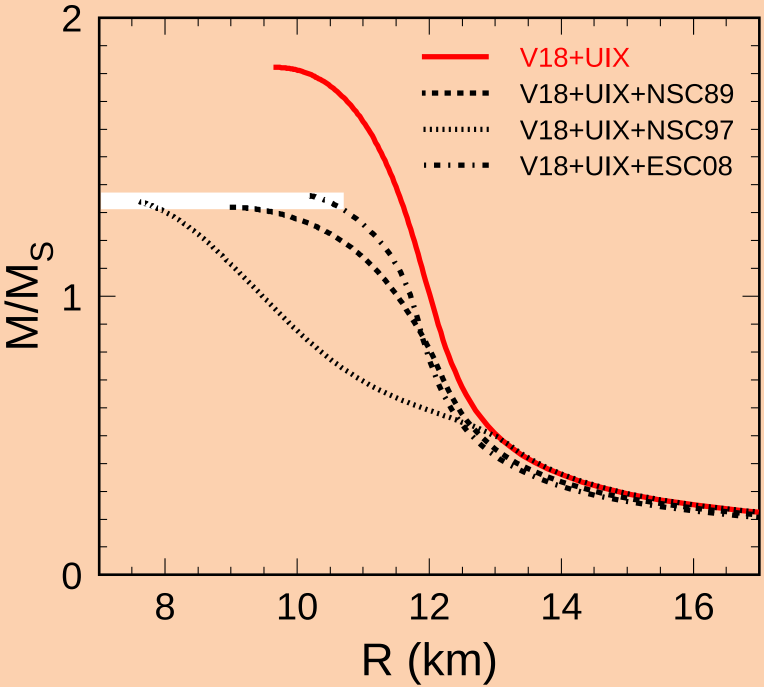
- Mass-radius relations using different NY, YY potentials:

