Relativistic Hartree-Fock calculation for nuclear matter

Tsuyoshi Miyatsu

Department of Physics, Faculty of Science and Technology,
Tokyo University of Science

Seminar @ Chiba Institute of Technology
October 28, 2017

Abstract

陽子や中性子を原子核に束縛する核力（強い力）理論をもとに、80年以上もの長い間にわたり、原子核や核物質の物性は現在でも盛んに研究され続けています。一方、近年では科学技術の向上による高精度天体観測などから、地上では実現不可能な極限状態における物理現象に関する情報を得ることが可能となりつつあります。

これまでの原子核研究は、主に非相対論枠組みに基づく計算で行われてきましたが、超巨大原子核とみなしうる中性子星などを想定した高密度核物質を研究する場合には、相対論的効果を考慮する必要があります。

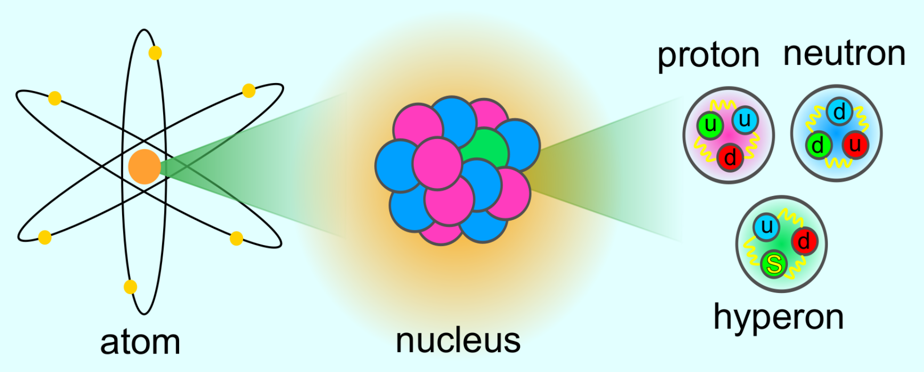
今回のセミナーでは、相対論的枠組みの中で場の理論による核子多体計算（HartreeからHartree-Fock近似）がどのように定式化されているかにフォーカスし、具体的な計算手法を説明します。応用として、相対論的Hartree-Fock計算を通常核子密度付近の核物質から中性子星物質に適応した計算結果を紹介したいと考えています。

Table of contents

Introduction

物質の最小単位としてのクォーク

- 物質の最小単位：クォークとグルーオン
- 全ての物質はクォークとグルーオンの組み合わせにより構成
  ⇒ ハドロン (バリオンと中間子)
- クォークはハドロン内に「閉じ込め」られている
- クォークやグルーオンの運動は量子色力学 (QCD) で記述



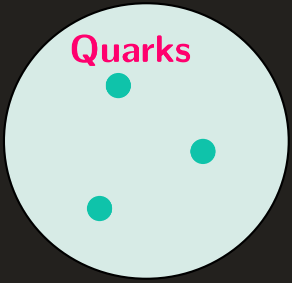
Fundamental interaction

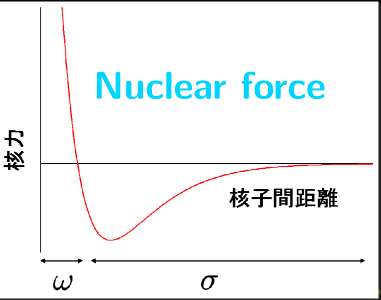
Properties of the Interactions

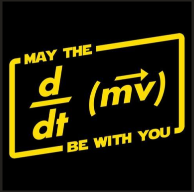
The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

