Seminar at Chiba Institute of Technology
November 24th, 2018, Shin-Narashino Campus, Chiba

Time-Dependent Density Functional Theory for Superfluid Dynamics in the Neutron Star Crust

Kazuyuki Sekizawa

Center for Interdisciplinary Research
Institute for Research Promotion, Niigata University, Japan

自己紹介（略歴）

1988年: 埼玉県出身

2006年3月: 埼玉県立伊奈学園総合高等学校 卒業

2010年3月: 東京理科大学 物理学科 卒業

2012年3月: 筑波大学大学院 博士前期課程 修了

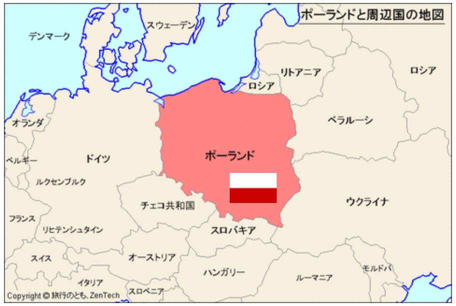
2015年3月: 筑波大学大学院 博士後期課程 修了（理学博士）

2015年4月~2017年8月: ポーランド・ワルシャワ工科大学（博士研究員）

2017年9月~2017年12月: 米国シアトル・ワシントン大学（博士研究員）

2018年1月~現在: 新潟大学 研究推進機構 超域学術院（特任助教・テニュアトラック）

専門は「原子核理論」



My recent studies: TDDFT for Nuclear Many-Body Problem

Multinucleon transfer and quasifission processes in heavy-ion reactions

Tsukuba, Tohoku, Kyushu

PRC**96**(2017)041601(R),
PRC**96**(2017)014615,
PRC**93**(2016)054616,
PRC**90**(2014)064614,
PRC**88**(2013)014614


K. Yabana


K. Hagino


K. Washiyama

BARC, Mumbai, India

B.J. Roy and his coworkers

B.J. Roy *et al.*,
PRC**97**(2018)034603,
PRC**92**(2015)024603

Dissipation and fluctuation mechanism in deep-inelastic collisions

ANU, Canberra, Australia
Texas A&M, USA

E. Williams, K. Sekizawa, D. Hinde *et al.*,
PRL**120**(2018)022501


E. Williams


D.J. Hinde


M. Dasgupta


C. Simenel


A. Wakhle

Superfluid dynamics in fermionic systems

WUT, Warsaw, Poland

PRL**120**(2018)253002
PRL**119**(2017)042501
PRL**117**(2016)232701


G. Wlazłowski

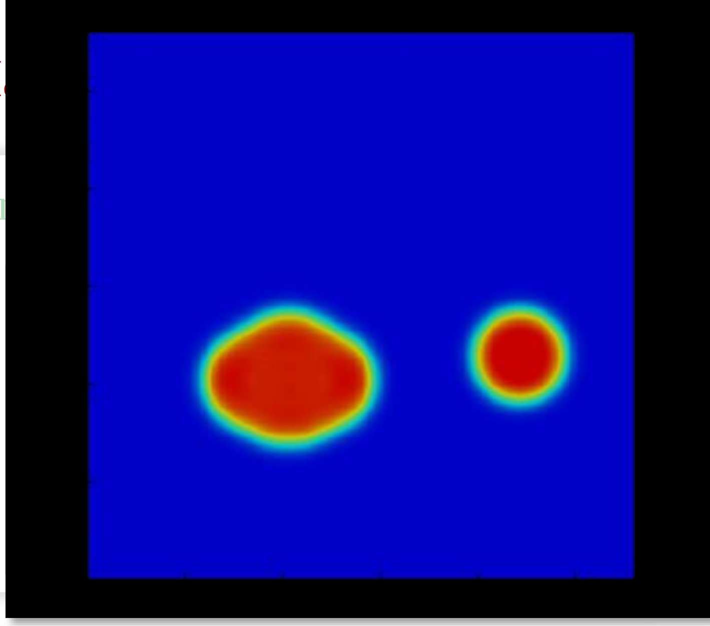

P. Magierski

UW, Seattle, USA


A. Bulgac


M.M. Forbes

My recent studies: TDDFT for Nuclear M

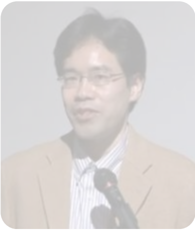


Multinucleon transfer and quasifission processes in heavy-io

Tsukuba, Tohoku, Kyushu

PRC**96**(2017)041601(R),
PRC**96**(2017)014615,
PRC**93**(2016)054616,
PRC**90**(2014)064614,
PRC**88**(2013)014614

K. Yabana



K. Hagino

K. Washiyama

Dissipation and fluctuation mechanism in deep-inelastic collisions

ANU, Canberra, Australia
Texas A&M, USA

E. Williams, K. Sekizawa, D. Hinde *et al.*,
PRL**120**(2018)022501

Today's talk

E. Williams

D.J. Hinde

M. Dasgupta

C. Simenel

A. Wakhle

Superfluid dynamics in fermionic systems

WUT, Warsaw, Poland

PRL**120**(2018)253002
PRL**119**(2017)042501
PRL**117**(2016)232701

G. Wlazłowski

P. Magierski

UW, Seattle, USA

A. Bulgac

M.M. Forbes

Outline

1. **Method:** DFT, TDDFT, and TDSLDA

2. **Dynamics of quantized vortices within TDSLDA:**
   Topological excitations in spin-imbalanced superfluid Fermi gas

3. **Application to Neutron Star physics:**
   Vortex-nucleus interaction in the neutron star crust

4. **Summary**

DFT, TDDFT, and TDSLDA

A theory which may provide the exact solution

Equivalent!
(w/ special EDF)

$$\hat{H}\Psi(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_N)=E\Psi(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_N)$$

Kohn-Sham equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2+v_{\mathrm{KS}}[\rho(\boldsymbol{r})]\right]\phi_i(\boldsymbol{r})=\varepsilon_i\,\phi_i(\boldsymbol{r})$$

EDF — This is the key!

$$v_{\mathrm{KS}}[\rho(\boldsymbol{r})]=\frac{\delta\mathcal{E}[\rho]}{\delta\rho}\qquad \rho(\boldsymbol{r})=\sum_{i=1}^{N}|\phi_i(\boldsymbol{r})|^2$$

Existence was proved, but "shape" is unknown :(

Energy can also be written as a functional of density

$$E[\rho]=\langle\Psi[\rho]|\hat{H}|\Psi[\rho]\rangle$$

w.f. is a functional of density

P. Hohenberg and W. Kohn, Phys. Rev. **136**, B864 (1964)

DFT in a nutshell (1/2) - Hohenberg-Kohn theorem

Assumption:

Two different wave functions Ψ and Ψ' provide the same one-body density $\rho(\boldsymbol{r})$

$$\hat{H} = \hat{T} + \hat{W} + \hat{V} \qquad \hat{H}' = \hat{T} + \hat{W} + \hat{V}'$$

$$\hat{H}|\Psi\rangle = E|\Psi\rangle \qquad \hat{H}'|\Psi'\rangle = E'|\Psi'\rangle$$

Now, one finds:

$$\begin{aligned} E &= \langle\Psi|\hat{H}|\Psi\rangle < \langle\Psi'|\hat{H}|\Psi'\rangle \\ &= \langle\Psi'|\hat{H}'|\Psi'\rangle + \langle\Psi'|\hat{V} - \hat{V}'|\Psi'\rangle \\ &= E' + \int \rho(\boldsymbol{r})\big[v(\boldsymbol{r}) - v'(\boldsymbol{r})\big]\,d\boldsymbol{r} \qquad \cdots \text{(a)} \end{aligned}$$

However, one also gets:

$$\begin{aligned} E' &= \langle\Psi'|\hat{H}'|\Psi'\rangle < \langle\Psi|\hat{H}'|\Psi\rangle \\ &= E + \int \rho(\boldsymbol{r})\big[v'(\boldsymbol{r}) - v(\boldsymbol{r})\big]\,d\boldsymbol{r} \qquad \cdots \text{(b)} \end{aligned}$$

(a) + (b) $\Rightarrow$ $E + E' < E + E'$?

P. Hohenberg and W. Kohn, Phys. Rev. **136**, B864 (1964)

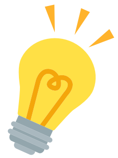
Assumption:

Two different wave functions Ψ and Ψ' provide the same one-body density $\rho(\boldsymbol{r})$

$$\hat{H} = \hat{T} + \hat{W} + \hat{V} \qquad \hat{H}' = \hat{T} + \hat{W} + \hat{V}'$$

$$\hat{H}|\Psi\rangle = E|\Psi\rangle \qquad \hat{H}'|\Psi'\rangle = E'|\Psi'\rangle$$

Our assumption was wrong!



Now, one finds:

"reductio ad absurdum"

$$\begin{aligned} E &= \langle\Psi|\hat{H}|\Psi\rangle < \langle\Psi'|\hat{H}|\Psi'\rangle \\ &= \langle\Psi'|\hat{H}'|\Psi'\rangle + \langle\Psi'|\hat{V} - \hat{V}'|\Psi'\rangle \\ &= E' + \int \rho(\boldsymbol{r})\big[v(\boldsymbol{r}) - v'(\boldsymbol{r})\big]\, d\boldsymbol{r} \qquad \cdots \text{(a)} \end{aligned}$$

However, one also gets:

$$\begin{aligned} E' &= \langle\Psi'|\hat{H}'|\Psi'\rangle < \langle\Psi|\hat{H}'|\Psi\rangle \\ &= E + \int \rho(\boldsymbol{r})\big[v'(\boldsymbol{r}) - v(\boldsymbol{r})\big]\, d\boldsymbol{r} \qquad \cdots \text{(b)} \end{aligned}$$

(a) + (b) $\Rightarrow$ $E + E' < E + E'$?

P. Hohenberg and W. Kohn, Phys. Rev. **136**, B864 (1964)

(1st) HK theorem:

There is a one-to-one correspondence between Ψ and ρ $\quad \Psi \leftrightarrow \rho(\boldsymbol{r})$

Now, one finds:

"reductio ad absurdum"

$$E = \langle \Psi | \hat{H} | \Psi \rangle < \langle \Psi' | \hat{H} | \Psi' \rangle$$
$$= \langle \Psi' | \hat{H}' | \Psi' \rangle + \langle \Psi' | \hat{V} - \hat{V}' | \Psi' \rangle$$
$$= E' + \int \rho(\boldsymbol{r}) \big[v(\boldsymbol{r}) - v'(\boldsymbol{r}) \big] \, d\boldsymbol{r} \quad \cdots \text{(a)}$$

However, one also gets:

$$E' = \langle \Psi' | \hat{H}' | \Psi' \rangle < \langle \Psi | \hat{H}' | \Psi \rangle$$
$$= E + \int \rho(\boldsymbol{r}) \big[v'(\boldsymbol{r}) - v(\boldsymbol{r}) \big] \, d\boldsymbol{r} \quad \cdots \text{(b)}$$

(a) + (b) $\Rightarrow \quad E + E' < E + E' \quad ?$

P. Hohenberg and W. Kohn, Phys. Rev. **136**, B864 (1964)

KS scheme makes DFT computationally solvable

(2nd) HK theorem:

$\hat{H} \overset{\text{SE}}{\longleftrightarrow} \Psi \overset{\text{HK}}{\longleftrightarrow} \rho(\boldsymbol{r})$

- Ψ can be written as a functional of ρ: $\Psi = \Psi[\rho]$
- Any observable can be written as a functional of ρ: $\mathcal{O}[\rho] = \langle \Psi[\rho] | \hat{\mathcal{O}} | \Psi[\rho] \rangle$
- **Minimization of EDF with respect to ρ will provide $E_{\text{g.s.}}$ and $\rho_{\text{g.s.}}$:**

$$E_{\text{g.s.}} = E[\rho_{\text{g.s.}}] = \min_{\rho} \left[\min_{\Psi \to \rho} E[\rho] \right]$$

$$E[\rho] = \langle \Psi[\rho] | \hat{H} | \Psi[\rho] \rangle \; : \; \textbf{EDF}$$

Kohn-Sham scheme:

W. Kohn and L.J. Sham, Phys. Rev. **140**, A1133 (1965)

✓ According to the HK theorem, **there exists an EDF associated with the *exact* solution Ψ**

- **The solution of *N* coupled nonlinear equations can be equivalent to the exact solution**

$$\hat{h}[\rho]\phi_i(\boldsymbol{r}) = \varepsilon_i \phi_i(\boldsymbol{r}) \; : \; \textbf{Kohn-Sham equation}$$

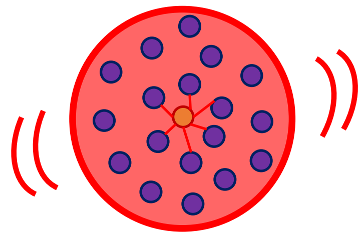
(often called Hartree-Fock eq.)

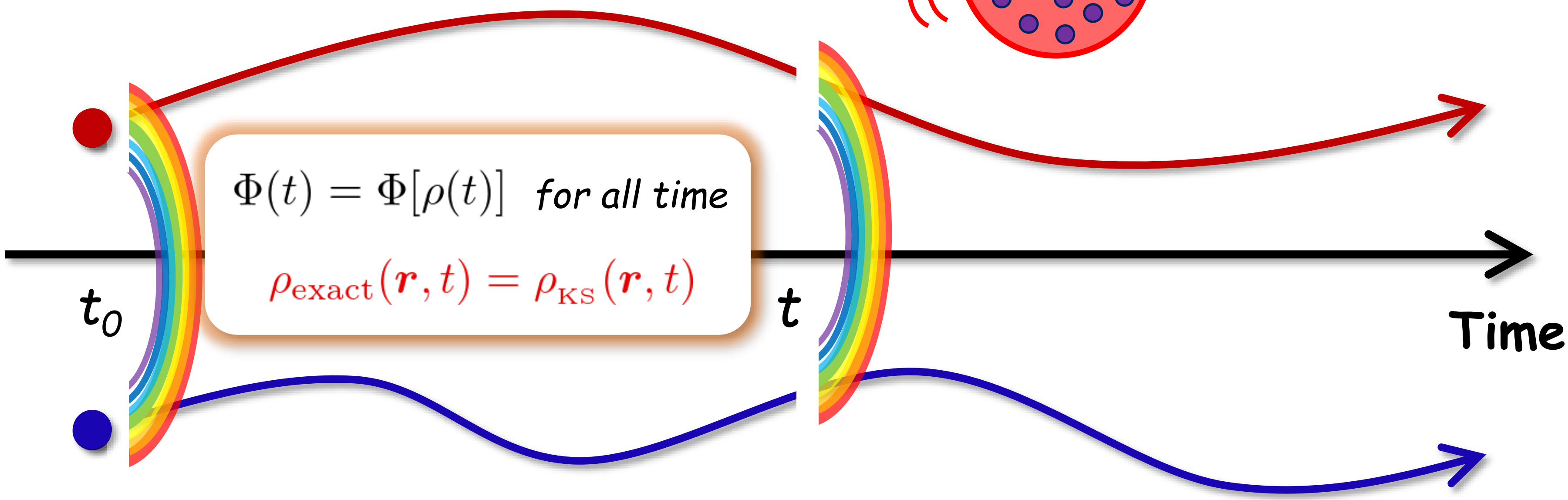
$$\frac{\delta}{\delta \phi_i^*} \left[E[\rho] - \sum_{kl} \varepsilon_{kl} \Big(\langle \phi_k | \phi_l \rangle - \delta_{kl} \Big) \right] = 0 \qquad \hat{h}[\rho] = \frac{\delta E[\rho]}{\delta \rho} \qquad \rho(\boldsymbol{r}) = \sum_{i=1}^{N} |\phi_i(\boldsymbol{r})|^2$$

TDDFT is a time-dependent extension of DFT

"Interacting" system

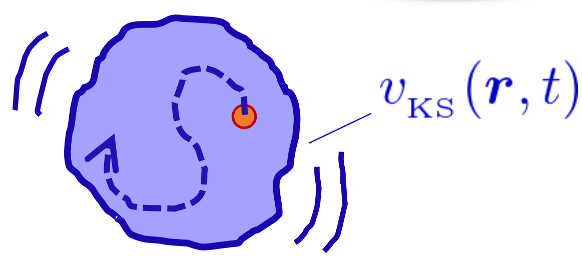
$$i\hbar\frac{\partial}{\partial t}\Phi(\boldsymbol{r}_1,\cdots,\boldsymbol{r}_N,t)=\hat{H}\Phi(\boldsymbol{r}_1,\cdots,\boldsymbol{r}_N,t)$$





"Non-interacting" system

$$i\hbar\frac{\partial\phi_i(\boldsymbol{r},t)}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+v_{\rm KS}[\rho(\boldsymbol{r},t)]\right]\phi_i(\boldsymbol{r},t)$$

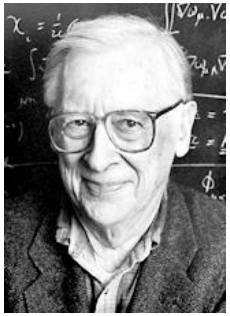


E. Runge and E.K.U. Gross, Phys. Rev. Lett. **52**, 997 (1984); R. van Leeuwen, Phys. Rev. Lett. **82**, 3863 (1999).

Great success of the Density Functional Theory

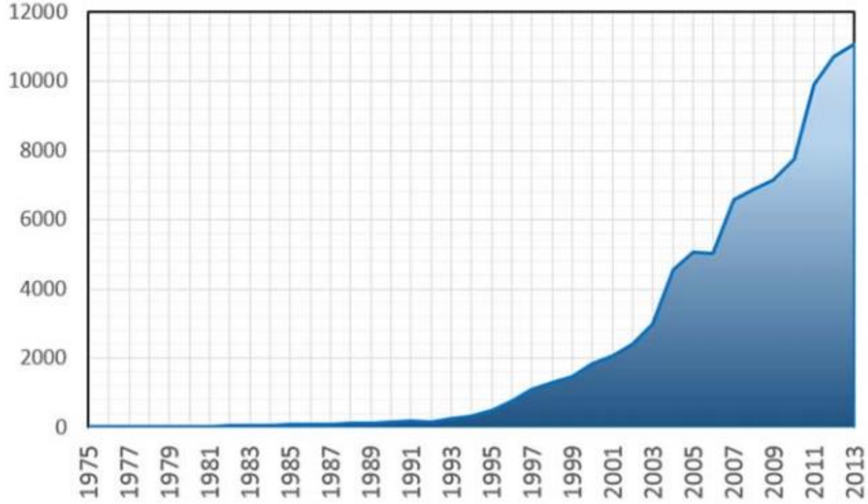
The Nobel Prize in Chemistry 1998

Walter Kohn



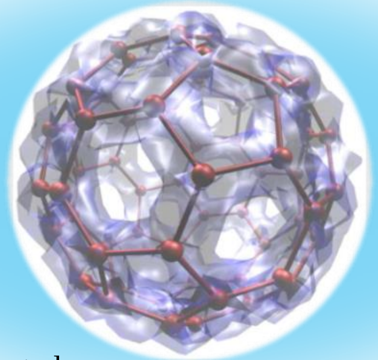
John Pople

Number of publications with “DFT” (till 2013)



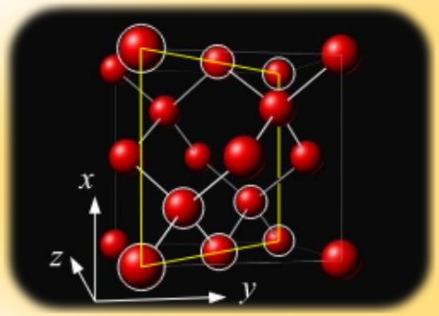
A. Galano and J.R. Alvarez-Idadoy, J. Compt. Chem. **35**, 2019 (2014)

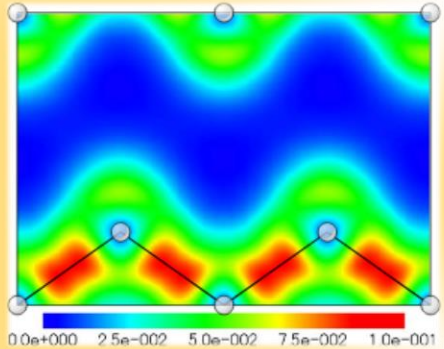
Fullerene: C_{60}



C-Z. Gao et al.,
J. Phys. B: At. Mol. Opt. Phys. **48**, 105102 (2015)