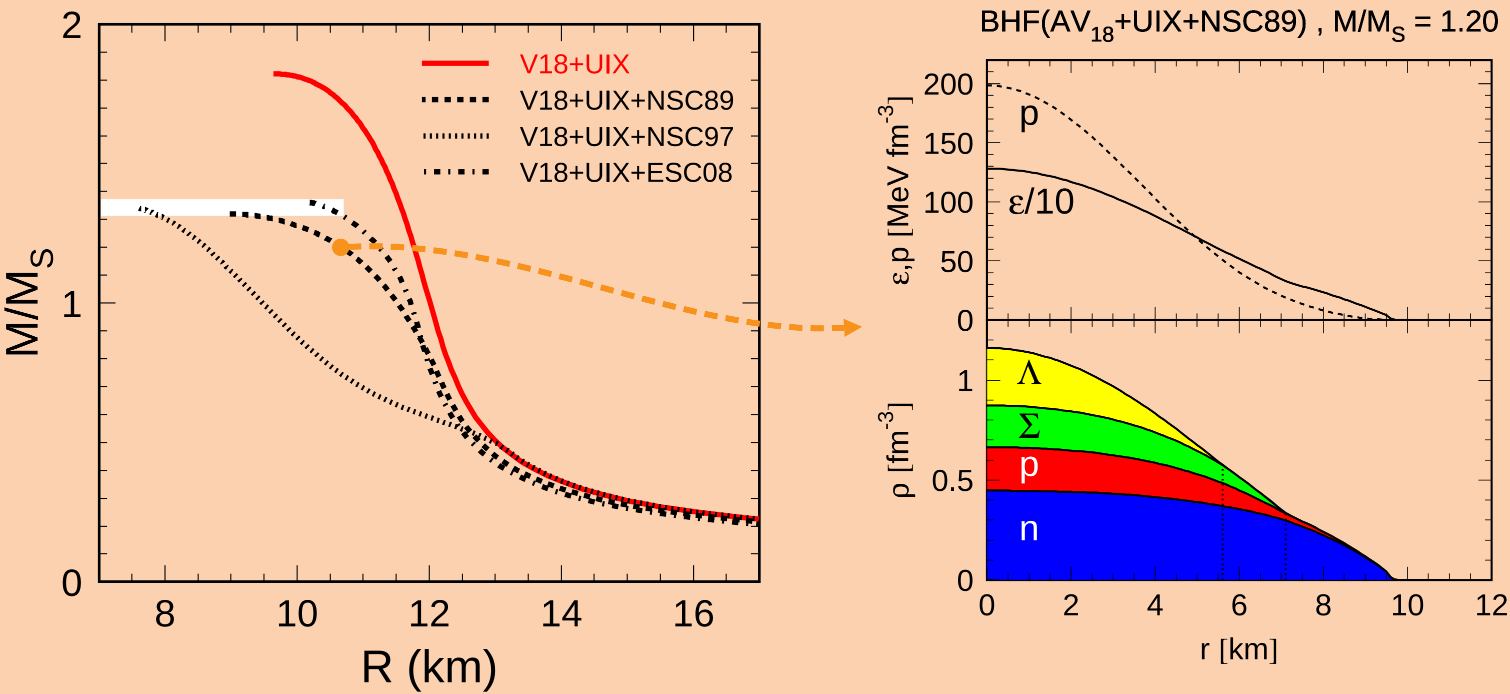


Maximum mass independent of potentials !

Maximum mass too low ($< 1.4\,M_\odot$) !

Proof for "quark" matter inside neutron stars ?

● Mass-radius relations using different NY, YY potentials:



Maximum mass independent of potentials !

Maximum mass too low ($< 1.4\,M_\odot$) !

Proof for "quark" matter inside neutron stars ?

Quark Matter EOS of Dense Matter:

- Problem: No “exact” results from QCD:

  Large theoretical uncertainties, limited predictive power

- Current strategy:

  Use available eff. quark models (MIT, NJL, CDM, DSM, ...)
  in combination with the hadronic EOS

- An important constraint (from heavy ion collisions):

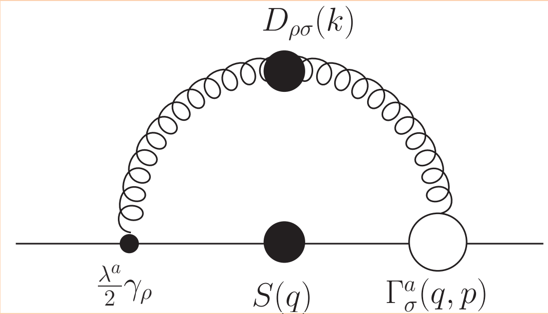
  In symmetric matter phase transition not below $\approx 3\rho_0$

↪ E.g., the simplest (MIT) quark model requires
a density-dependent bag “constant”:

$$\epsilon_Q = B + \epsilon_{\rm kin} + \alpha_s \times \ldots$$

$$B(\rho) = B_\infty + (B_0 - B_\infty) \exp\left[-\beta \left(\rho/\rho_0\right)^2\right]$$

A more sophisticated approach: Dyson-Schwinger model:

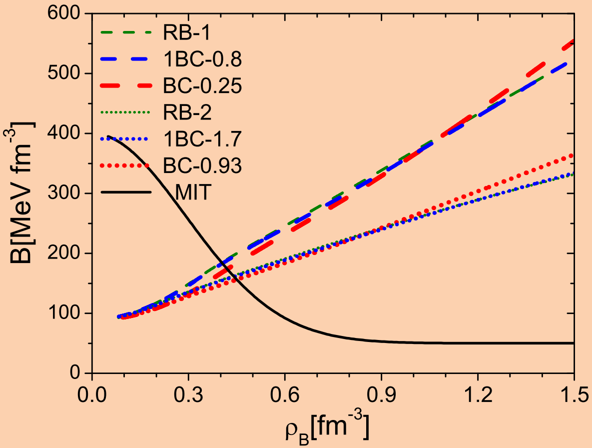


H. Chen et al.,
PRD 84, 105023 (2011)
PRD 86, 045006 (2012)
PRD 91, 105002 (2015)
EPJA 52, 291 (2016)
PRD 96, 043008 (2017)