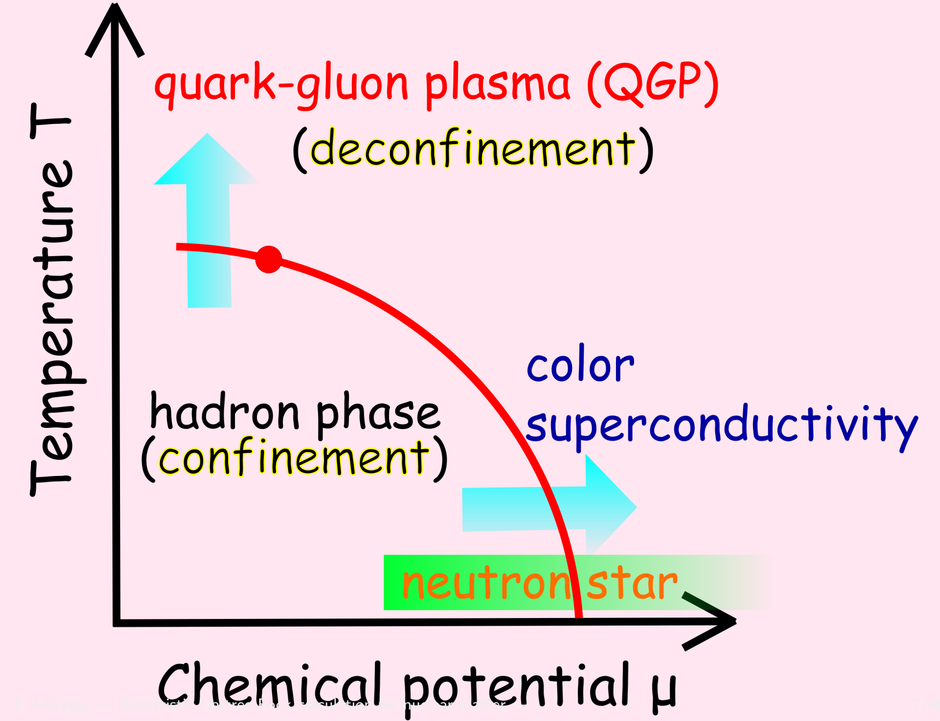
Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at 10^{-18} m	10^{-41}	0.8	1	25
Strength at 3×10^{-17} m	10^{-41}	10^{-4}	1	60

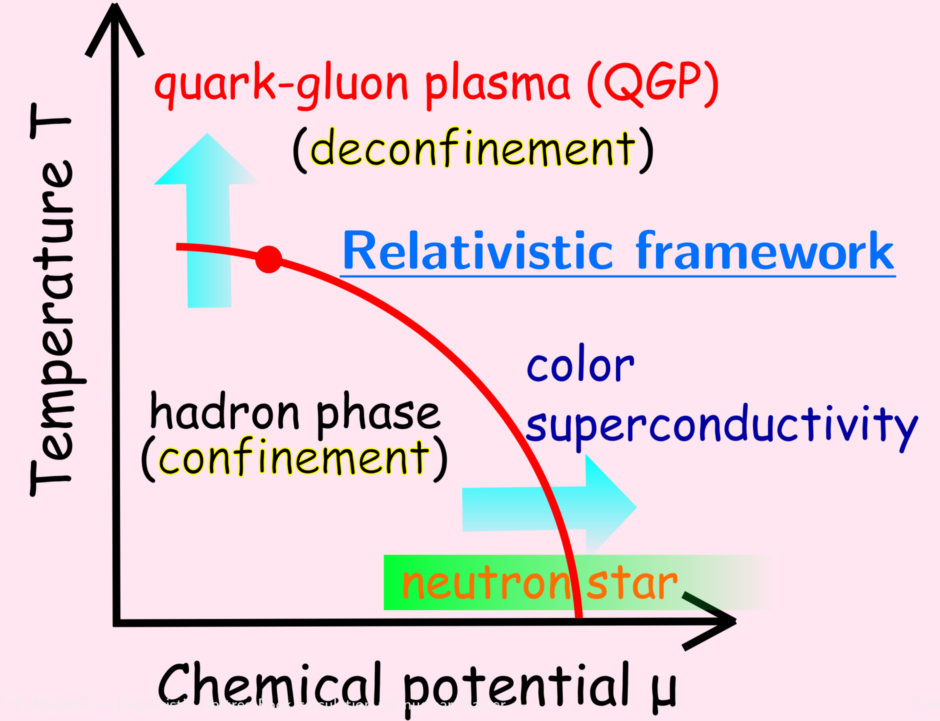
http://cpepweb.org








quark-gluon plasma (QGP)
(deconfinement)
Temperature T
color
superconductivity
hadron phase
(confinement)
neutron star
Chemical potential μ


quark-gluon plasma (QGP)
(deconfinement)
Relativistic framework
color superconductivity
hadron phase
(confinement)
neutron star
Temperature T
Chemical potential μ