Si crystal



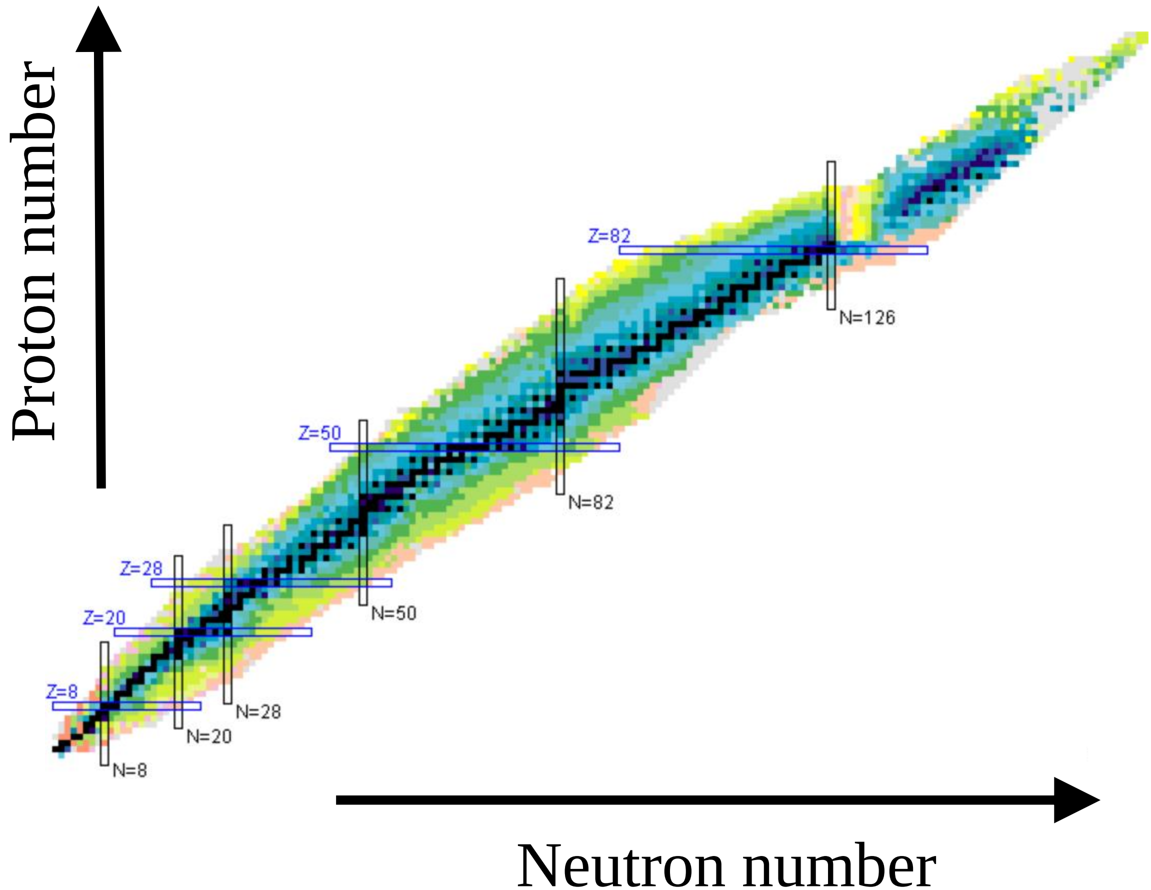


Y. Shinohara, K. Yabana, Y. Kawashita, J.-I. Iwata, T. Otobe, and G. F. Bertsch, Phys. Rev. B **82**, 155110 (2010)

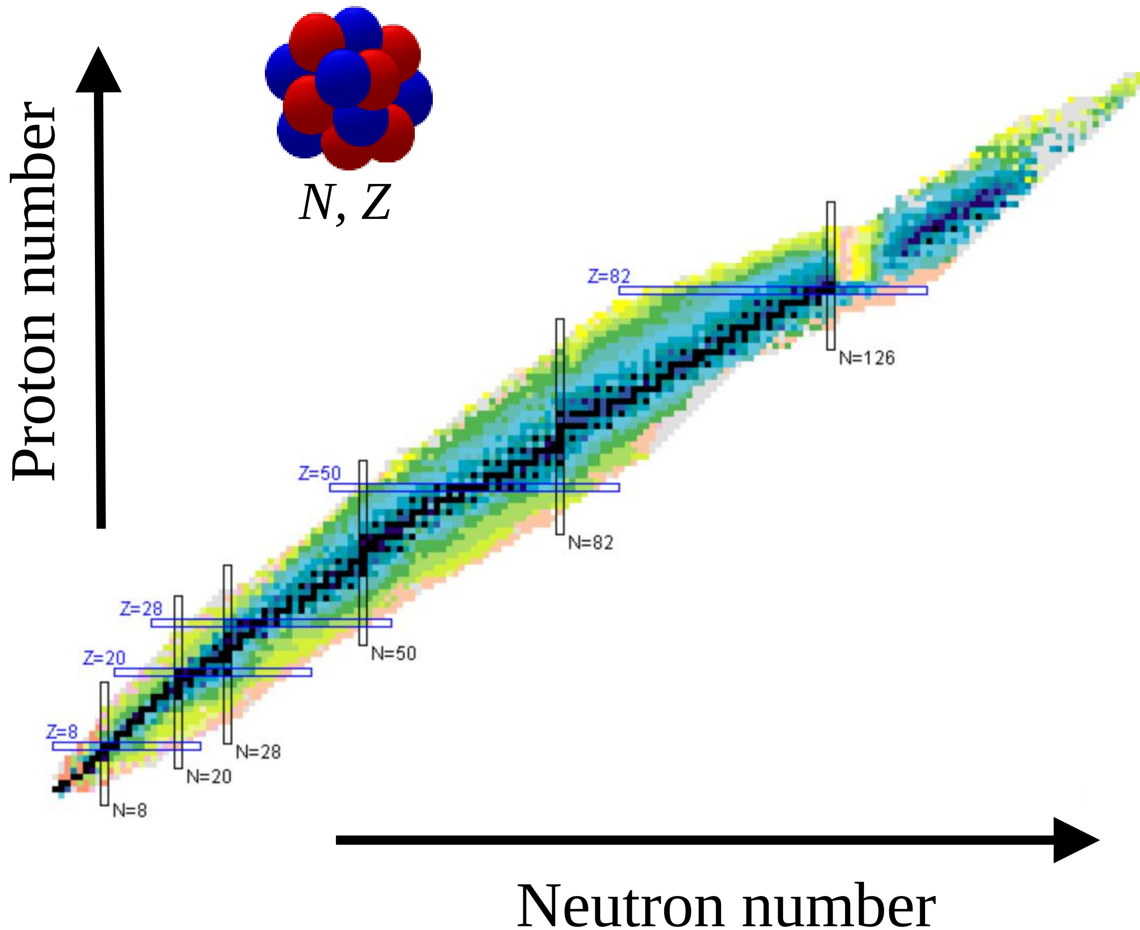
The seminal papers on DFT
- P. Hohenberg and W. Kohn, Phys. Rev. **136**, B864 (1964) ➔ **19,015 citations!**
- W. Kohn and L.J. Sham, Phys. Rev. **140**, A1133 (1965) ➔ **24,384 citations!**

All nuclei can be described with a single EDF

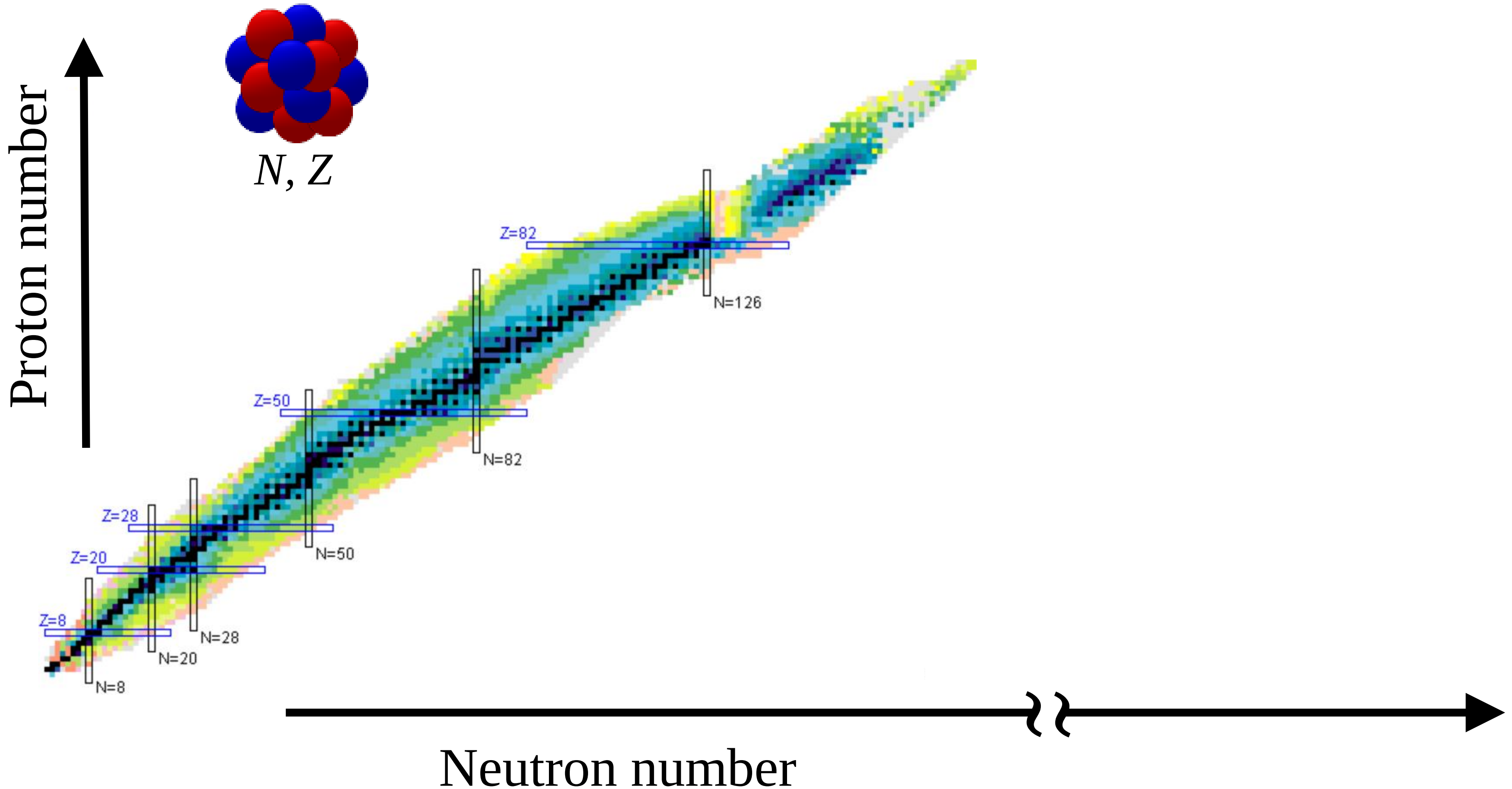
All nuclei can be described with a single EDF



All nuclei can be described with a single EDF



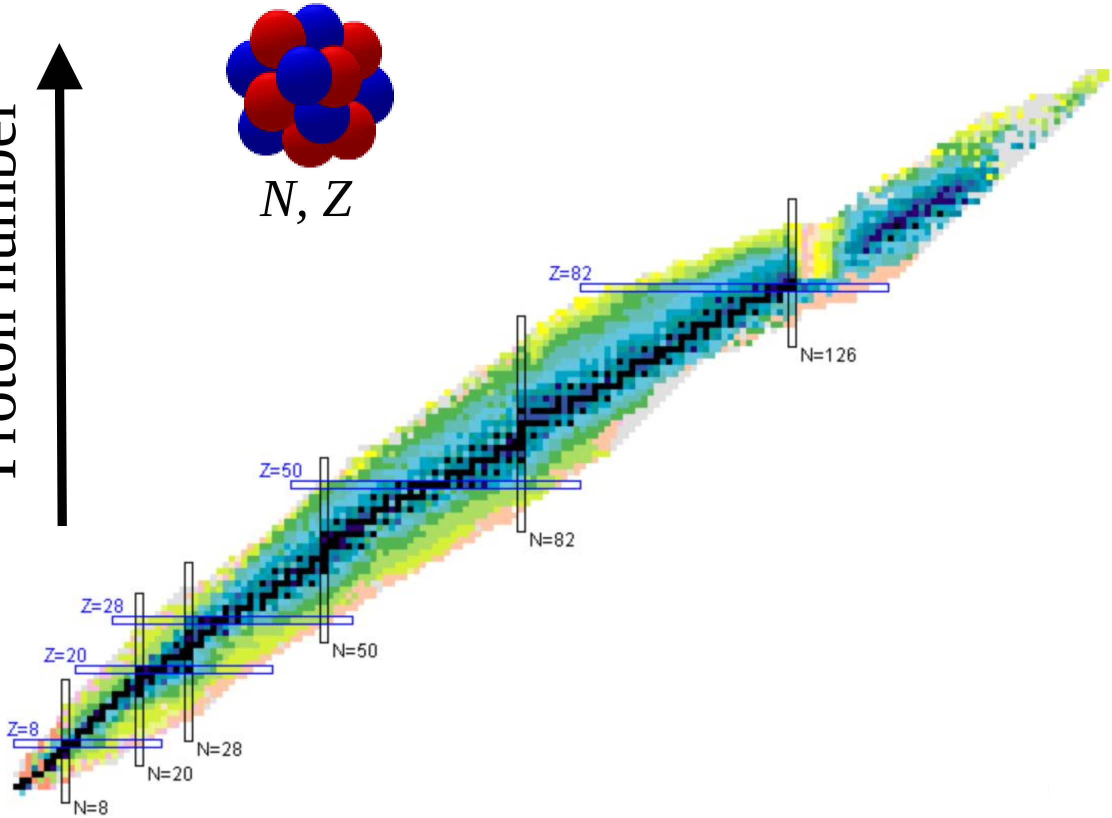
All nuclei can be described with a single EDF



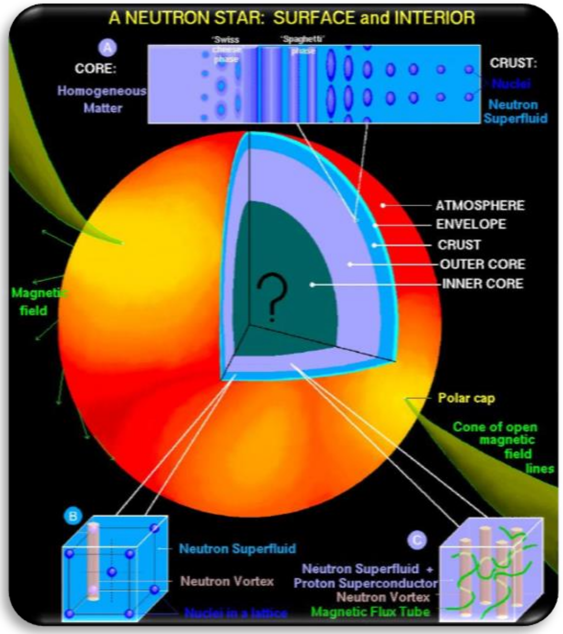
All nuclei can be described with a single EDF

N, *Z*

Proton number



Neutron star



Neutron number

TDSLDA: TDDFT with local treatment of pairing

Kohn-Sham scheme is extended for non-interacting quasiparticles

➤ TDSLDA equations (formally equivalent to TDHFB or TD-BdG equations)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow\uparrow}(\boldsymbol{r},t) & h_{\uparrow\downarrow}(\boldsymbol{r},t) & 0 & \Delta(\boldsymbol{r},t) \\ h_{\downarrow\uparrow}(\boldsymbol{r},t) & h_{\downarrow\downarrow}(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & & -h^*_{\uparrow\downarrow}(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & & & -h^*_{\downarrow\downarrow}(\boldsymbol{r},t) \end{pmatrix} \begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix}$$

Supercomputing!!

$$h_\sigma = \frac{\delta E}{\delta n_\sigma}$$: s.p. Hamiltonian

$$\Delta = -\frac{\delta E}{\delta \nu^*}$$: pairing field

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_k<E_c} |v_{k,\sigma}(\boldsymbol{r},t)|^2$$: number density

$$\nu(\boldsymbol{r},t) = \sum_{E_k<E_c} u_{k,\uparrow}(\boldsymbol{r},t)v^*_{k,\downarrow}(\boldsymbol{r},t)$$: anomalous density

$$\boldsymbol{j}_\sigma(\boldsymbol{r},t) = \hbar\sum_{E_k<E_c} \mathrm{Im}[v^*_{k,\sigma}(\boldsymbol{r},t)\boldsymbol{\nabla}v_{k,\sigma}(\boldsymbol{r},t)]$$: current

A large number (10^4-10^6) of 3D coupled non-linear PDEs have to be solved!!

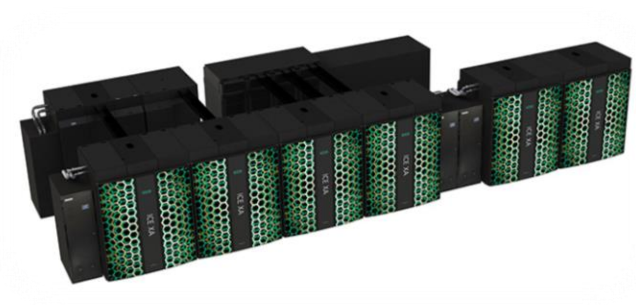
# of qp orbitals ~ # of grid points

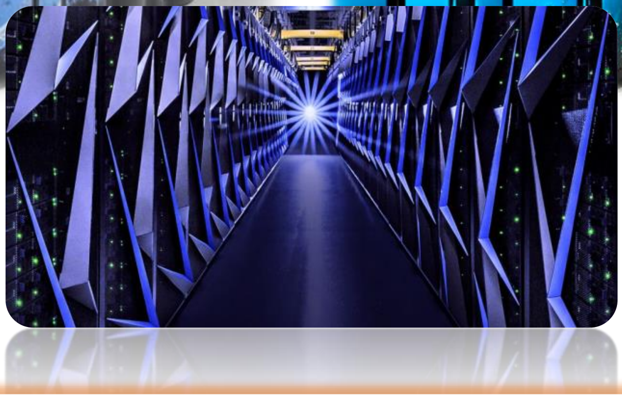
*The number indicates the rank according to the TOP500 list (June 2018)

Piz Daint, CSCS, Switzerland (No. 6)

TITAN, ORNL, USA (No. 7)

TSUBAME3.0, Japan (No. 19)

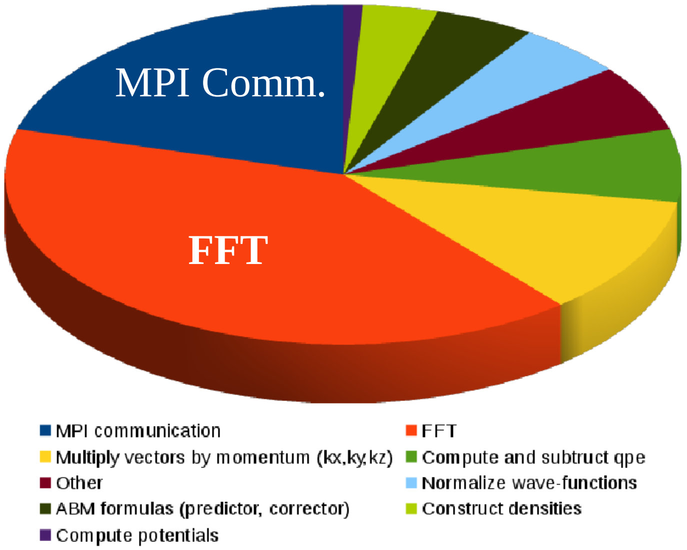




The fastest machine:
Summit, ORNL, USA
GPU, 188 PFlops/s

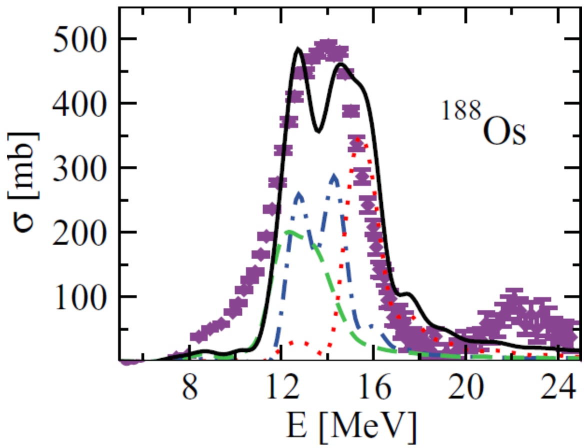
Present computing capabilities:

- ✓ Full 3D (w/o symmetry restrictions)
- ✓ Volume as large as 100^3 lattice points
- ✓ Evolution up to 10^6 time steps (as long as 10^{-19} sec)



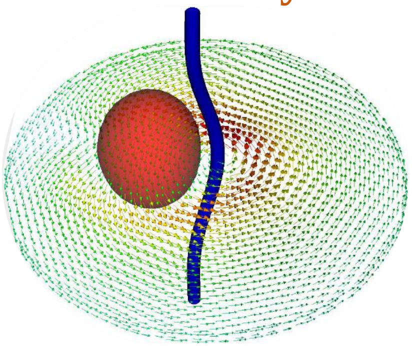
TDSLDA is a versatile tool!!

IVGDR



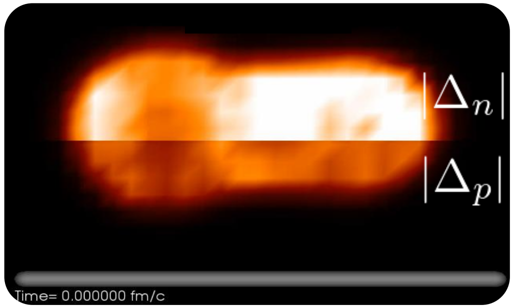
Phys. Rev. C **84**, 051309(R) (2011)
I. Stetcu, A. Bulgac, P. Magierski, and K.J. Roche

Vortex-nucleus dynamics



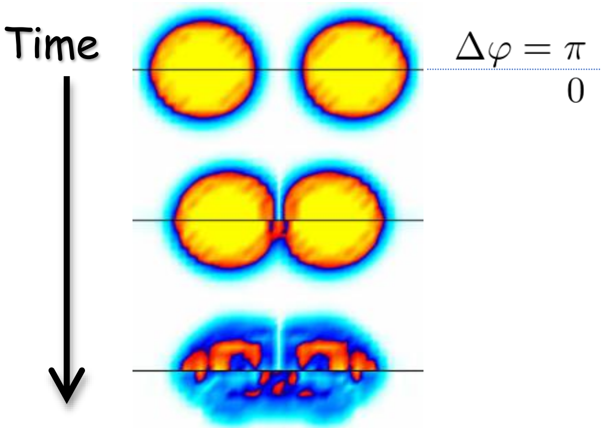
Phys. Rev. Lett. **117**, 232701 (2016)
G. Wlazłowski, K.S., P. Magierski, A. Bulgac, and M.M. Forbes

Induced fission of ^{240}Pu



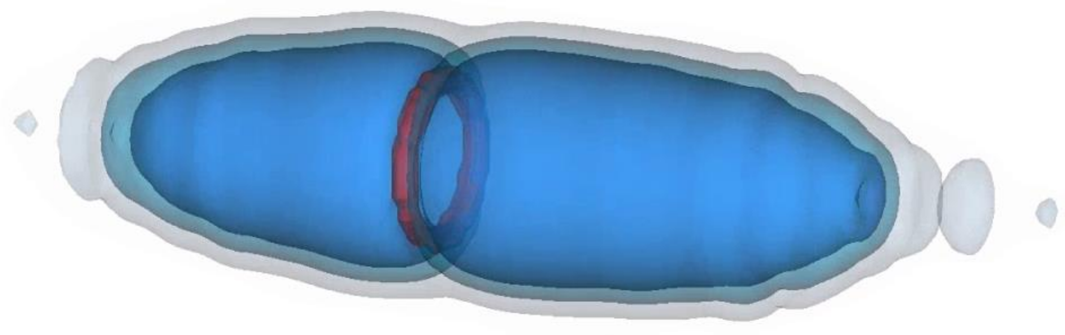
Phys. Rev. Lett. **116**, 122504 (2016)
A. Bulgac, P. Magierski, K.J. Roche, and I. Stetcu

Low-energy heavy ion reactions



Phys. Rev. Lett. **119**, 042501 (2017)
P. Magierski, K.S., and G. Wlazłowski

Solitonic cascades in polarized Fermi gas



G. Wlazłowski, K. Sekizawa, M. Marchwiany, and P. Magierski,
Phys. Rev. Lett. **120**, 253002 (2018)

TDSLDA has been successfully applied for both UFG and nuclear systems

Unitary Fermi Gas (UFG)

$$k_{\mathrm{F}}a \to \infty, \; k_{\mathrm{F}}r_{\mathrm{eff}} \to 0$$

a : s-wave scattering length
r_{eff} : effective range

A. Bulgac and S. Yoon, PRL**102**(2009)085302.
A. Bulgac *et al.*, Science **332**(2011)1288.
A. Bulgac *et al.*, PRL**108**(2012)150401.
A. Bulgac *et al.*, PRL**112**(2014)025301.
G. Wlazłowski *et al.*, PRA**91**(2015)031602(R).