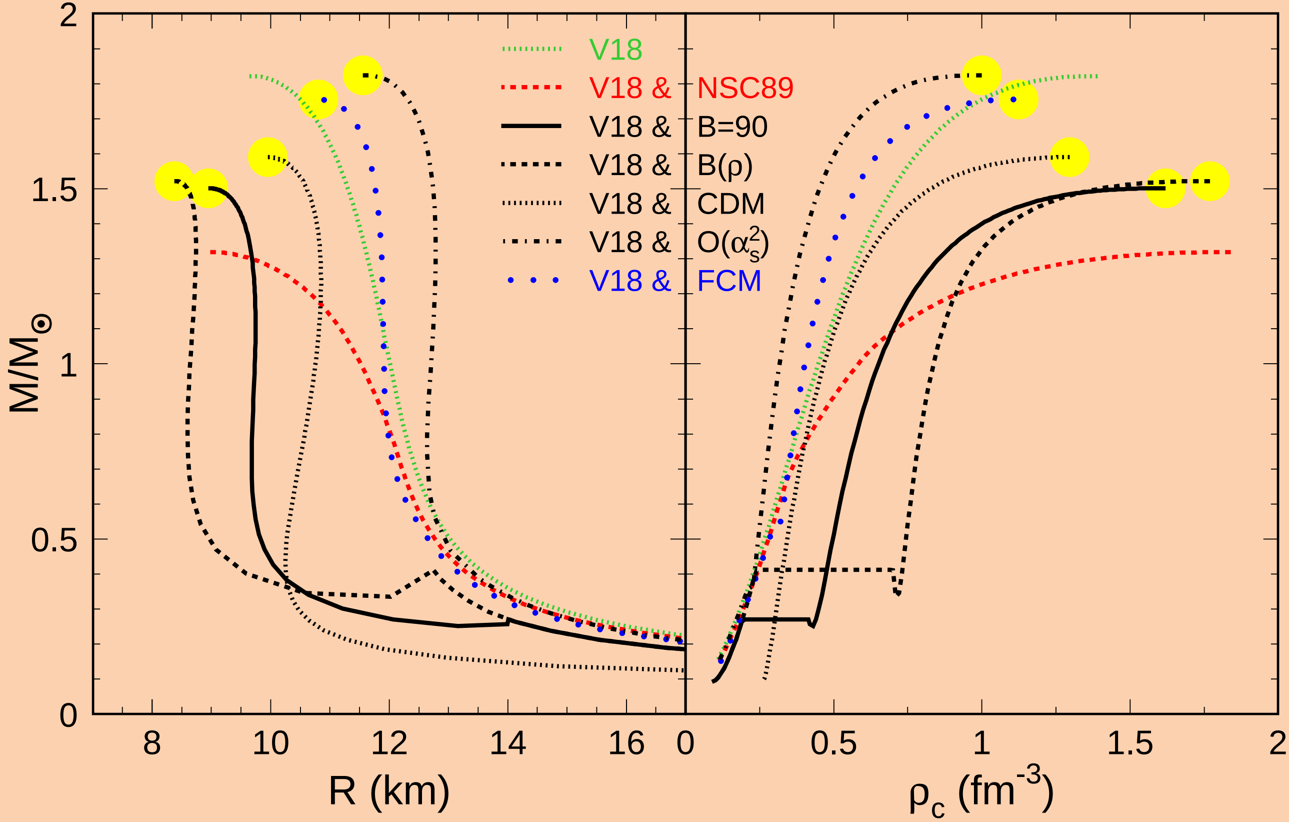
$$\Sigma(p) = \int \frac{d^4 p}{(2\pi)^4} S(q) \frac{\lambda^a}{2} \gamma_\rho D_{\rho\sigma}(k) \Gamma^a_\sigma(q,p)$$