Relativistic many-body calculations I

Lagrangian density

Meson theory of nuclear matter

For the NN interaction at low energy, there are essentially only three boson fields which are of relevance

- **the pseudoscalar (ps) field**,
- **the scalar (s) field**, and
- **the vector (v) field**.

The modern point of view is to consider these fields as effective (nonfundamental) fields.

Guided by symmetry principles, simplicity, and physical intuition, the most commonly used interaction Lagrangian which couple these fields to the nucleon (ψ) are

$$\mathcal{L}_{ps} = -g_{ps}\bar{\psi}i\gamma_5\psi\varphi^{(ps)},$$

$$\mathcal{L}_{s} = +g_{s}\bar{\psi}\psi\varphi^{(s)},$$

$$\mathcal{L}_{v} = -g_{v}\bar{\psi}\gamma^{\mu}\psi\varphi^{(v)} - \frac{f_v}{4M}\bar{\psi}\sigma^{\mu\nu}\psi\left(\partial_\mu\varphi_\nu^{(v)} - \partial_\nu\varphi_\mu^{(v)}\right).$$

R. Machleidt, 'The Meson theory of nuclear forces and nuclear structure,' Adv. Nucl. Phys. **19**, 189 (1989).

Pseudoscalar and/or pseudovector

For the ps field, there is also **the so-called pseudovector (pv) or gredient coupling to the nucleon**, which is suggested as effective coupling by chiral symmetry:

$$\mathcal{L}pv = -\frac{f_{ps}}{m_{ps}}\bar{\psi}\gamma_5\gamma^{\mu}\psi\partial_{\mu}\varphi^{(ps)}.$$

The ps and pv couplings are equivalent for on-mass-shell nucleons if coupling constants are related by $f_{ps} = \left(\frac{m_{ps}}{2M}\right) g_{ps}$. However, for off-shell cases, the predictions are rather different. Anti-particle contributions turn out to be huge using the pseudoscalar coupling whereas they are suppressed by the gradient coupling.

One boson exchange description

According to **the one boson exchange (OBE) description of the NN interaction**, we start from an effective Lagrangian density constructed from the degrees of freedom associated with two isoscalr mesons (σ and ω) and two isovector mesons ($\boldsymbol{\pi}$ and $\boldsymbol{\rho}$) with the following quantum numbers (J^π, T):

$$\sigma(0^+,0), \quad \omega(1^-,0), \quad \boldsymbol{\pi}(0^-,1), \quad \boldsymbol{\rho}(1^-,1).$$

A. Bouyssy, J. F. Mathiot, V. G. Nguyen, and S. Marcos, Phys. Rev. C **36**, 380 (1987).

Simple Lagrangian density

The Lagrangian density is written as ($U_{\rm NL} = \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 - \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2$)

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_N + \mathcal{L}_M + \mathcal{L}_{\rm int} - U_{\rm NL} \\ &= \sum_{N=p,n} \bar{\psi}_N \left(i\gamma_\mu\partial^\mu - M_N\right)\psi_N + \mathcal{L}_M + \mathcal{L}_{\rm int} - U_{\rm NL},\end{aligned}$$