Large-amplitude pairing field dynamics
Dynamics of quantum vortices
Quantum shock waves and domain walls
Dynamics of vortex rings
Quantum turbulence

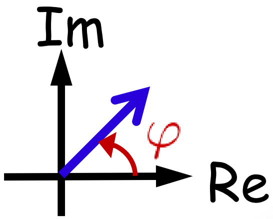
Nuclear systems

I. Stetcu *et al.*, PRC**84**(2011)051309(R).
I. Stetcu *et al.*, PRL**114**(2015)012701.
A. Bulgac *et al.*, PRL**116**(2016)122504.
G. Wlazłowski *et al.*, PRL**117**(2016)232701.

Isovector giant dipole resonance (IVGDR)
Relativistic coulomb excitation
Induced fission of ^{240}Pu
Vortex-nucleus interaction

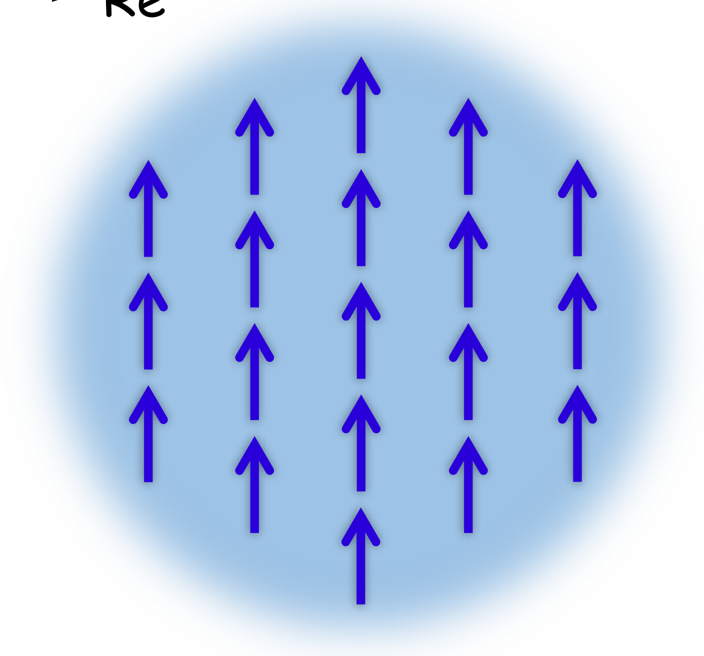
The **Pairing field** provides a variety of **dynamic** excitation modes

$$|\Delta(\boldsymbol{r},t)|e^{\mathrm{i}\varphi(\boldsymbol{r},t)}$$

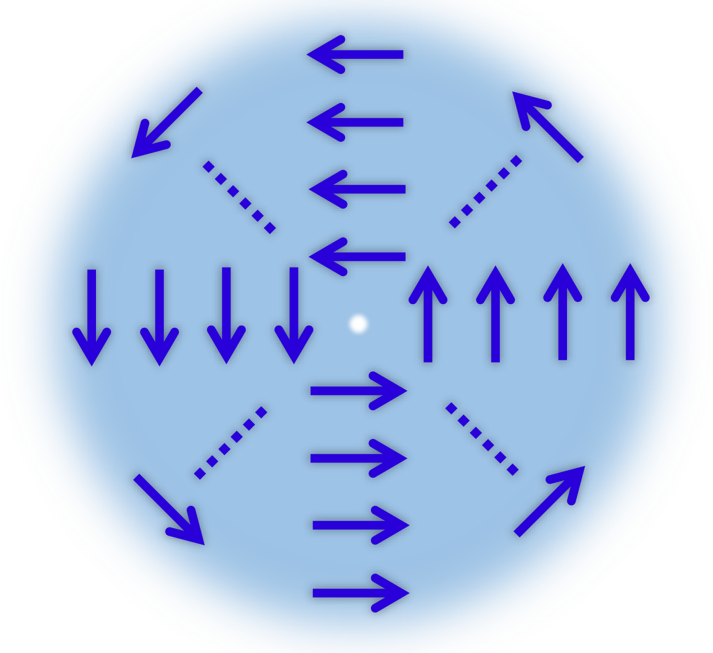


**Superfluid velocity*

$$\boldsymbol{v}_s(\boldsymbol{r},t) \propto \boldsymbol{\nabla}\varphi(\boldsymbol{r},t)$$



Collective rotation
(of the phase)

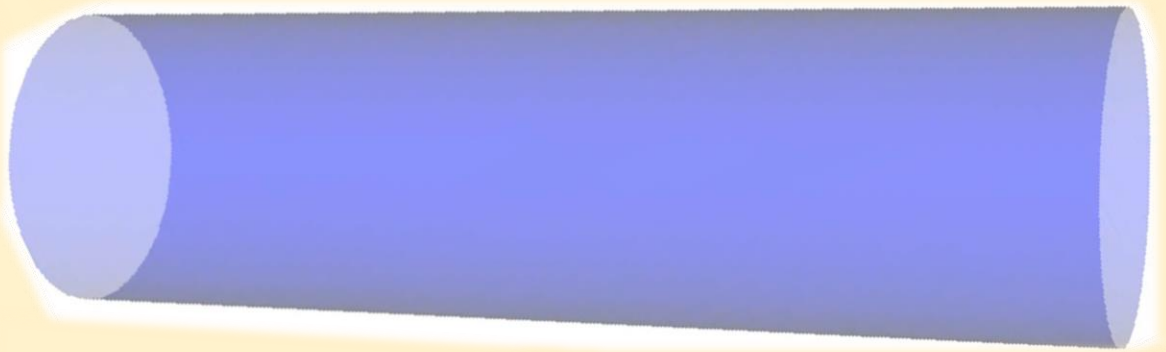


Quantum vortex

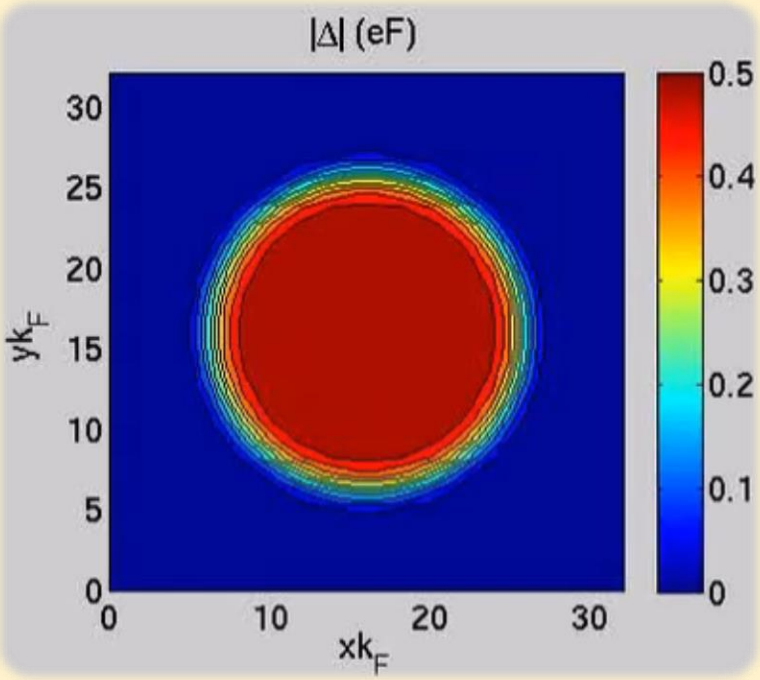
Applications of TDSLDA: Unitary Fermi Gas

Real-Time Dynamics of Quantized Vortices in a Unitary Fermi Superfluid
A. Bulgac, Y.-L. Luo, P. Magierski, K.J. Roche, Y. Yu
Science ***332****, 1288 (2011)*

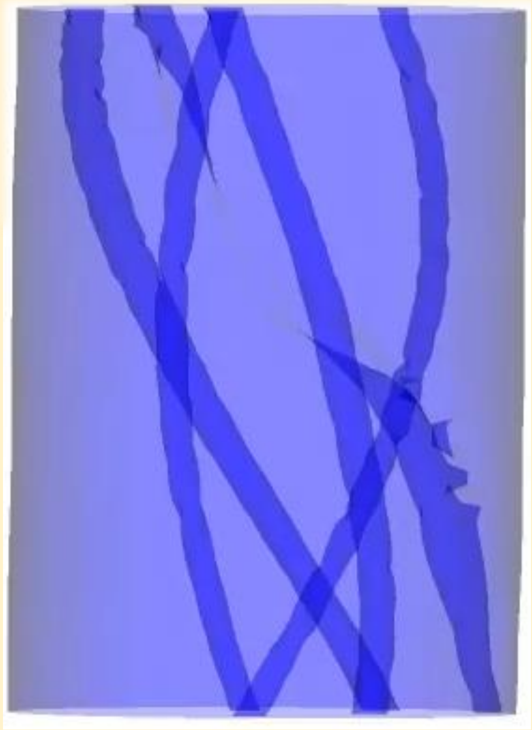
Throwing a spherical projectile ➡ Vortex rings/lines



Stirring ➡ Quantized vortices



Twisted stirring
⬇
Vortex crossings
& reconnections
(quantum turbulence)

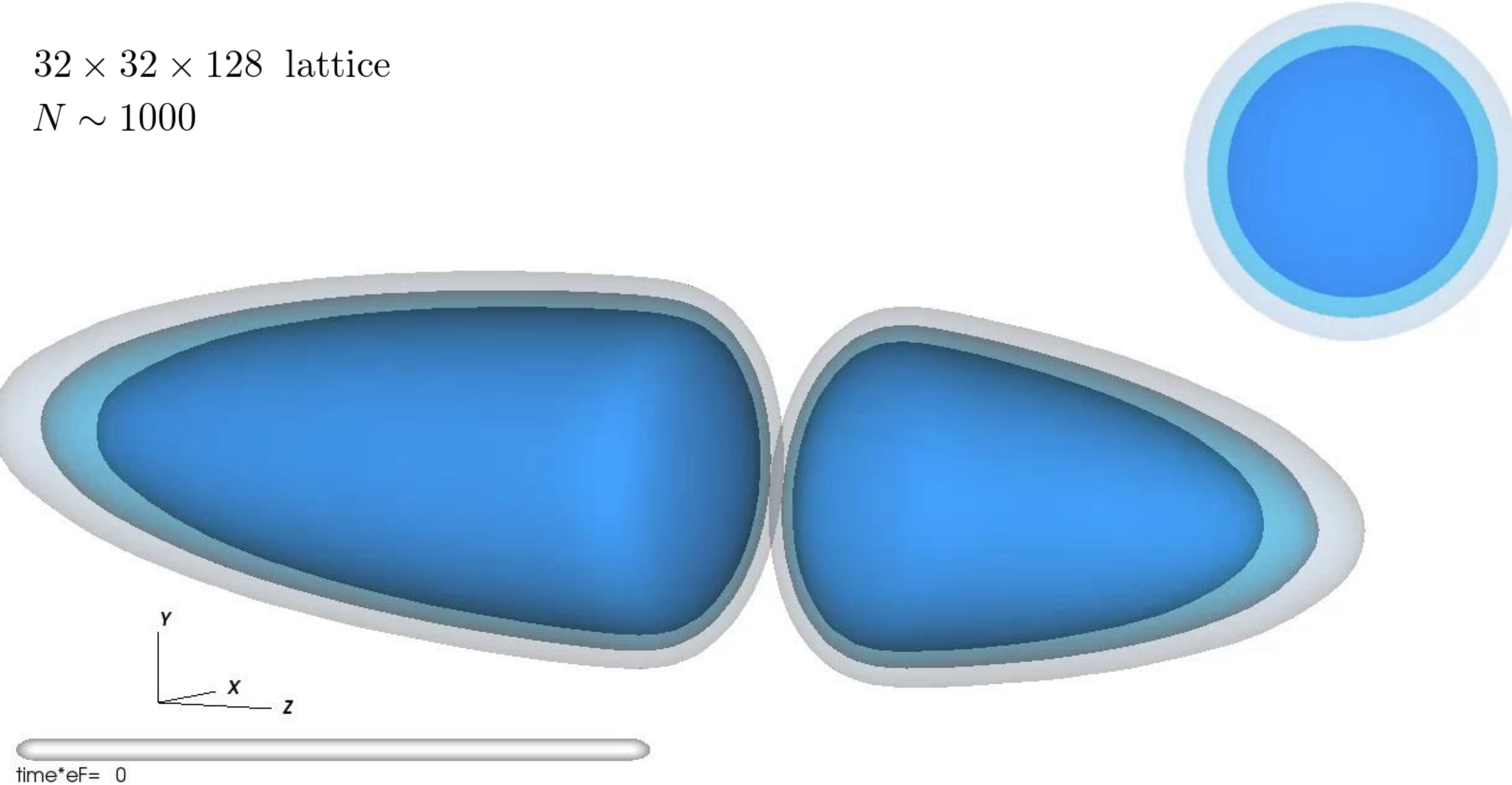


Result of TDSLDA simulation:

Phase discontinuity creates a vortex ring which decays into a vortex line

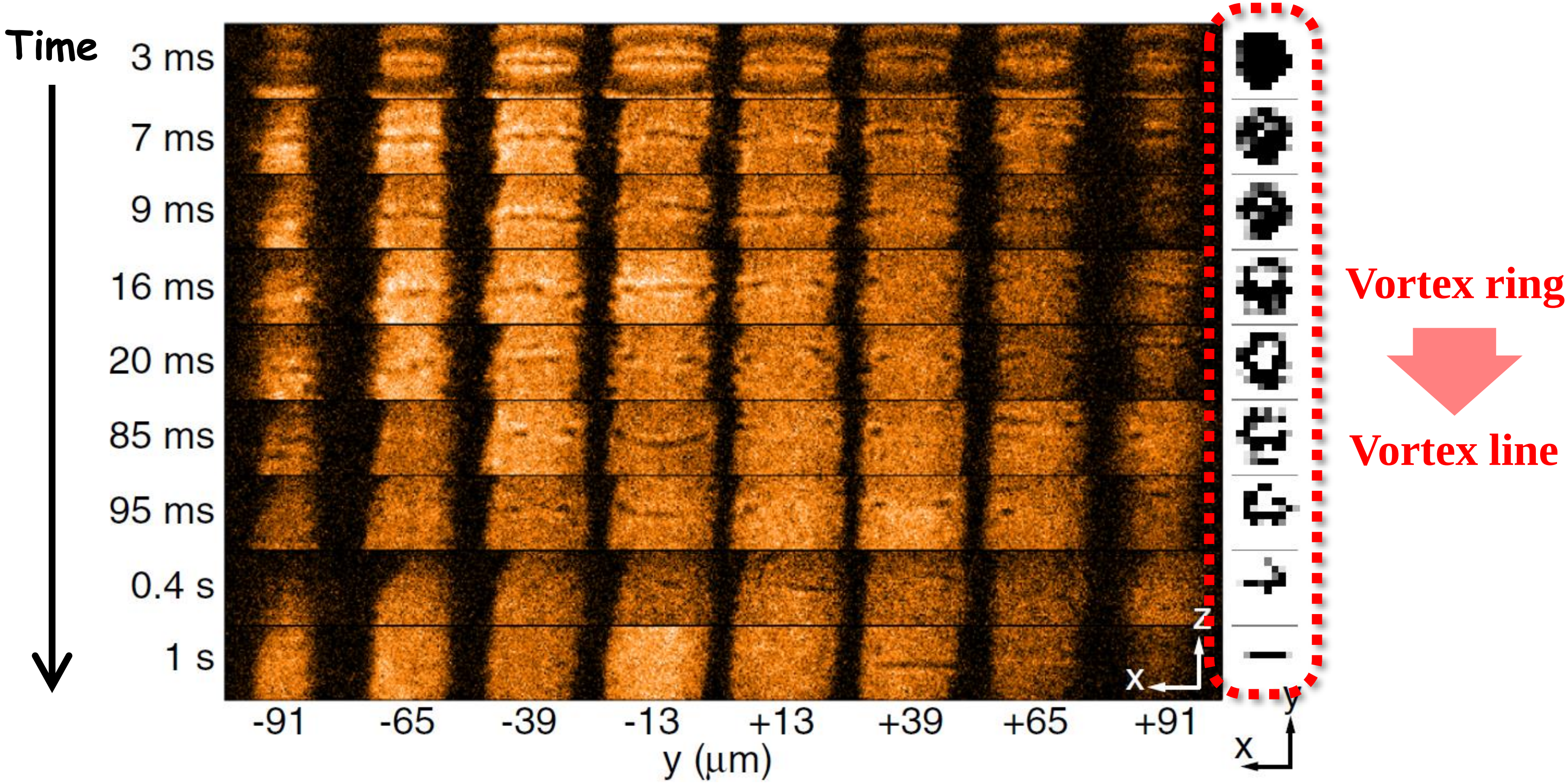
$32 \times 32 \times 128$ lattice

$N \sim 1000$



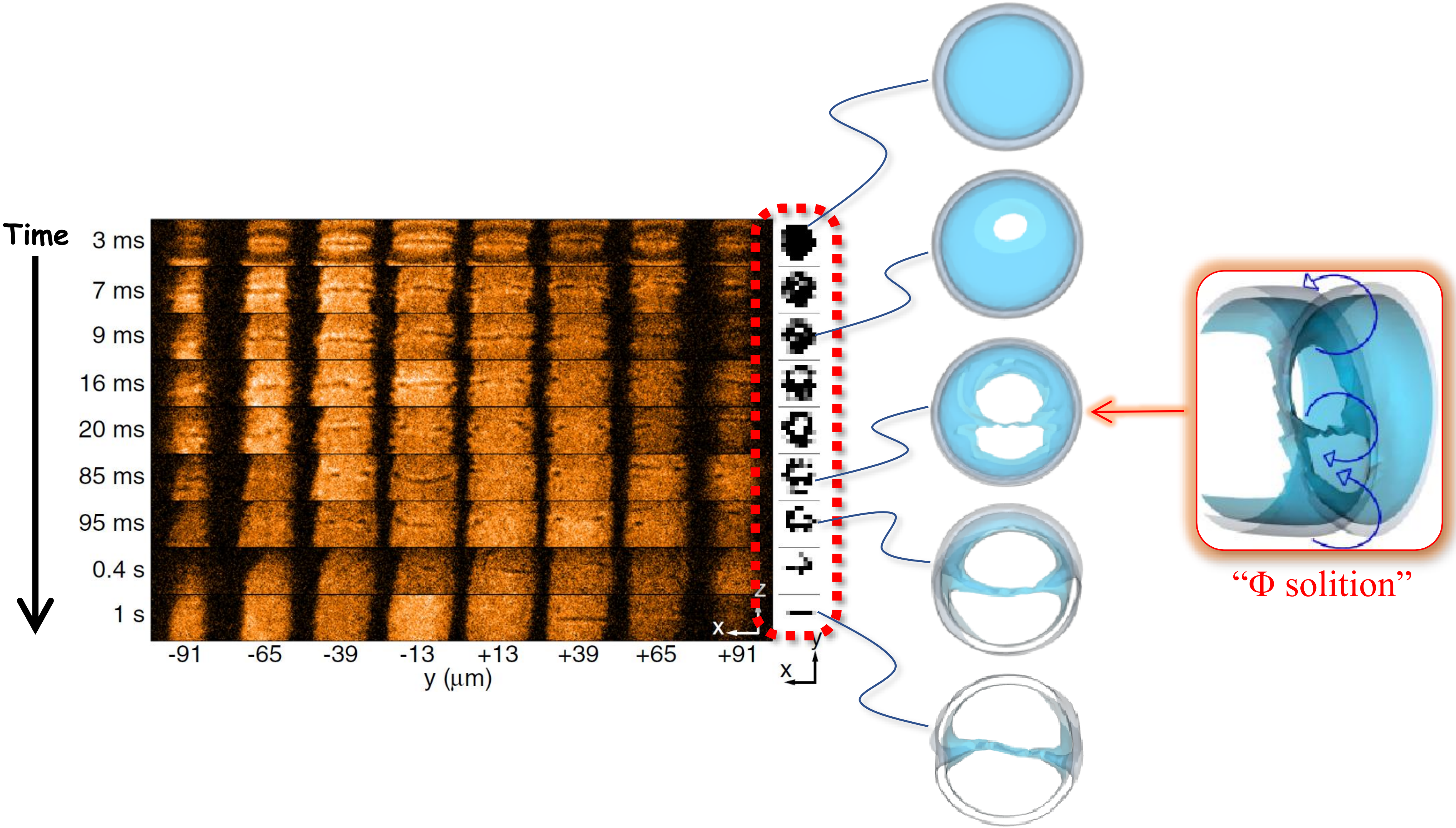
G. Wlazłowski, A. Bulgac, M.M. Forbes, and K.J. Roche, Phys. Rev. A **91**, 031602 (2015)

The cascades of solitonic excitations have been identified experimentally



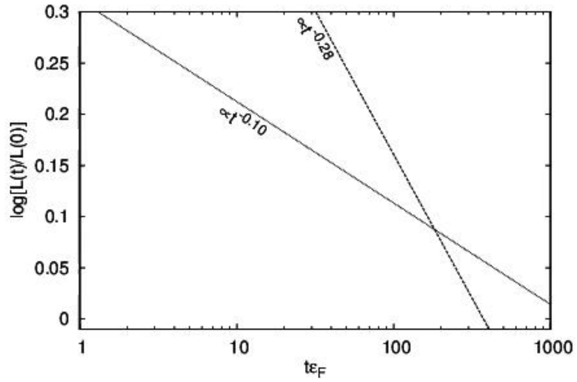
M.J.H. Ku, B. Mukherjee, T. Yefsah, and M.W. Zwierlein, Phys. Rev. Lett. **116**, 045304 (2016)

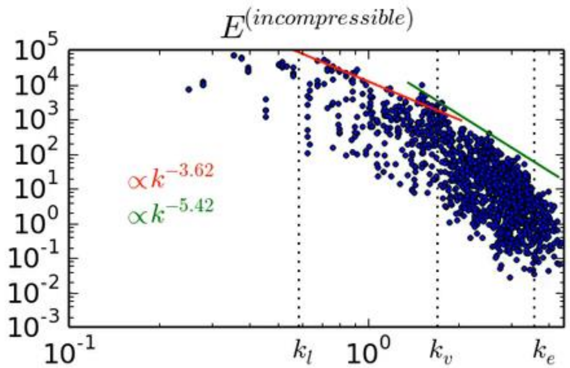
Each stage of solitonic cascade could be reproduced with TDSLDA!