Compute the quark propagator $S(q)$ from QCD

Allows to *calculate*
the bag constant:



• Different quark EOS's: bag models, color dielectric model:

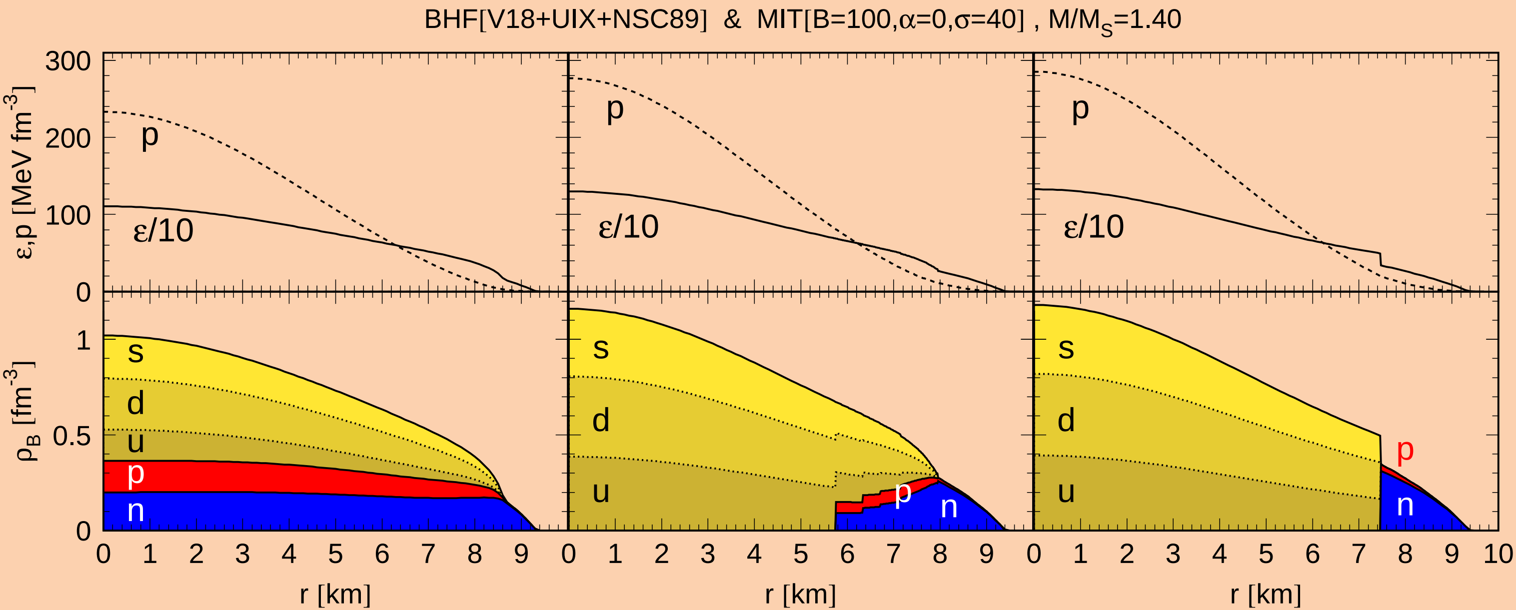


NJL, FCM, Dyson-Schwinger models: hyperons prevent phase transition

↪ Maximum masses: 1.5...1.9 $M_\odot$, Radii are different !