where the meson and interaction terms read

$$\begin{aligned}\mathcal{L}_M &= \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}W_{\mu\nu}W^{\mu\nu} \\ &\quad + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu - \frac{1}{4}\boldsymbol{R}_{\mu\nu}\cdot\boldsymbol{R}^{\mu\nu} + \frac{1}{2}\left(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - m_\pi^2\boldsymbol{\pi}^2\right), \\ \mathcal{L}_{\rm int} &= \sum_{N=p,n}\bar{\psi}_N\Bigg(g_{\sigma N}\sigma - g_{\omega N}\gamma_\mu\omega^\mu + \frac{f_{\omega N}}{2\mathcal{M}}\sigma_{\mu\nu}\partial^\nu\omega^\mu \\ &\quad - g_{\rho N}\gamma_\mu\boldsymbol{\rho}^\mu\cdot\boldsymbol{\tau}_N + \frac{f_{\rho N}}{2\mathcal{M}}\sigma_{\mu\nu}\partial^\nu\boldsymbol{\rho}^\mu\cdot\boldsymbol{\tau}_N - \frac{f_{\pi N}}{m_\pi}\gamma_5\gamma_\mu\partial^\mu\boldsymbol{\pi}\cdot\boldsymbol{\tau}_N\Bigg)\psi_N,\end{aligned}$$

with

$$W_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu, \quad \boldsymbol{R}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu, \quad \sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu,\gamma_\nu\right].$$

Relativistic many-body calculations I

Eular-Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial \Phi_\alpha} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \Phi_\alpha \right)} \right] = 0 \quad (\alpha = N, \sigma, \omega, \boldsymbol{\rho}, \boldsymbol{\pi}) \, .$$

Dirac fields

Equations of motion for **Dirac fields**:

Nucleon:

$$(i\gamma_\mu \partial^\mu - M_N)\,\psi_N = -g_{\sigma N}\sigma\psi_N + \left(g_{\omega N}\gamma_\mu\omega^\mu - \frac{f_{\omega N}}{2\mathcal{M}}\sigma_{\mu\nu}\partial^\nu\omega^\mu \right)\psi_N$$

$$+ \left(g_{\rho N}\gamma_\mu \boldsymbol{\rho}^\mu \cdot \boldsymbol{\tau}_N - \frac{f_{\rho N}}{2\mathcal{M}}\sigma_{\mu\nu}\partial^\nu \boldsymbol{\rho}^\mu \cdot \boldsymbol{\tau}_N \right)\psi_N + \frac{f_{\pi N}}{m_\pi}\gamma_5\gamma_\mu\partial^\mu \boldsymbol{\pi}\cdot\boldsymbol{\tau}_N \psi$$

Anti-nucleon:

$$\bar{\psi}_N \left(i\gamma_\mu \overleftarrow{\partial}^\mu + M_N \right) = \bar{\psi}_N g_{\sigma N}\sigma - \bar{\psi}_N \left(g_{\omega N}\gamma_\mu\omega^\mu - \frac{f_{\omega N}}{2\mathcal{M}}\sigma_{\mu\nu}\partial^\nu\omega^\mu \right)$$

$$- \bar{\psi}_N \left(g_{\rho N}\gamma_\mu \boldsymbol{\rho}^\mu \cdot \boldsymbol{\tau}_N - \frac{f_{\rho N}}{2\mathcal{M}}\sigma_{\mu\nu}\partial^\nu \boldsymbol{\rho}^\mu \cdot \boldsymbol{\tau}_N \right) - \frac{f_{\pi N}}{m_\pi}\bar{\psi}_N\gamma_5\gamma_\mu\partial^\mu \boldsymbol{\pi}$$

Meson fields

Equations of motion for **meson fields**:

σ
$$\left[\partial_\mu\partial^\mu + m_\sigma^{*2}(\sigma)\right]\sigma = \sum_{N=p,n} g_{\sigma N}\bar{\psi}_N\psi_N, \quad \text{with} \quad m_\sigma^{*2}(\sigma) = m_\sigma^2 + g_2\sigma + g_3\sigma^2,$$

ω
$$\partial_\lambda W^{\lambda\kappa} + m_\omega^{*2}(\omega)\omega^\kappa = \sum_{N=p,n}\left[g_{\omega N}\bar{\psi}_N\gamma^\kappa\psi_N - \frac{f_{\omega N}}{2\mathcal{M}}\partial_\lambda\left(\bar{\psi}_N\sigma^{\lambda\kappa}\psi_N\right)\right],$$
$$\text{with} \quad m_\omega^{*2}(\omega) = m_\omega^2 + c_3\left(\omega_\mu\omega^\mu\right)\omega^\kappa,$$