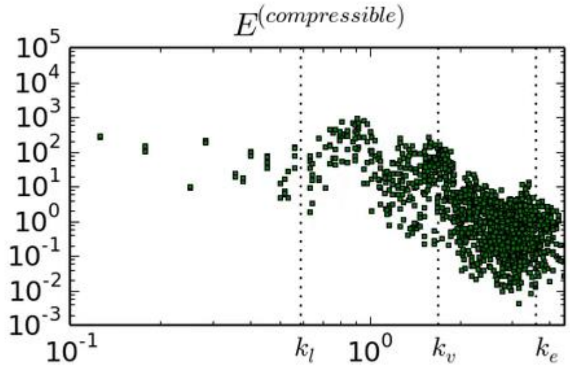


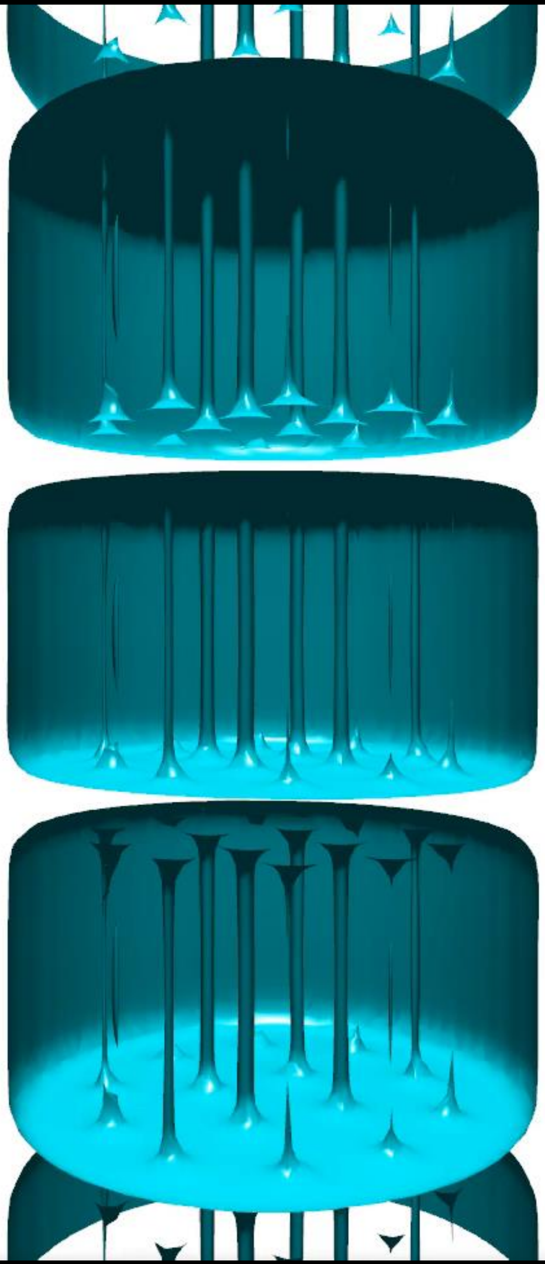
Computational/theoretical challenge

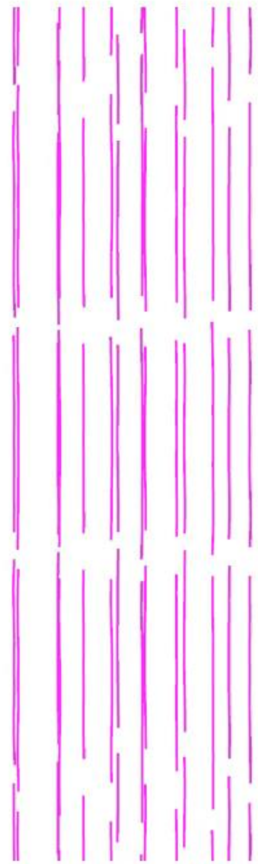
Microscopic simulation of quantum turbulence in superfluid fermi gas







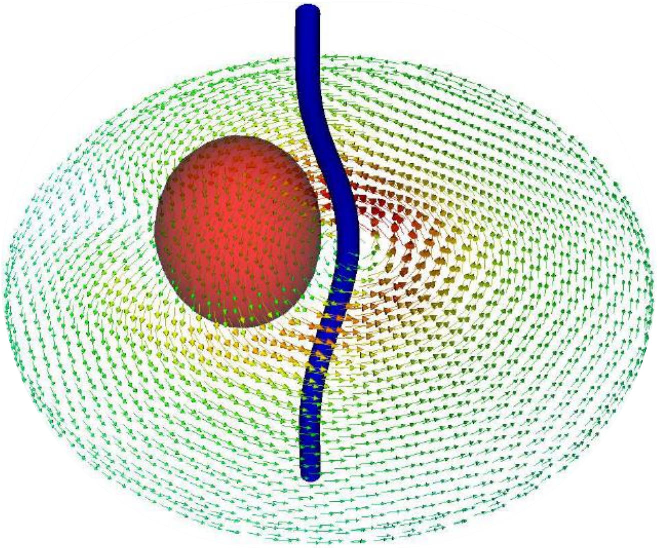




time = 0 $[\varepsilon_F^{-1}]$

Very preliminary

Vortex-nucleus dynamics in the NS crust



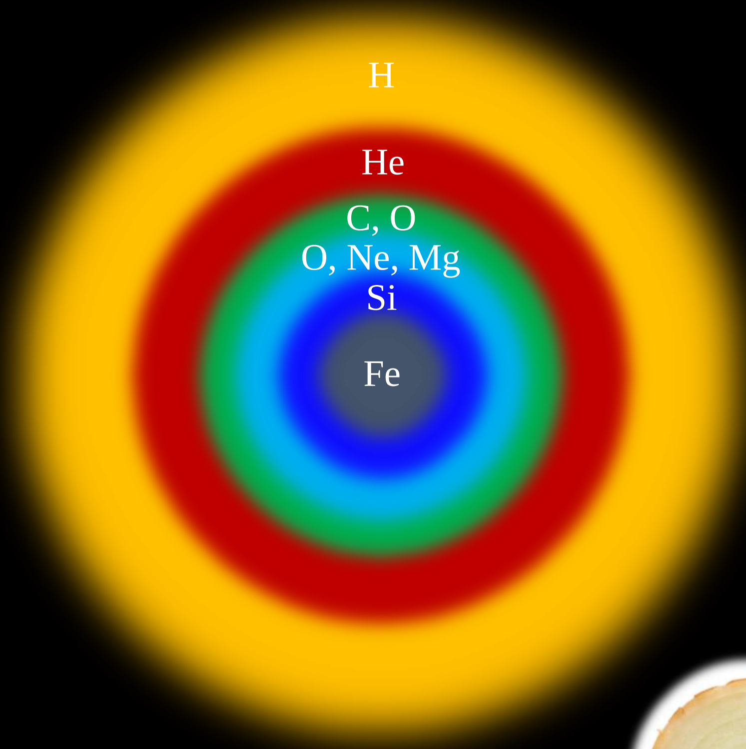
G. Wlazłowski, K.S., P. Magierski, A. Bulgac, and M.M. Forbes,
Phys. Rev. Lett. **117**, 232701 (2016)

The fate of a massive star

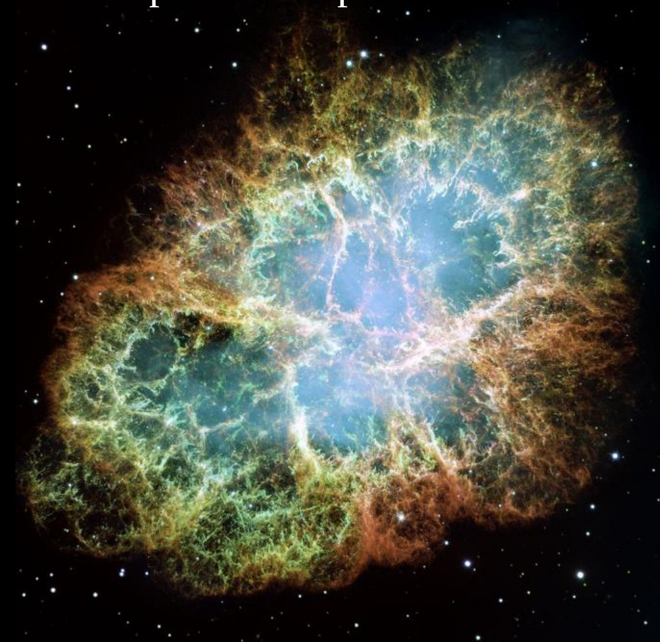
Nuclear reactions:

$^{1}H \rightarrow {}^{4}He \rightarrow {}^{12}C \rightarrow {}^{16}O \rightarrow {}^{20}Ne \rightarrow {}^{24}Mg \rightarrow {}^{28}Si \rightarrow \dots \rightarrow {}^{56}Fe$

"Onion structure"

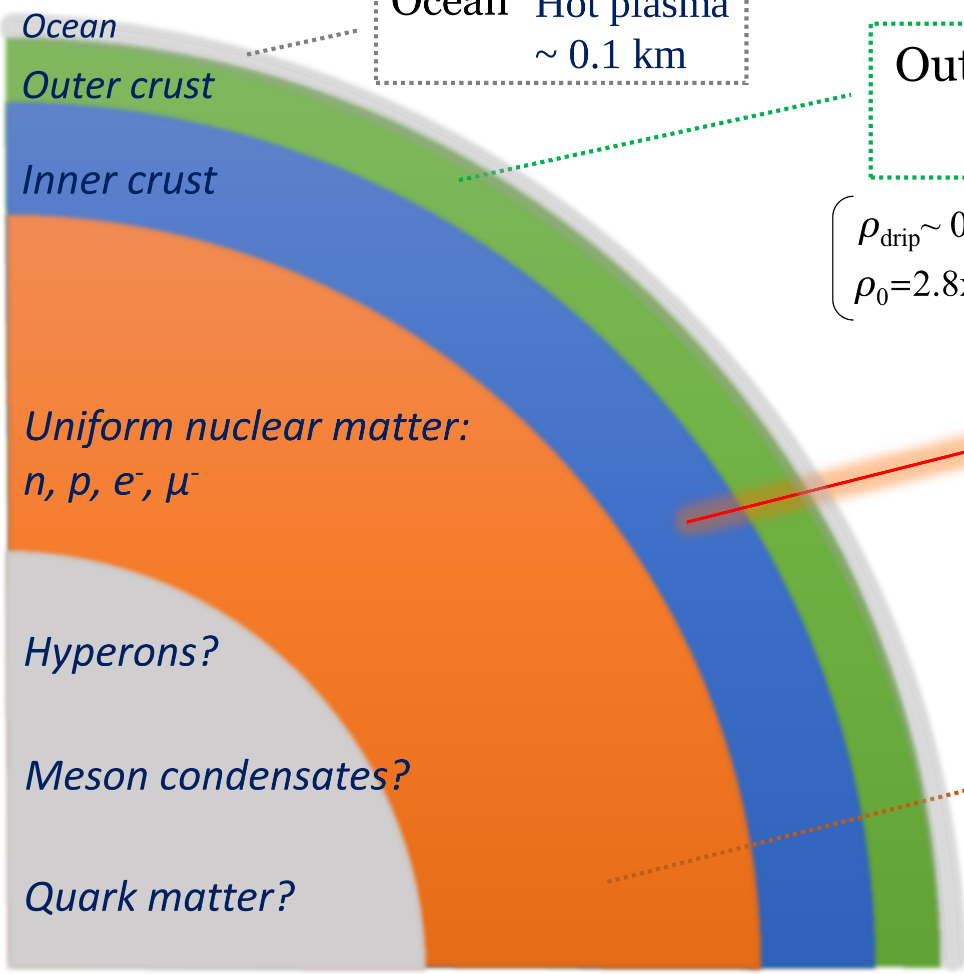


After forming the iron core…
→ no more fuel
→ gravitational collapse
→ supernova explosion



The Crab Nebula
Remnant of the SN in 1054

Neutron star is a great playground for nuclear theorists



Ocean Hot plasma ~ 0.1 km

Outer crust Nuclei (Fe), electrons

0.3-0.6 km $\rho < \rho_{drip}$

$\rho_{drip} \sim 0.0014\rho_0$; Above ρ_{drip} unbound neutrons exist outside nuclei

$\rho_0 = 2.8\text{x}10^{14}$ g/cm^3=0.16 fm^{-3} ; Nuclear saturation density

Inner crust Nuclei, electrons, "dripped" neutrons

0.5-0.8 km $\rho_{drip} \leq \rho \leq 0.6\rho_0$

Outer/inner core

9-12 km $0.6\rho_0 < \rho \leq 3\text{-}5\rho_0$

Structure of the inner crust

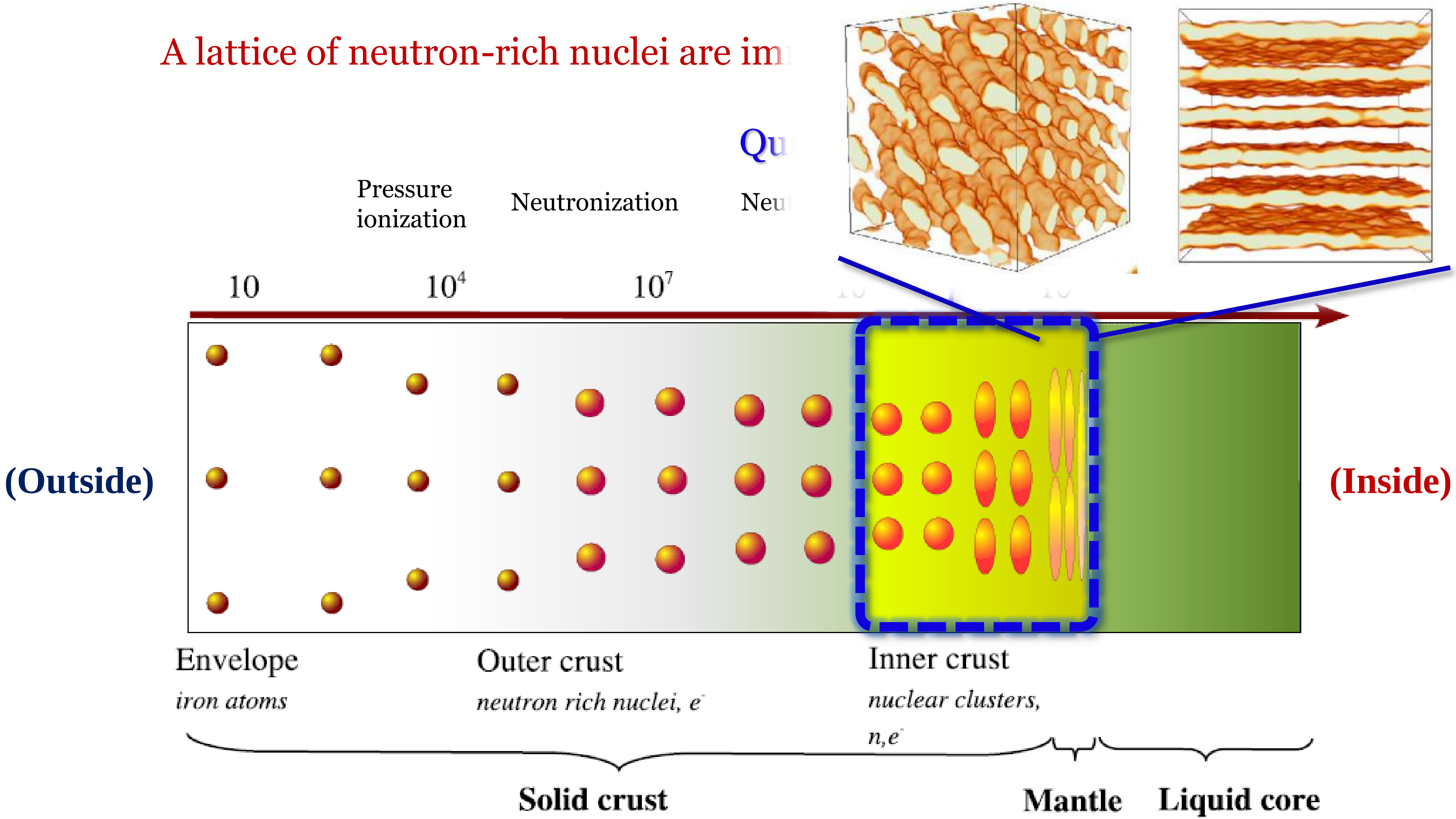


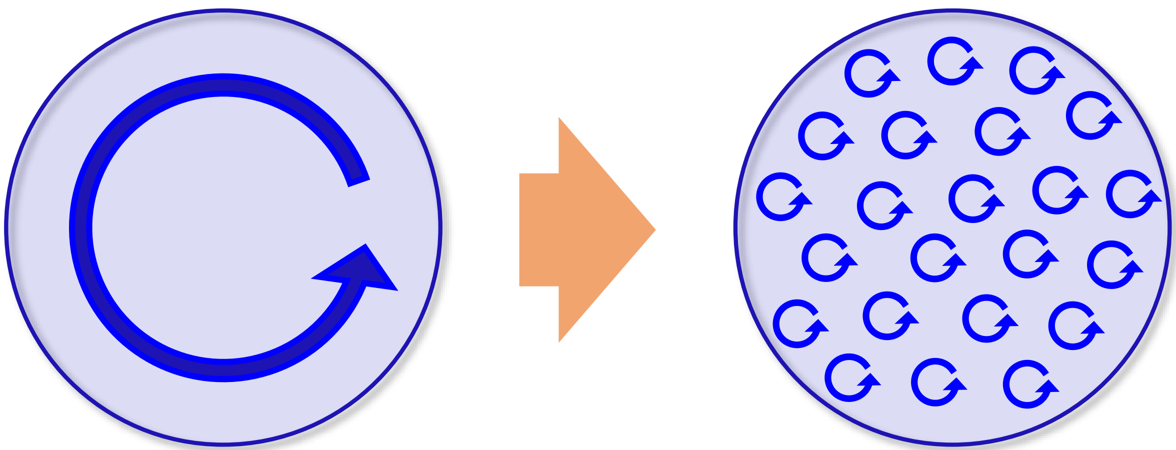
Fig.4 in N. Chamel and P. Haensel, Living Rev. Relativity **11**, 10 (2008)

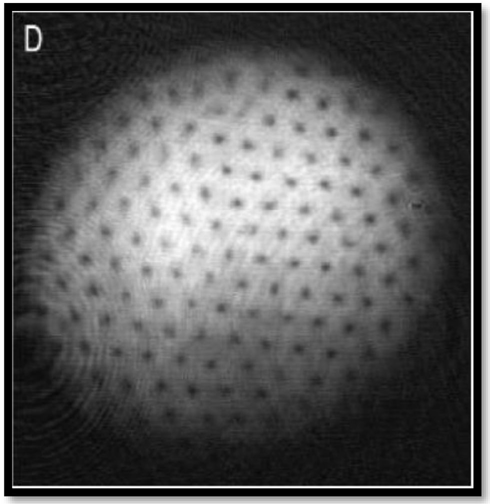
In rotating superfluid, an arra

- Observation in ultra-cold atomic gases

The Nobel Prize in Physics 2003

A.A. Abrikosov V.L. Ginzburg A.J. Leggett





"Abrikosov lattice"