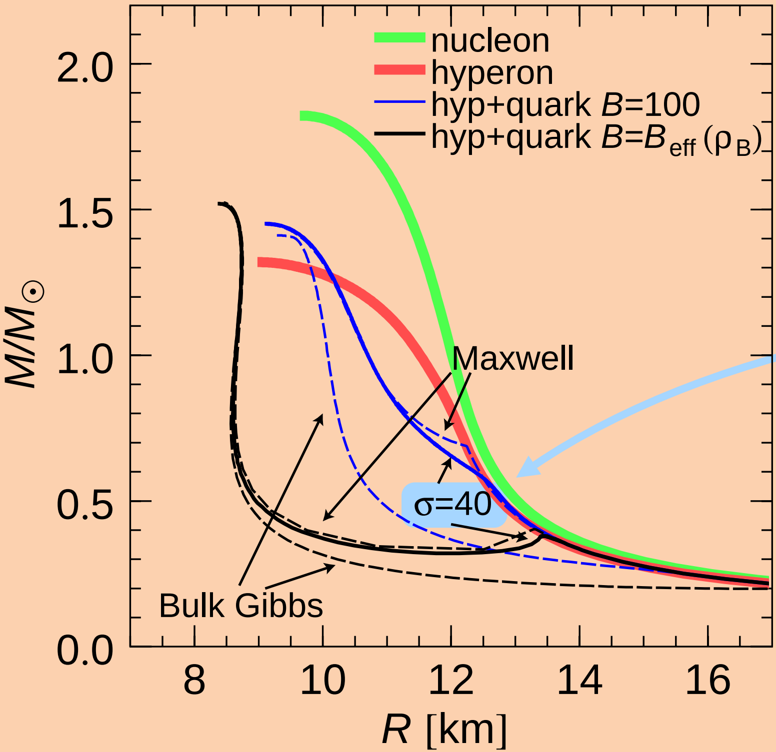
• Details of the phase transition: neutron star profiles:

Bulk Gibbs   Screened Gibbs   Maxwell

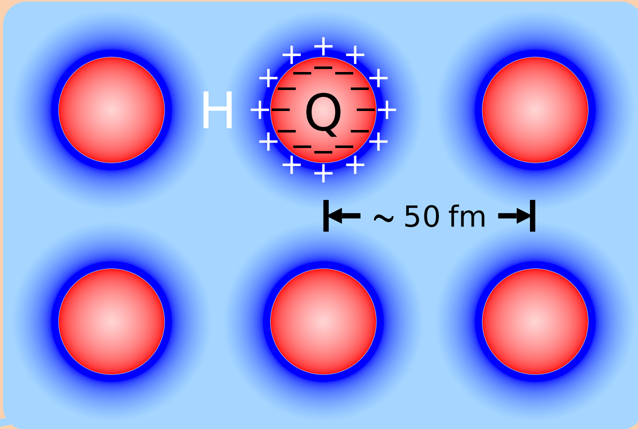


- Hyperons replaced by strange quark matter
- Very different possible internal structures
- Surface tension + screening enforce ‘quasi’ Maxwell construction (exact for $\sigma \gtrsim 70$ MeV/fm^2)

- Mass-radius relations with different hadron-quark phase transition constructions:



e.m. interaction vs. surface tension :



- Screened Gibbs constr. very close to Maxwell construction
- Maximum mass independent of phase transition

Summary:

- Neutron star physics probes the 4 fundamental interactions:
  - Gravitation: Densest object in the Universe
  - Strong: Nuclear EOS
  - Weak: Beta-equilibrium of matter, Neutrino physics
  - EM: Charge-neutrality, Mixed-phase structures, Crust

Conclusions:

- Hyperons cannot be ignored !
- BHF EOS with hyperons predicts M_{max} not above $\sim 1.7\, M_\odot$
- Need “quark matter” to reach higher masses
- Currently $M_{max} \approx 1.9\, M_\odot$ for hybrid stars in this approach

However:

However:

We do not know dark matter.

However:

We do not know dark matter.

We do not know dark energy.

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We do not know dark matter.

We do not know dark energy.

Do we know GR at $10\,\rho_0$?