W. Ketterle, MIT Physics Annual. 2001

In rotating superfluid, an array of quantum vortices is generated

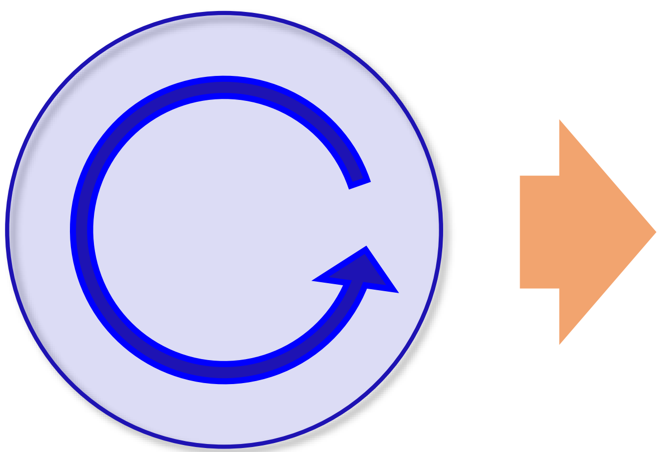
- Observation in ultra-cold atomic gases

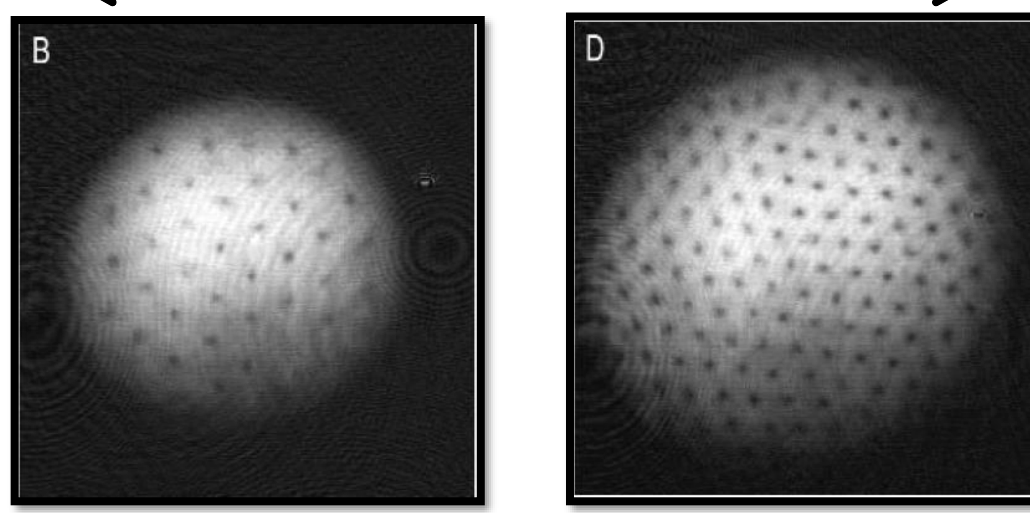
Vortex-number density

$\propto$

Rotation speed

Low ⟷ High

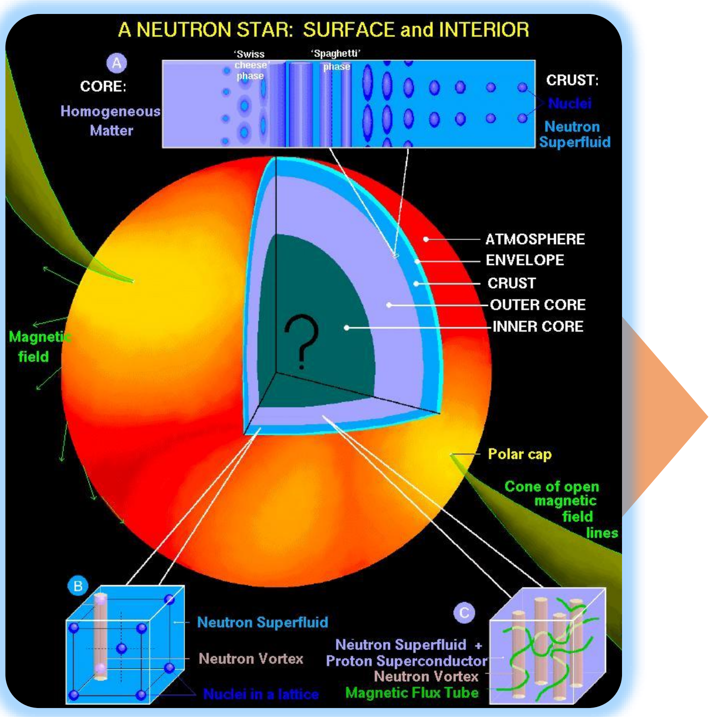




W. Ketterle, MIT Physics Annual. 2001

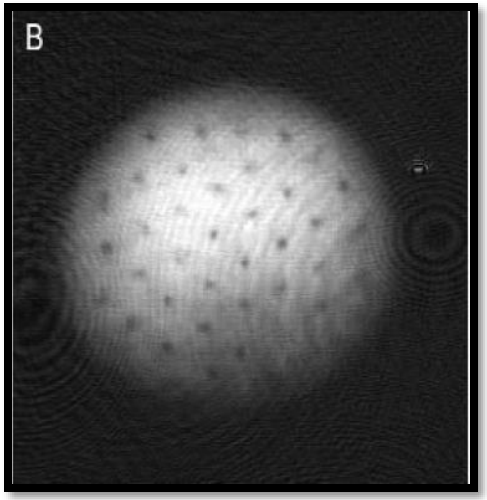
In rotating superfluid, an array of quantum vortices is generated

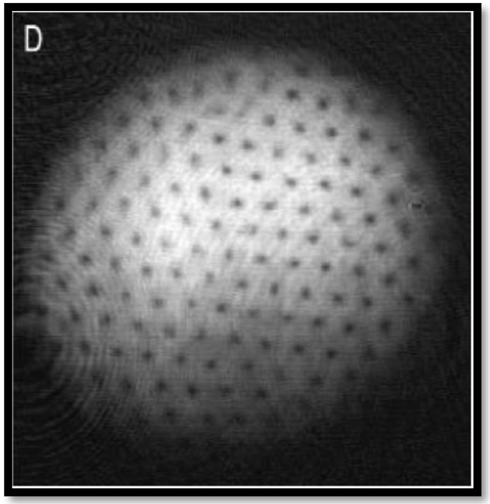
- Observation in ultra-cold atomic gases



Vortex-number density
$\propto$
Rotation speed

Low ⟷ High

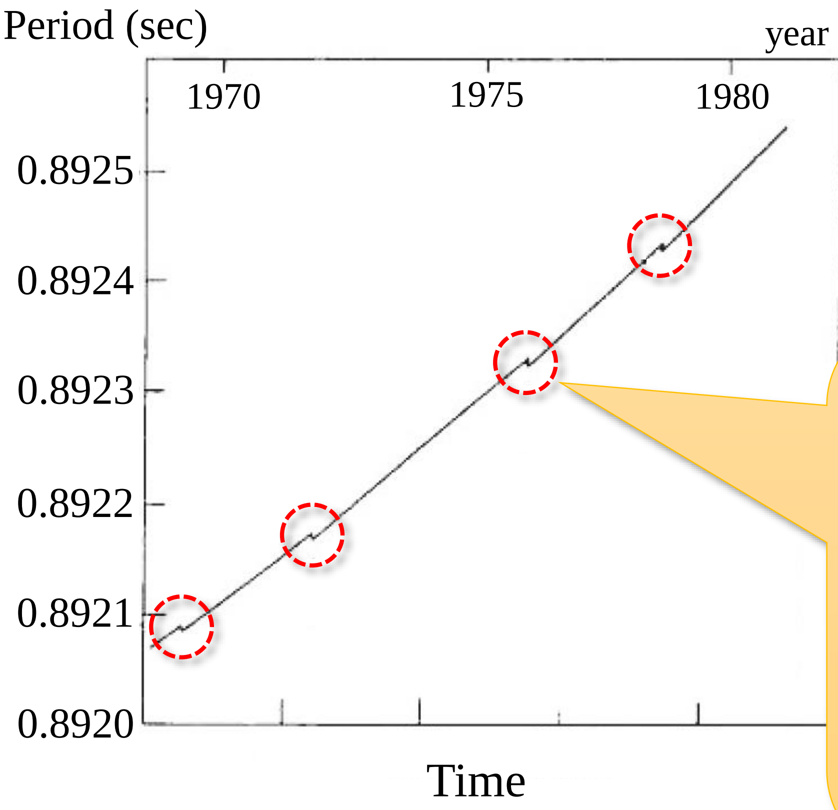




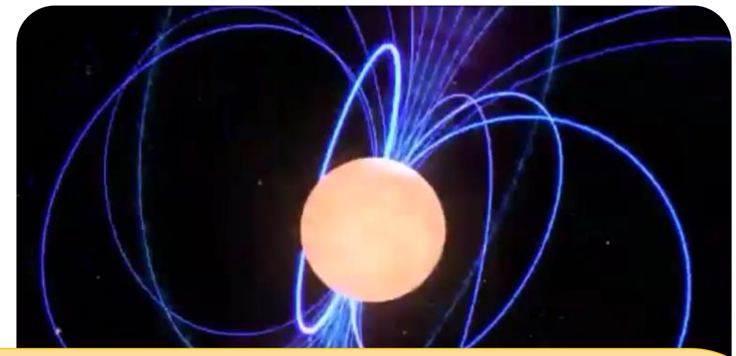
W. Ketterle, MIT Physics Annual. 2001

Glitch: a sudden increase of the rotational frequency

Glitches in the Vela pulsar

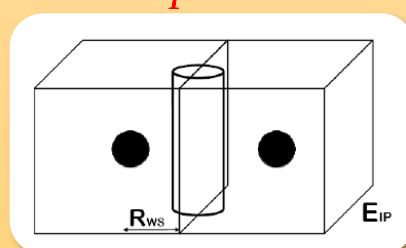


- Pulsar: a rotating neutron star



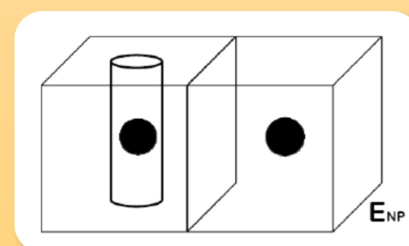
A conjectured scenario:
Vortex *pinning/unpinning* in the neutron star crust

Repulsive?



or

Attractive?



P.W. Anderson and N. Itoh, Nature **256**, 25 (1975)

V.B. Bhatia, A Textbook of Astronomy and Astrophysics with Elements of Cosmology, Alpha Science, 2001.

Representative studies of the pinning force

- Hartree-Fock-Bogoliubov theory

  P. Avogadro, F. Barranco, R.A. Broglia, and E. Vigezzi,
  PRC**75**(2007)012805(R); NPA**788**(2007)130; NPA**811**(2008)378

- Thomas-Fermi + LDA

  P.M. Pizzochero, L. Viverit, and R. A. Broglia, PRL**79**(1997)3347
  P. Donati and P.M. Pizzochero, PRL**90**(2003)211101; NPA**742**(2004)363; PLB**640**(2006)74
  S. Seveso, P.M. Pizzochero, F. Grill, and B. Haskell, MNRAS**455**(2016)3952

- Hydrodynamics + Ginzburg-Landau (for pairing)

  M.A. Alpar et al. Astrophys. J. **213**(1977)527; **276**(1984)325
  R.I. Epstein, G. Baym, Astrophys. J. **328**(1988)680
  R.K. Link, R.I. Epstein, Astrophys. J. **373**(1991)592

Density dependence and asymptotic behavior of the force are predicted

$$E = E_{\text{tension}} + \frac{1}{2}M^* u^2 + 2\pi R^3 \frac{\rho_{\text{out}}(\rho_{\text{in}} - \rho_{\text{out}})}{2\rho_{\text{out}} + \rho_{\text{in}}}\left(\frac{\kappa}{2\pi r}\right)^2 + \mathcal{O}(1/r^3) \qquad (r \gg \xi)$$

Interaction energy between a vortex line and an impurity

$\rho_{\text{in}} < \rho_{\text{out}}$: attraction

$\rho_{\text{in}} > \rho_{\text{out}}$: repulsion

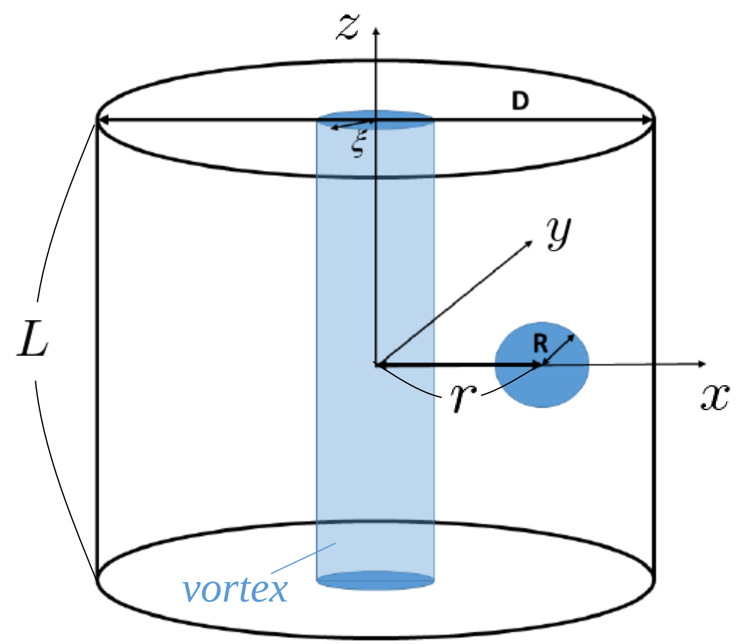
*$\rho_{\text{in/out}}$: superfluid density inside/outside a nucleus

$$F = -\frac{\mathrm{d}E}{\mathrm{d}r} \propto \frac{1}{r^3}$$

$$E_{\text{tension}} = \frac{1}{4\pi}\rho_{\text{out}}\kappa^2 L \ln\left(\frac{D}{2\xi}\right)$$

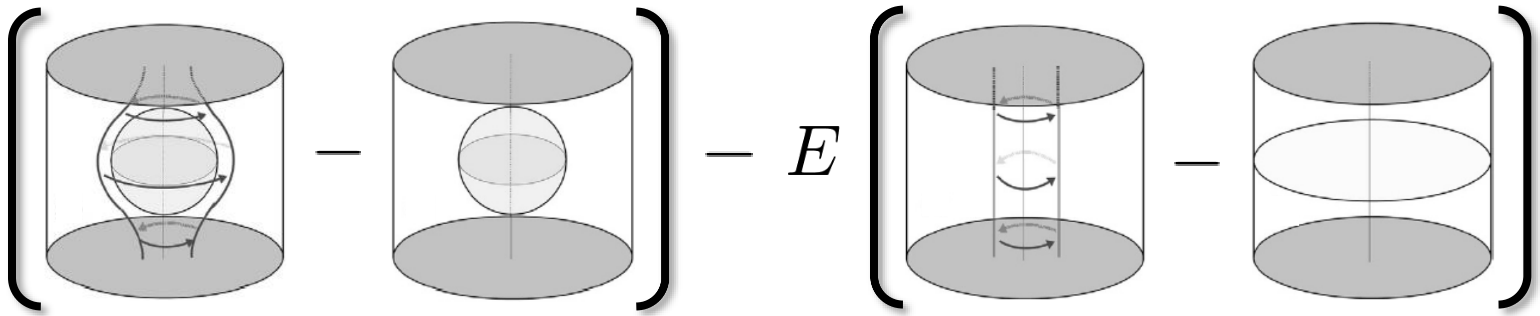
$$M^* = \frac{4\pi}{3}R^3 \frac{(\rho_{\text{out}} - \rho_{\text{in}})^2}{2\rho_{\text{out}} + \rho_{\text{in}}}$$

$$\kappa = \frac{2\pi\hbar}{2m_n}$$



Microscopic, static HFB calculations were performed assuming axial symmetry

$$E_{\rm pin} = E\left[\;\cdot\;-\;\cdot\;\right] - E\left[\;\cdot\;-\;\cdot\;\right]$$



Energy to create a vortex line on a nuclear impurity

Energy to create a vortex line in a uniform matter

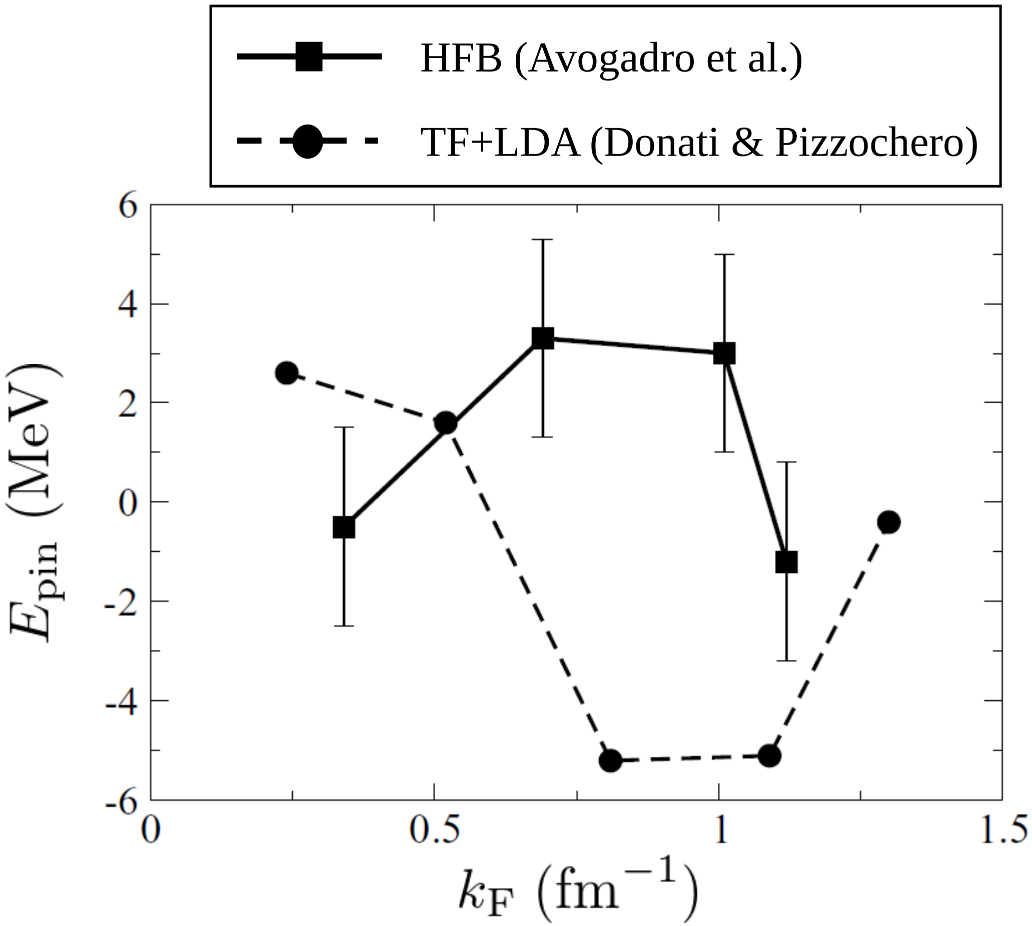
E.g.) 0.026 fm^{-3} (SLy4)

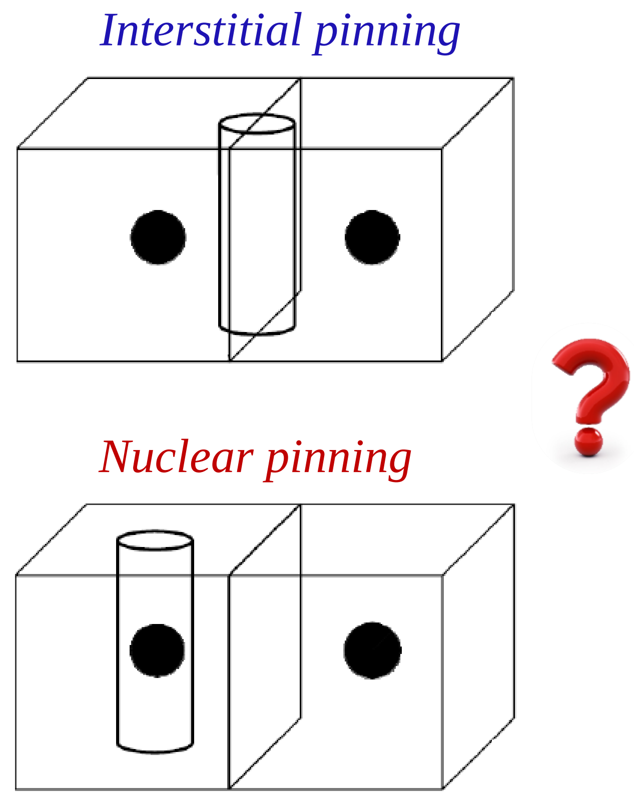
6.19 MeV	13058.04	12954.02	13714.88	13617.05

P. Avogadro, F. Barranco, R.A. Broglia, and E. Vigezzi, PRC**75**(2007)012805(R); NPA**788**(2007)130; NP**A811**(2008)378

What was the state-of-the-art?

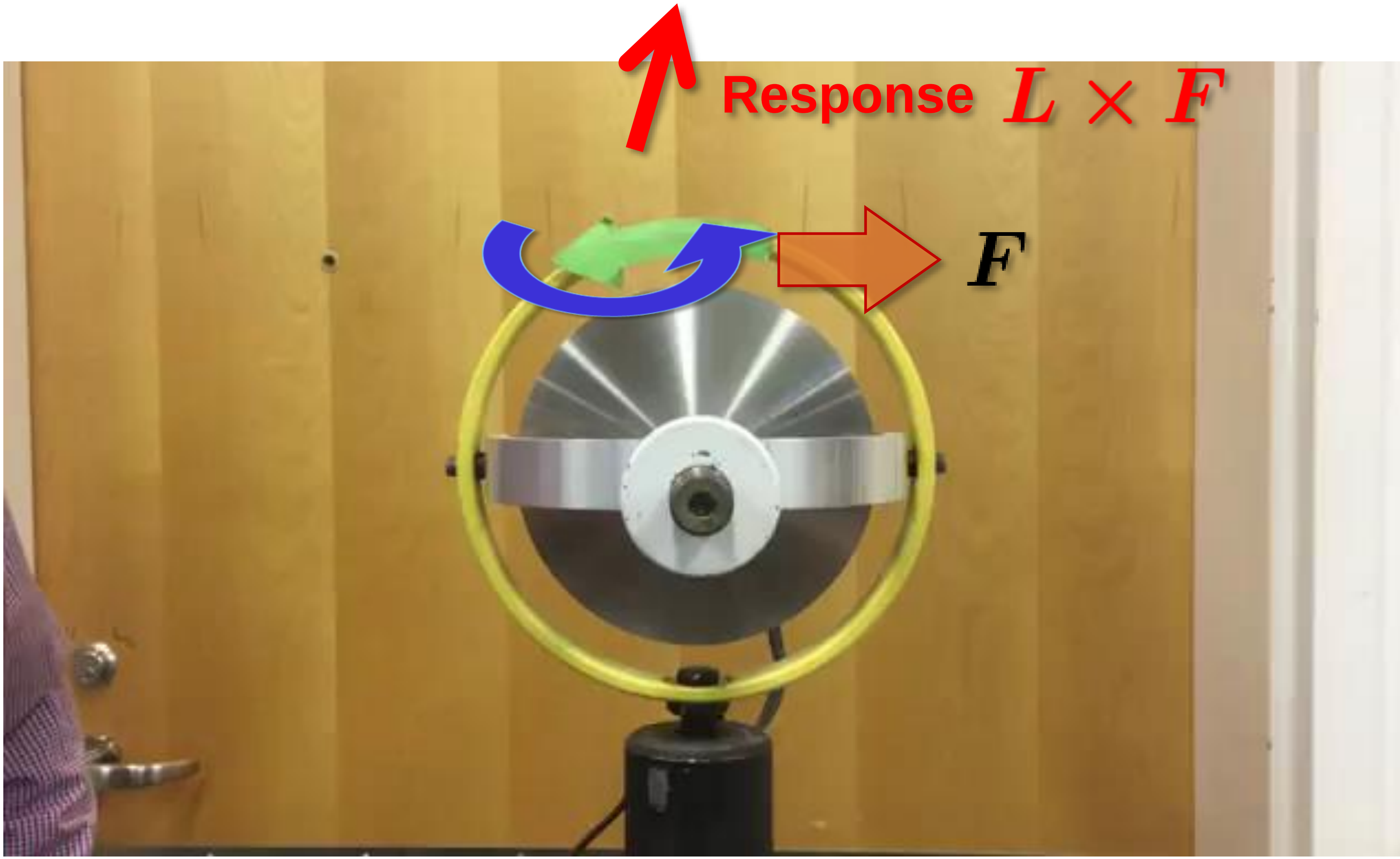
Property of the pinning force was unclear





P. Avogadro, F. Barranco, R.A. Broglia, and E. Vigezzi, PRC**75**(2007)012805(R); NP**A811**(2008)378

Response of a spinning gyroscope when pushed



We performed 3D, dynamical simulations by TDDFT with superfluidity

TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h(\mathbf{r}) \end{pmatrix} \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}$$

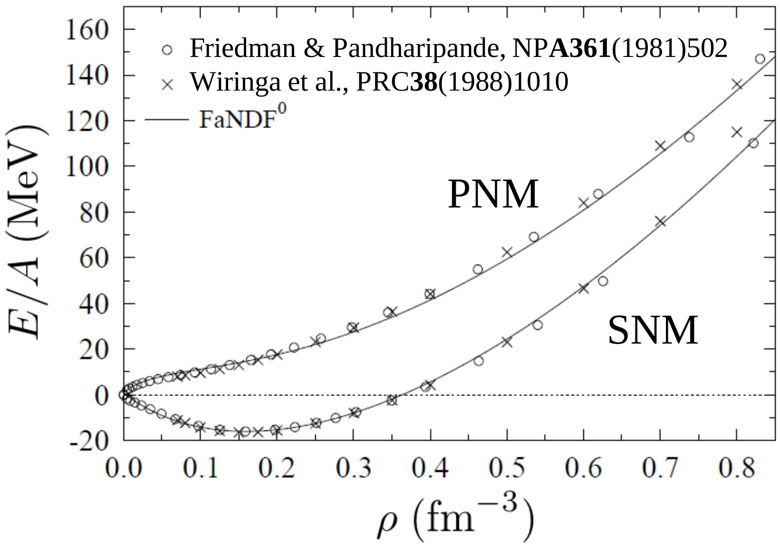
Energy density functional (EDF)

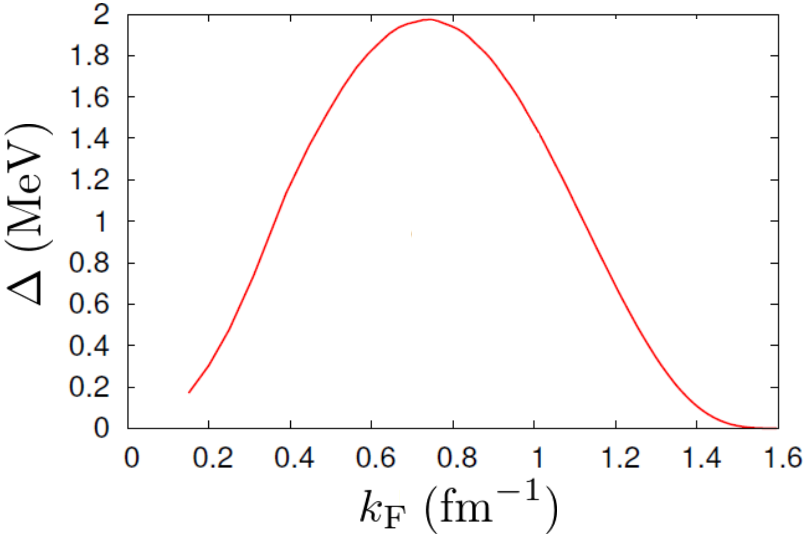
$$\mathcal{E}(\mathbf{r}) = \mathcal{E}_0(\mathbf{r}) + \mathcal{E}_{\rm pair}(\mathbf{r})$$

$\mathcal{E}_0(\mathbf{r})$: Fayans EDF (FaNDF^0) w/o LS

$$\mathcal{E}(\boldsymbol{r}) = \sum_{q=n,p} g[\rho_q(\boldsymbol{r})]|\nu_q(\boldsymbol{r})|^2$$

S.A. Fayans, JETP Lett. **68**, 169 (1998)





We performed 3D, dynamical simulations by TDDFT with superfluidity

TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}u_i(\mathbf{r})\\ v_i(\mathbf{r})\end{pmatrix}=\begin{pmatrix}h(\mathbf{r}) & \Delta(\mathbf{r})\\ \Delta^*(\mathbf{r}) & -h(\mathbf{r})\end{pmatrix}\begin{pmatrix}u_i(\mathbf{r})\\ v_i(\mathbf{r})\end{pmatrix}$$

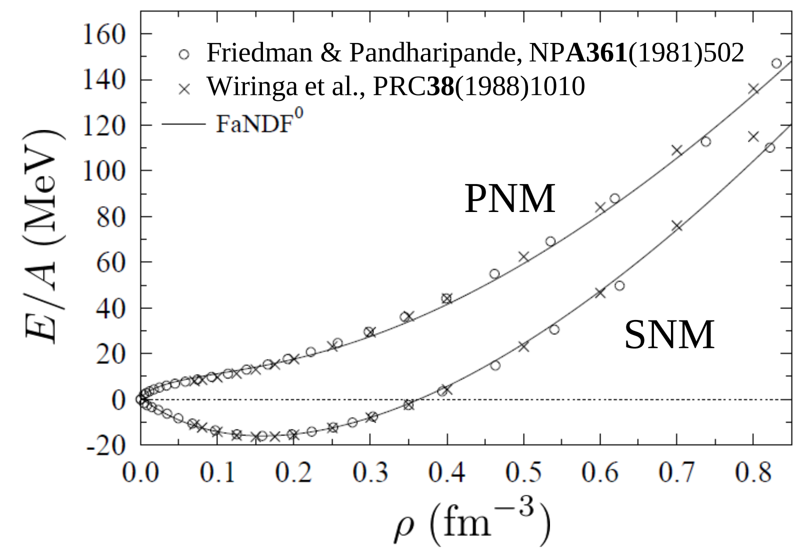
Energy density functional (EDF)

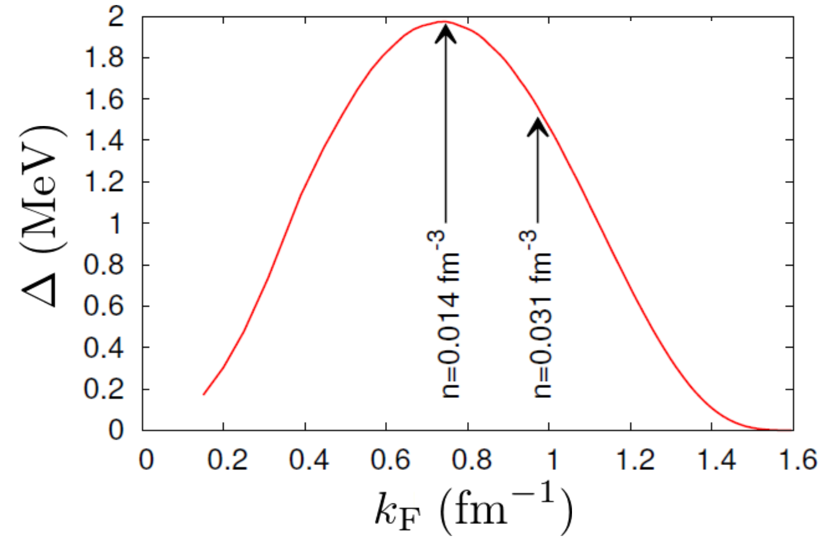
$$\mathcal{E}(\mathbf{r})=\mathcal{E}_0(\mathbf{r})+\mathcal{E}_{\rm pair}(\mathbf{r})$$

$\mathcal{E}_0(\mathbf{r})$: Fayans EDF (FaNDF^0) w/o LS

$$\mathcal{E}(\boldsymbol{r})=\sum_{q=n,p} g[\rho_q(\boldsymbol{r})]|\nu_q(\boldsymbol{r})|^2$$

S.A. Fayans, JETP Lett. **68**, 169 (1998)





We performed 3D, dynamical simulations by TDDFT with superfluidity

TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h(\mathbf{r}) \end{pmatrix} \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}$$

Computational details

$75\ \mathrm{fm} \times 75\ \mathrm{fm} \times 60\ \mathrm{fm}$

$(50 \times 50 \times 40,\ \Delta x = 1.5\ \mathrm{fm})$

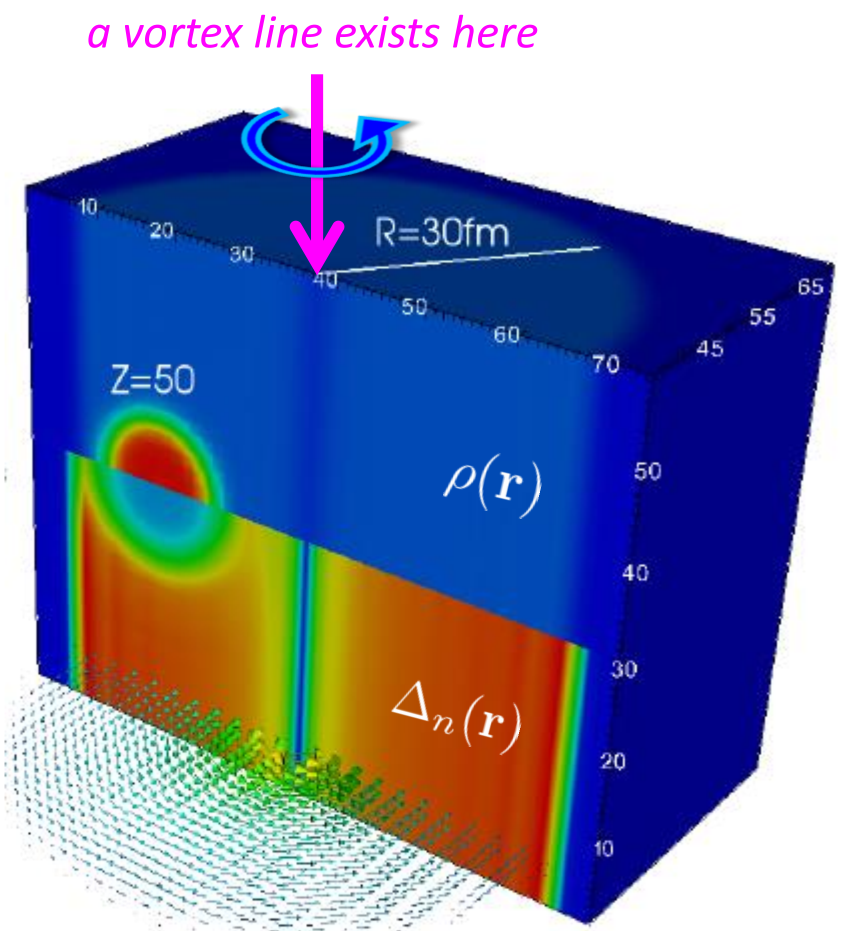
$k_\mathrm{c} = \pi/\Delta x > k_\mathrm{F} \qquad k_\mathrm{F} = (3\pi^2 \rho_n)^{1/3}$

Nuclear impurity: $Z = 50$

$\rho_n \simeq 0.014\ \mathrm{fm}^{-3}\ (N \simeq 2{,}530)$

$\rho_n \simeq 0.031\ \mathrm{fm}^{-3}\ (N \simeq 5{,}714)$

\# of quasi-particle w.f. $\approx 100,000$



$\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$

We performed 3D, dynamical simulations by TDDFT with superfluidity

☐ TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h(\mathbf{r}) \end{pmatrix}\begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}$$

☐ Computational details

$75\ \mathrm{fm} \times 75\ \mathrm{fm} \times 60\ \mathrm{fm}$

$(50 \times 50 \times 40,\ \Delta x = 1.5\ \mathrm{fm})$

$k_\mathrm{c} = \pi/\Delta x > k_\mathrm{F} \qquad k_\mathrm{F} = (3\pi^2 \rho_n)^{1/3}$

Nuclear impurity: $Z = 50$

$\rho_n \simeq 0.014\ \mathrm{fm}^{-3}\ (N \simeq 2{,}530)$

$\rho_n \simeq 0.031\ \mathrm{fm}^{-3}\ (N \simeq 5{,}714)$

# of quasi-particle w.f. $\approx 100{,}000$

MPI+GPU
→ 48h w/ 200GPUs
for 10,000 fm/c

TITAN, Oak Ridge

NERSC Edison, Berkeley

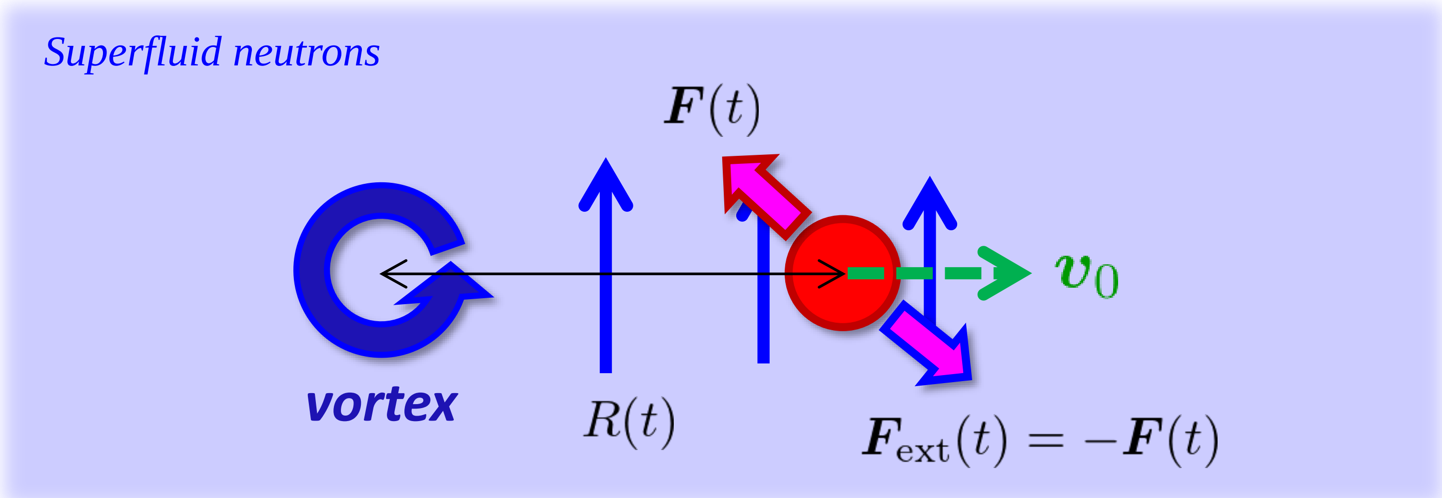
HA-PACS, Tsukuba

We directly measure the force $\boldsymbol{F}(R)$ in dynamical simulation

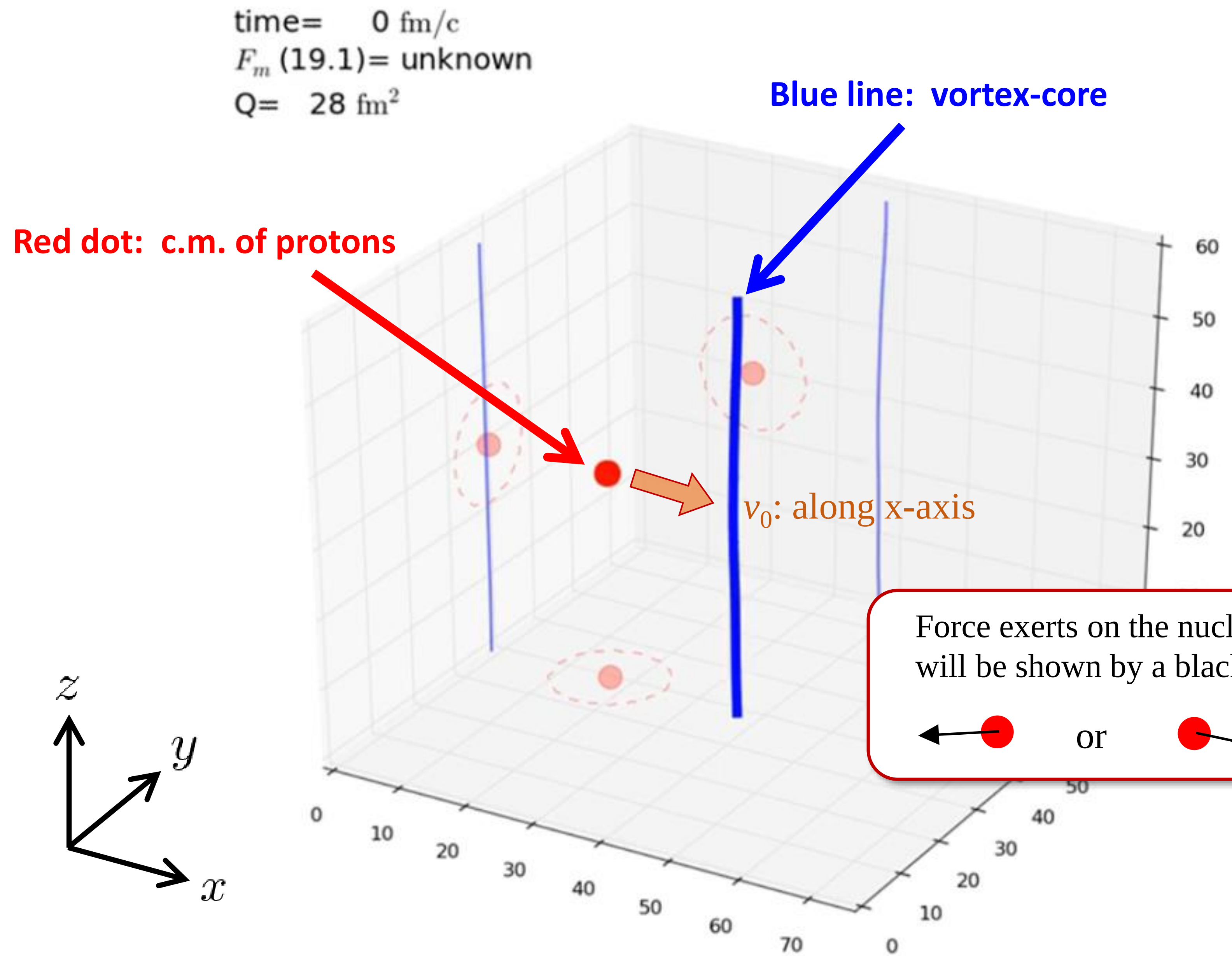
- Newton's law

$$\boldsymbol{F} = M\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \quad \Rightarrow \quad \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = 0 \quad \text{if} \quad \boldsymbol{F} = 0$$

- We keep a nuclear motion in a constant velocity v_0 $(\ll v_{\mathrm{crit}})$



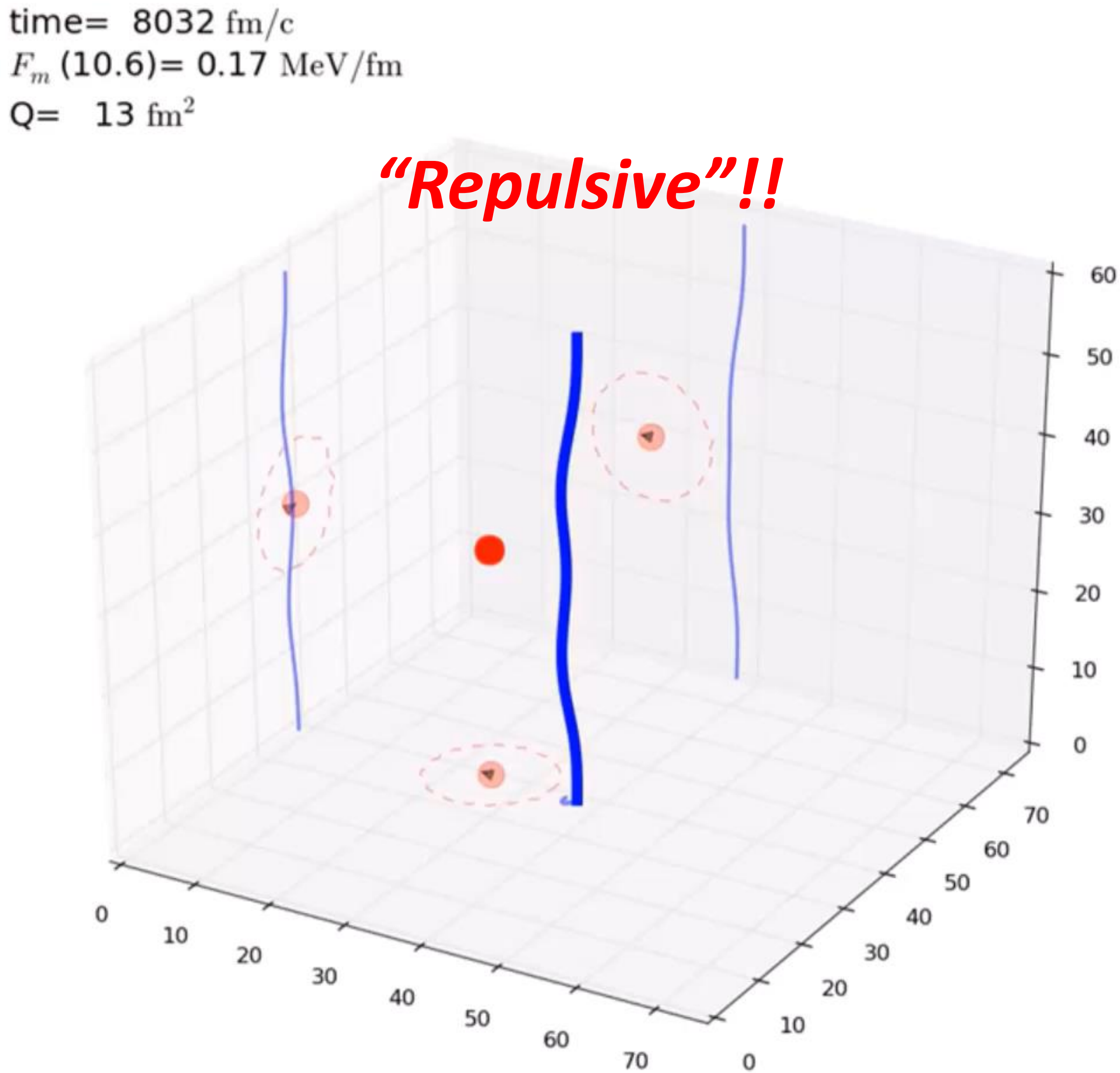
Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$



Force exerts on the nucleus will be shown by a black arrow

or ?

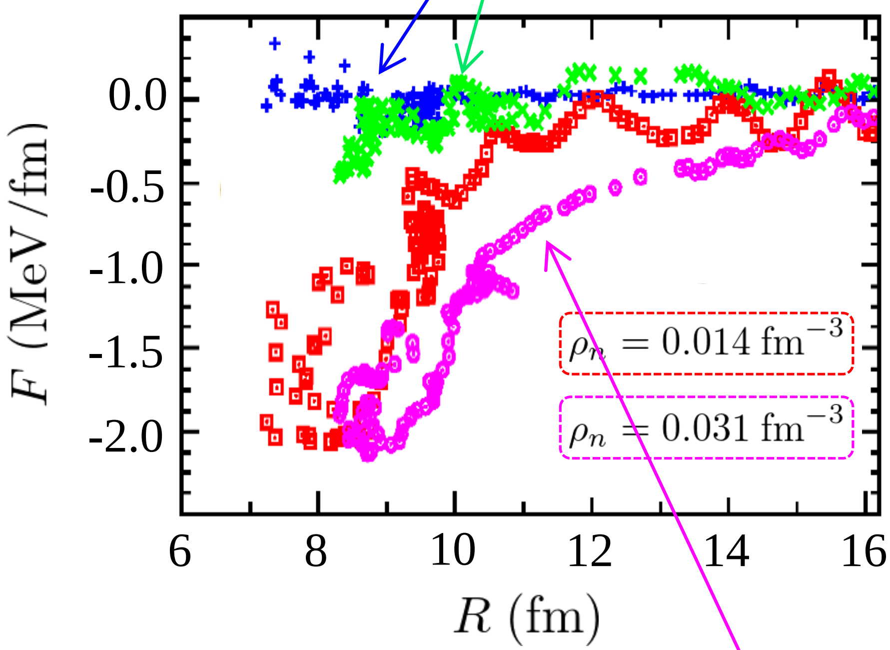
Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$



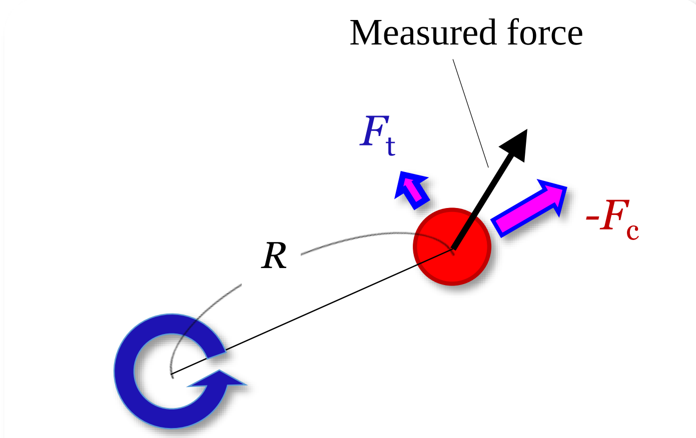
Measured force

The force is essentially central, not a simple function of *R*

F_t is negligibly small



Long-ranged for higher density
(smaller Δ, larger ξ)



F_c : negative for repulsive
F_t : positive for anti-clockwise

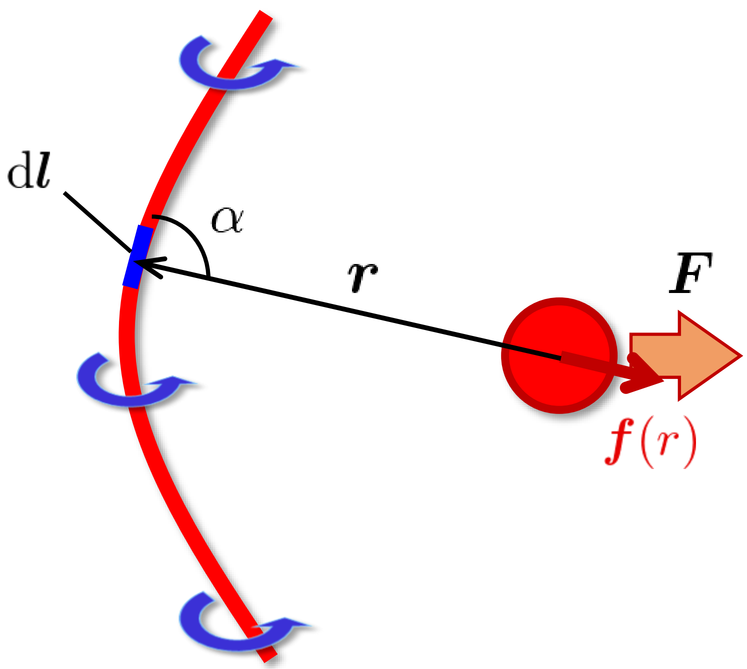
Force per unit length

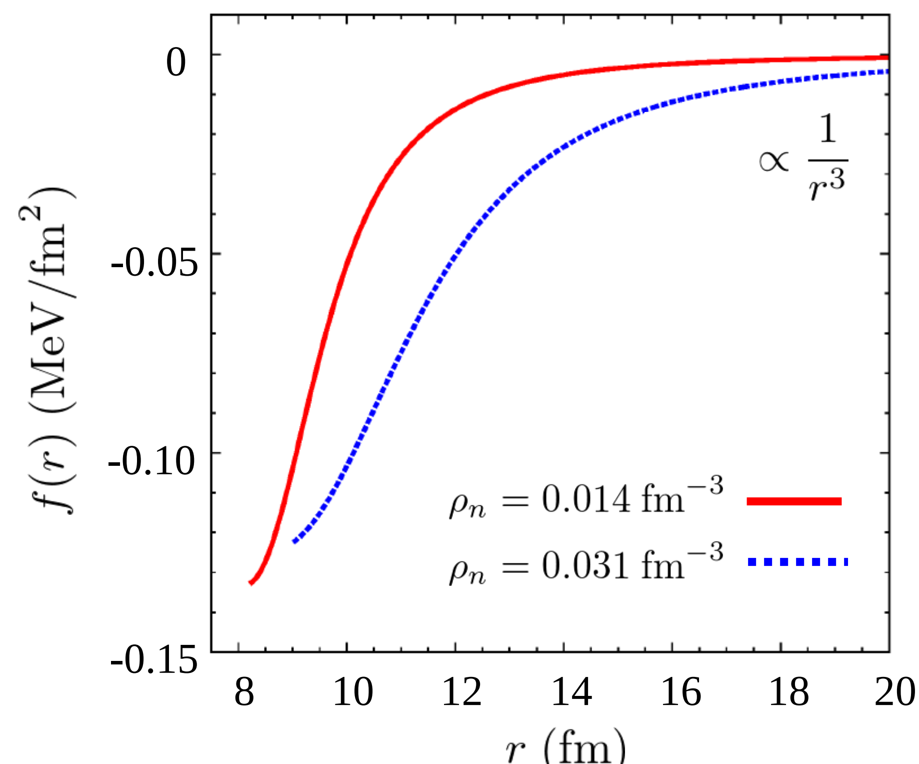
We can predict the force for any vortex-nucleus configuration

- Force per unit length

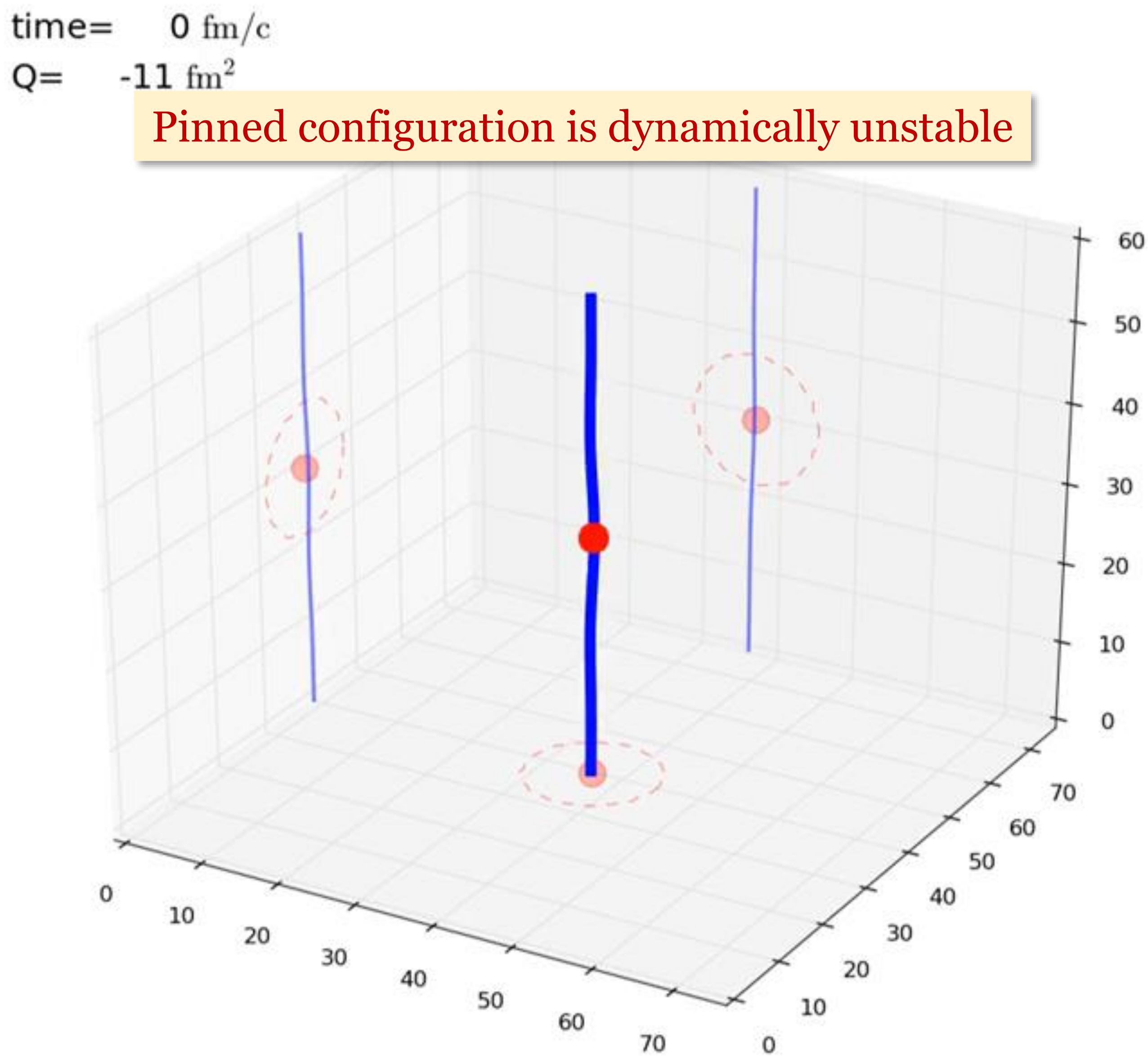
$$\boldsymbol{F} = \int_L f(r)\sin\alpha\,\boldsymbol{e}_r\,\mathrm{d}l \qquad f(r) = \frac{\sum_{k=0}^{n} a_k r^k}{1+\sum_{k=1}^{n+3} b_k r^k}$$

Padé approximant
(n=2 was used)





Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$

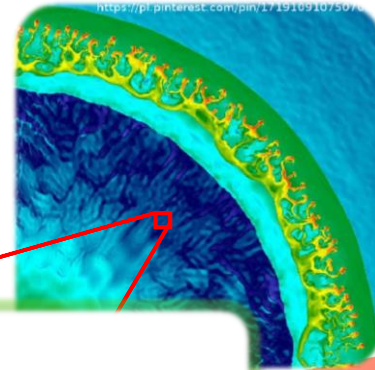


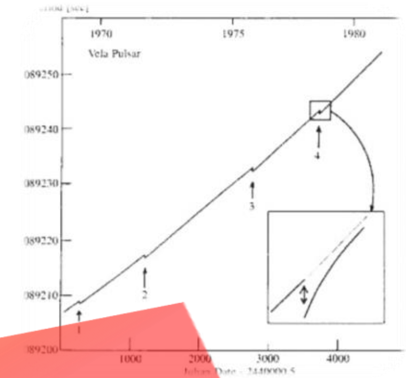
Pinned configuration is dynamically unstable

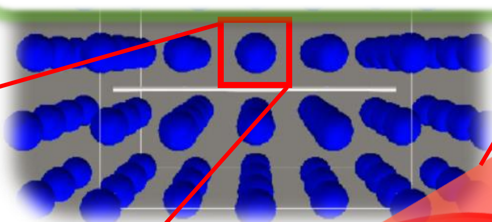
Our goal and strategy

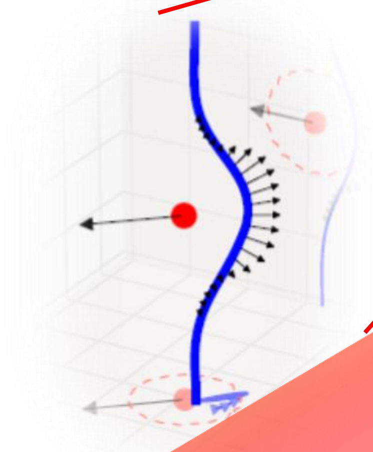
Goal: Unveil the mechanism of glitches

New collaboration started:
Nicolaus Copernicus Astronomical Centre
B. Haskell et al.









10^{4}m

Macroscopic

- observations
- hydrodynamics

$\sim 10^{-10}$m

Mesoscopic

- dynamics of *vortices* in a lattice of *nuclei* (e.g. filament model)

Provide model ingredients

10^{-15}-10^{-13}m

Microscopic

Nuclear Physics!!

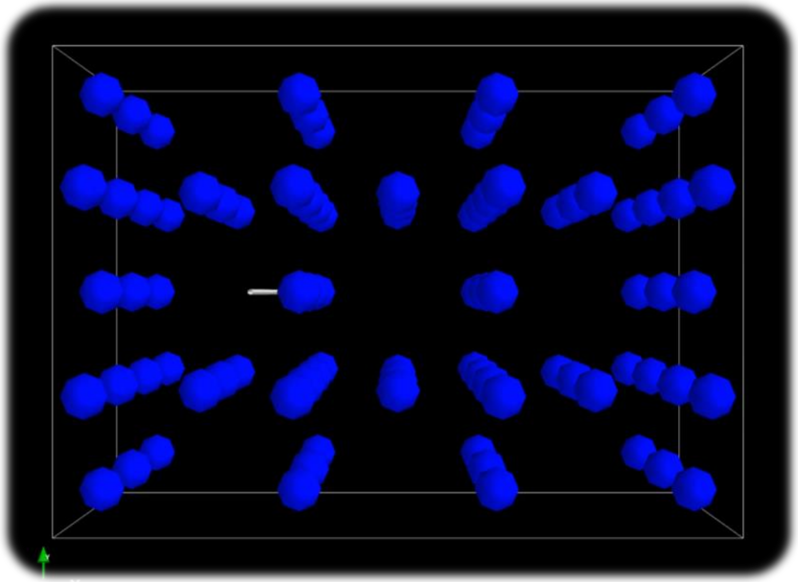
- vortex-nucleus dynamics from *neutrons and protons*

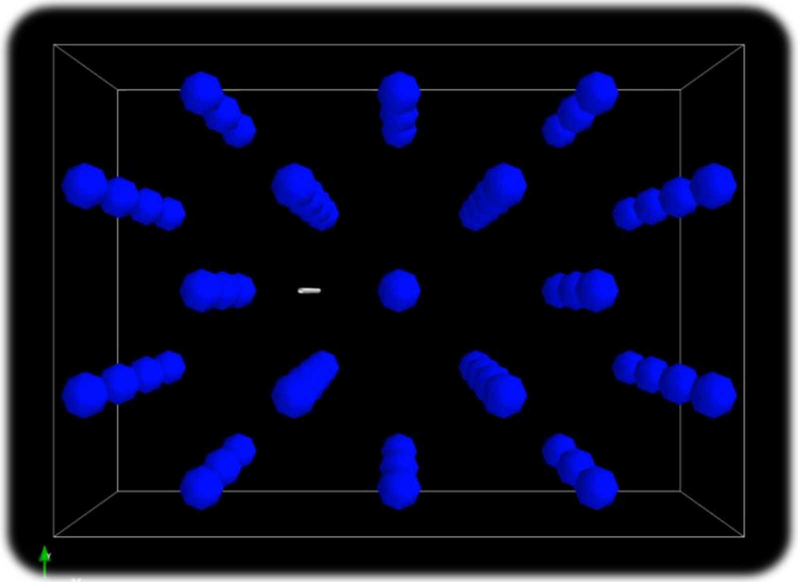
Ongoing project

Mesoscopic simulation of pinning force with the vortex filament model

Simulations by K. Kobczewski (PhD student at WUT)

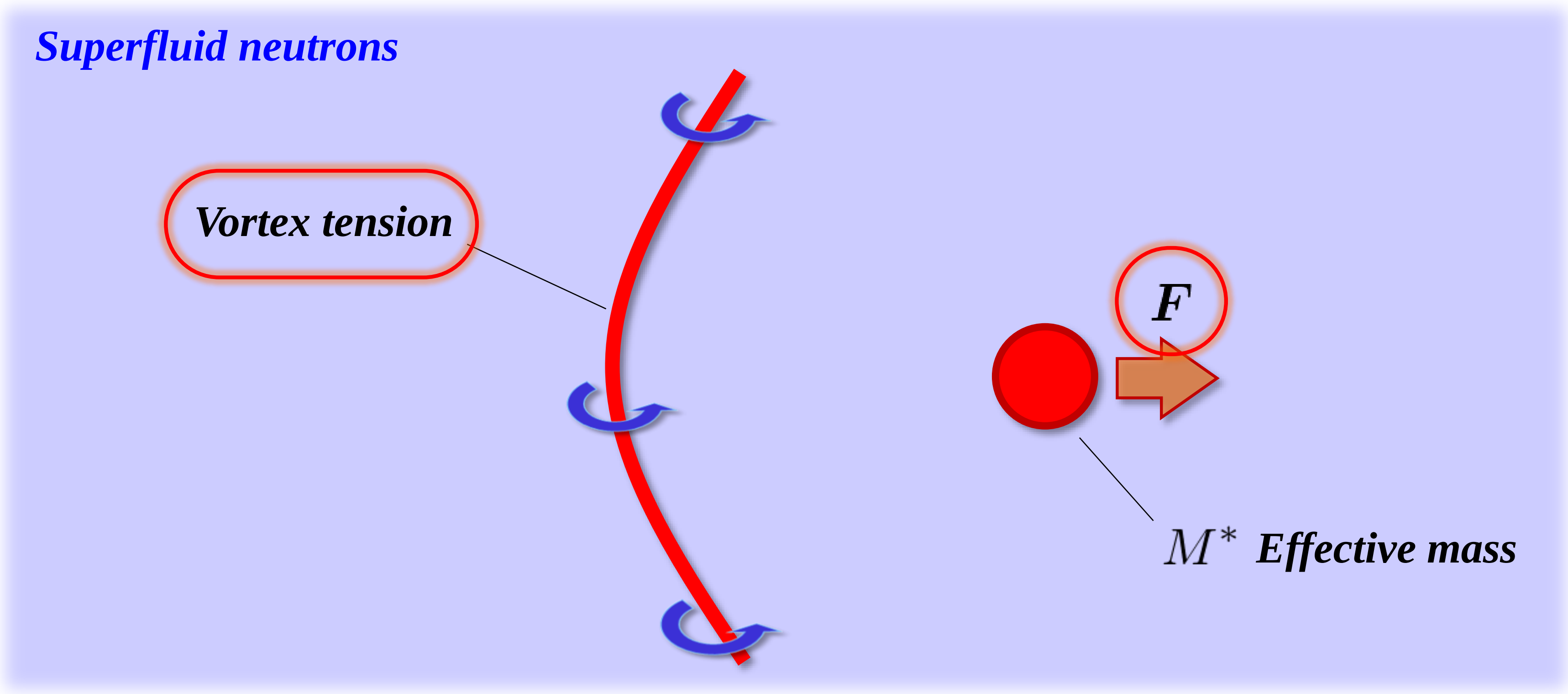
$$n_s \boldsymbol{\kappa} \times (\boldsymbol{v}_{vor} - \boldsymbol{v}_{ext}) = \boldsymbol{f}_{VN} + \boldsymbol{f}_{tension} + \boldsymbol{f}_{dissipation}$$





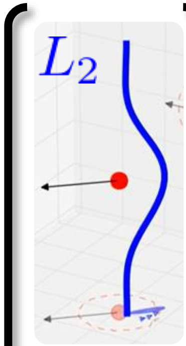
Talk by K. Kobczewski at POLNS18, March 26-28, 2018: https://indico.camk.edu.pl/event/10/contribution/8

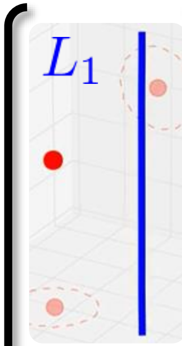
Very preliminary


Superfluid neutrons
Vortex tension
F
M^* Effective mass

We can evaluate the vortex tension from the dynamical simulations

$$T \lesssim \frac{E^*}{\Delta L} = \left(E\left[\text{(image 1)} \right] - E\left[\text{(image 2)} \right] \right) \Big/ \Delta L$$





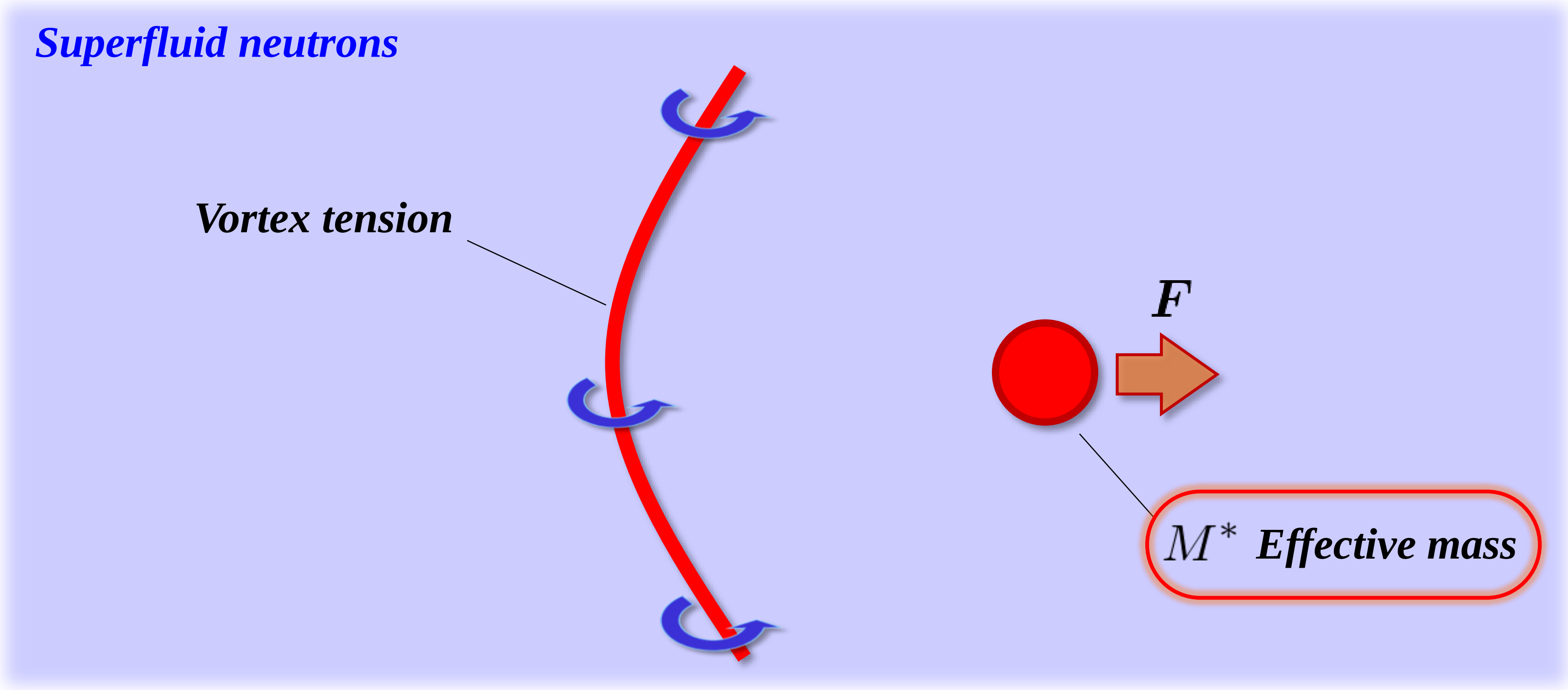
Work done by $\boldsymbol{F}_{\text{ext}}$

$$\int_{t_0}^{t_1} \boldsymbol{F}(t) \cdot \boldsymbol{v}(t) dt$$

$$T \lesssim \frac{5}{3.5} = 1.4\ \text{MeV/fm} \qquad n = 0.014\ \text{fm}^{-3}$$

$$T \lesssim \frac{11}{1.5} = 7.3\ \text{MeV/fm} \qquad n = 0.031\ \text{fm}^{-3}$$

$$\frac{1.4}{7.3} = 0.19$$ cf. hydrodynamic approx.: 0.77

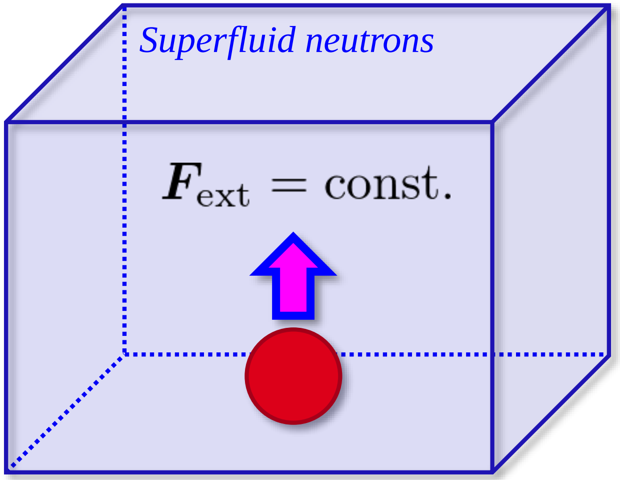

Superfluid neutrons
Vortex tension
F
M^* Effective mass

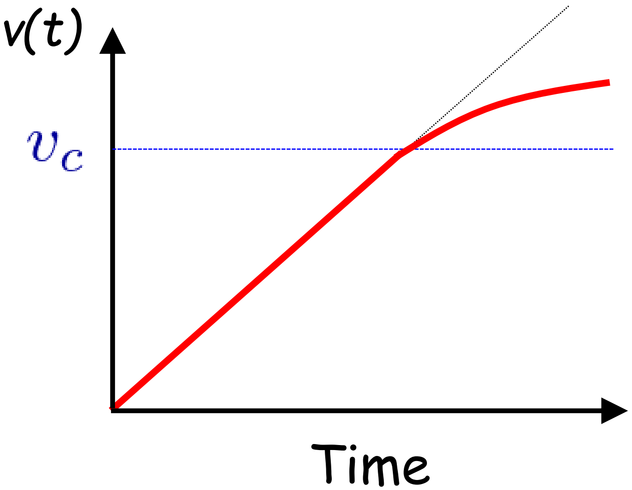
Dragging by a constant force provides the effective mass

- Newton's law

$$\boldsymbol{F} = M\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \quad \Rightarrow \quad M = F\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)^{-1}$$

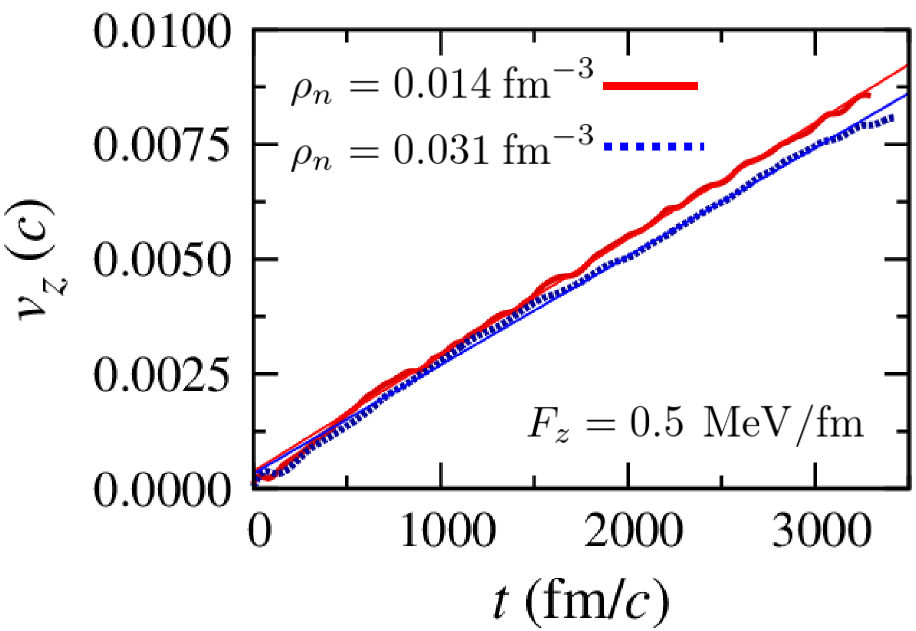
- We accelerate a nuclear impurity by a constant force





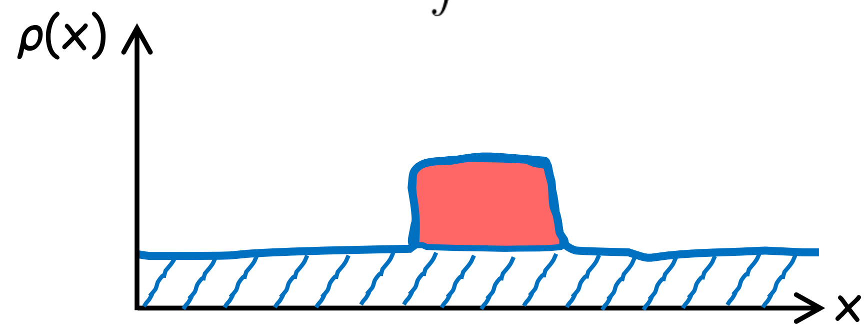
Dynamical effects may reduce the effective mass

Very preliminary



$$\frac{M^*}{M_{\rm nucl}} = \begin{cases} 0.96 & n = 0.014\ {\rm fm}^{-3} \\ 0.87 & n = 0.031\ {\rm fm}^{-3} \end{cases}$$

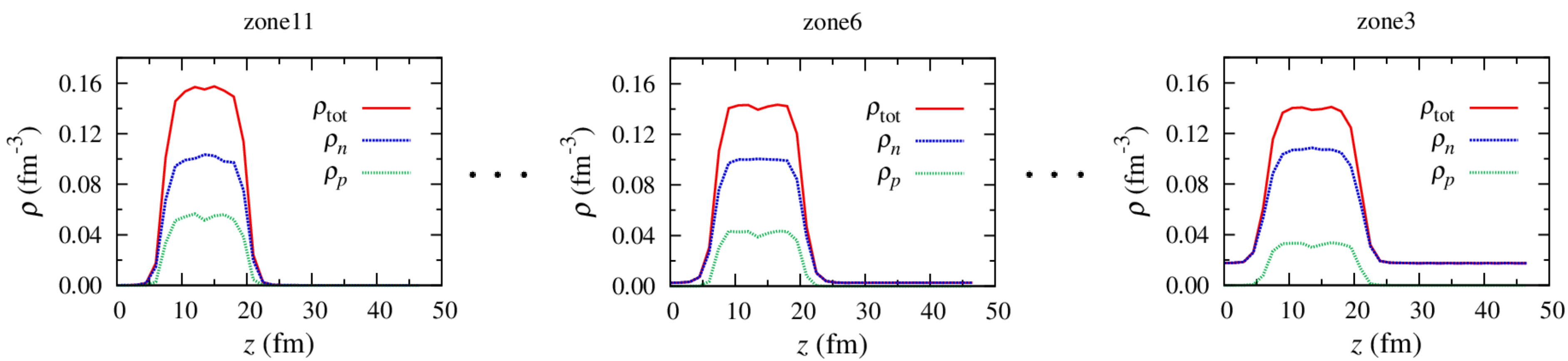
$$M_{\rm nucl} = m \int \left(\rho(\boldsymbol{r}) - \rho_n^{\rm b.g.}\right) d\boldsymbol{r}$$



Effective mass: future work

We are going to calculate M^* and v_c through out the inner crust

✓ We have prepared initial states for dynamical simulations

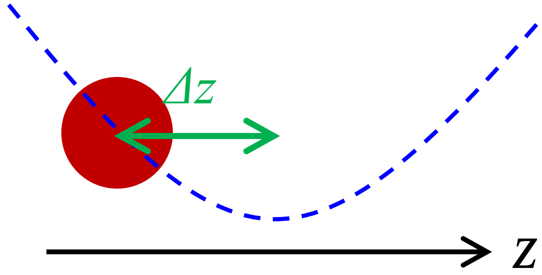


Coupling between the nuclear lattice and superfluid may be extracted

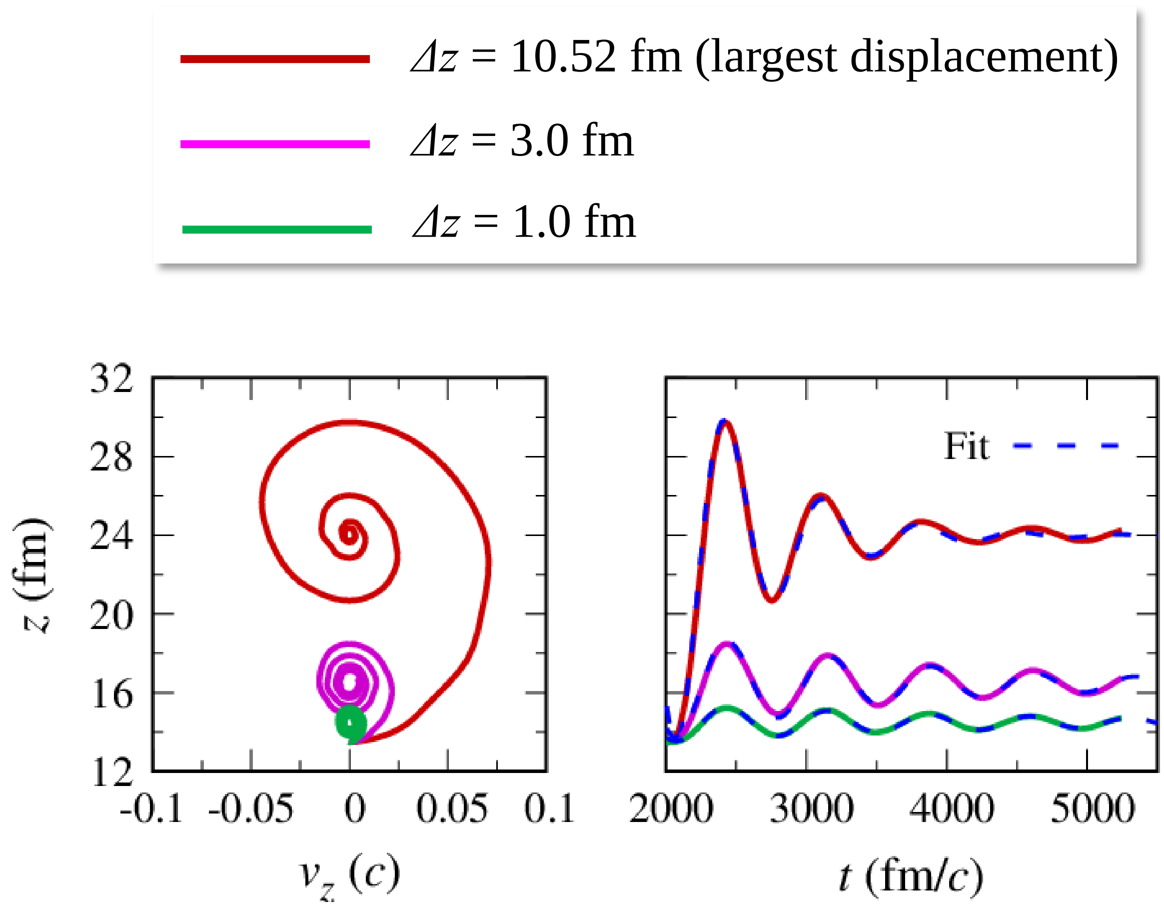
ρ_n~ 0.024 fm^{-1}: Z=50, R_{WS}=33 fm

Very preliminary

$$V(z) = \frac{1}{2} M^* \omega_p^2 z^2$$



$$\omega_p = \sqrt{\frac{3Z^2e^2}{M^* R_{\rm WS}^3}}$$



Γ = 0.330 MeV; τ = 598.46 fm/c

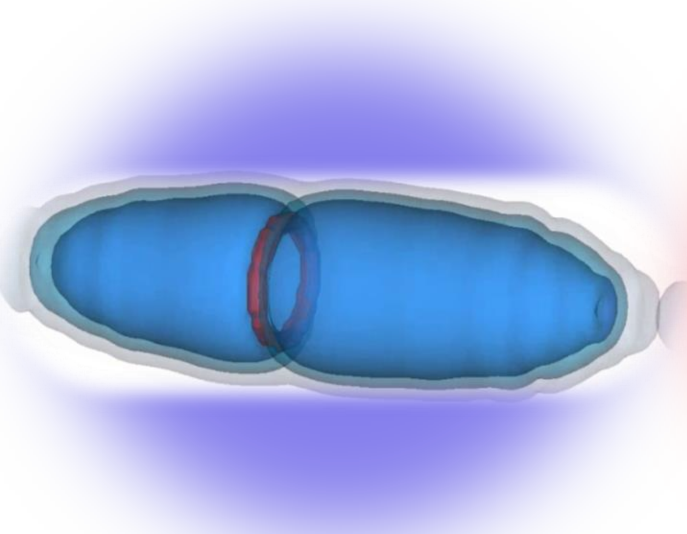
Γ = 0.123 MeV; τ = 1600.09 fm/c

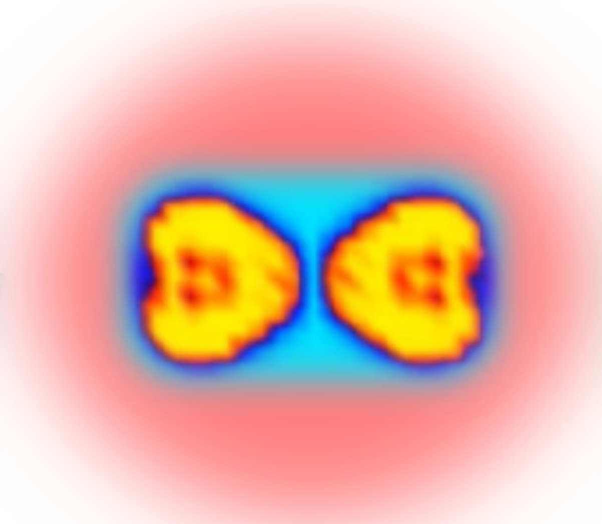
Γ = 0.083 MeV; τ = 2370.53 fm/c

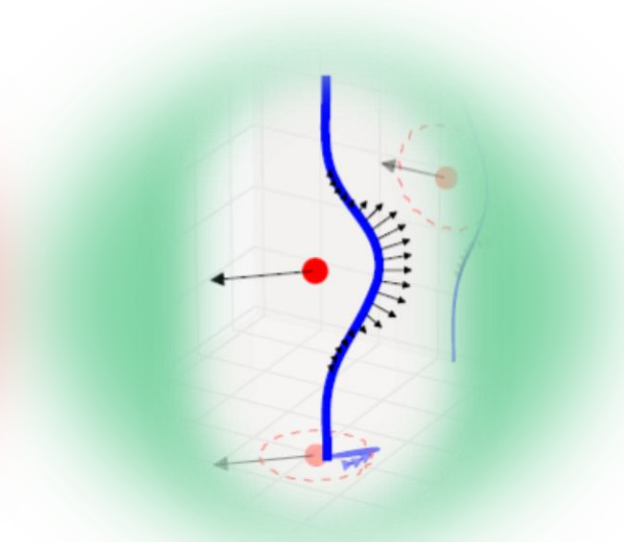
Conclusion

Takeaway message

✓ TDSLDA is a powerful tool to study a variety of dynamics in superfluid Fermi systems!







Cold atoms:
UFG

- solitonic cascades
- quantum turbulence

Atomic nuclei:
Nuclear dynamics

- solitonic excitations
- fusion hindrance

Neutron star:
Inner crust

- vortex-nucleus interaction
- tension, M^*, dissipations

Kazuyuki Sekizawa
Specially Appointed Assistant Professor
Center for Transdisciplinary Research
Institute for Research Promotion, Niigata University
8050, Ikarashi Ninoho, Nishi-ku, Niigata City, Niigata 950-2181, Japan
sekizawa @ phys.sc.niigata-u.ac.jp
http://sekizawa.fizyka.pw.edu.pl/english/