ρ
$$\partial_\lambda \boldsymbol{R}^{\lambda\kappa} + m_\rho^2\boldsymbol{\rho}^\kappa = \sum_{N=p,n}\left[g_{\rho N}\bar{\psi}_N\gamma^\kappa\boldsymbol{\tau}_N\psi_N - \frac{f_{\rho N}}{2\mathcal{M}}\partial_\lambda\left(\bar{\psi}_N\sigma^{\lambda\kappa}\boldsymbol{\tau}_N\psi_N\right)\right],$$

π
$$\left(\partial_\lambda\partial^\lambda + m_\pi^2\right)\boldsymbol{\pi} = \sum_{N=p,n}\frac{f_{\pi N}}{m_\pi}\partial_\lambda\left(\bar{\psi}_N\gamma_5\gamma^\lambda\boldsymbol{\tau}_N\psi_N\right).$$

e.g. Hartree approximation

In **the mean-field approximation**, the meson fields are replaced by the constant mean-field (classical expectation) values.
⇒ **Hartree approximation** (**tadpole diagram**).

$$\langle\sigma\rangle = \bar{\sigma} + \delta\sigma = \bar{\sigma},$$
$$\langle\omega_\mu\rangle = \delta_{\mu 0}\bar{\omega} + \delta\omega = \delta_{\mu 0}\bar{\omega},$$
$$\langle\rho_\mu\rangle = \delta_{\mu 0}\bar{\rho} + \delta\rho = \delta_{\mu 0}\bar{\rho}, \quad \bar{\rho} = \rho^0,$$
$$\langle\pi\rangle = \bar{\pi} + \delta\pi = 0.$$

Therefore

$$\bar{\sigma} = \sum_{N=p,n} \frac{g_{\sigma N}}{m_\sigma^{*2}(\bar{\sigma})} \langle\bar{\psi}_N \psi_N\rangle \quad \text{or} \quad \bar{\sigma} = \sum_{N=p,n} \frac{g_{\sigma N}}{m_\sigma^2} \langle\bar{\psi}_N \psi_N\rangle - \frac{1}{m_\sigma^2}\left(g_2\bar{\sigma}^2 + g_3\bar{\sigma}^3\right),$$

$$\bar{\omega} = \sum_{N=p,n} \frac{g_{\omega N}}{m_\omega^{*2}(\bar{\omega})} \langle\bar{\psi}_N \gamma^0 \psi_N\rangle \quad \text{or} \quad \bar{\omega} = \sum_{N=p,n} \frac{g_{\omega N}}{m_\omega^2} \langle\bar{\psi}_N \gamma^0 \psi_N\rangle - \frac{c_3}{m_\omega^2}\bar{\omega}^3,$$

$$\bar{\rho} = \sum_{N=p,n} \frac{g_{\rho N}}{m_\rho^2} \left(\boldsymbol{\tau}_N\right)_z \langle\bar{\psi}_N \gamma^0 \psi_N\rangle, \qquad \bar{\pi} = 0.$$

Relativistic many-body calculations II

Green's function

Green's function

The general definition of **2n-point Green's function** is the set of average diagonal matrix elements [1]

$$G^n_{N_1\cdots N_n N_{1'}\cdots N_{n'}}\left(x_1,\cdots,x_n;x'_1,\cdots,x'_n\right) = (-i)^n \langle\Psi_0| T\left[\psi_{N_1}(x_1)\cdots\psi_{N_n}(x_n)\bar{\psi}_{N_{n'}}(x'_n)\cdots\bar{\psi}_{N_1}(x'_1)\right]|\Psi_0\rangle\,,$$

where x_i or i ($i \in \{1,\cdots,n\}$) includes the spin and isospin wight for nucleon, N, and $|\Psi_0\rangle$ is the grand state of infinite nuclear matter.

The two-point Green's function of nucleon:

$$\begin{aligned} iG_{NN'}\left(x,x'\right) &:= \langle\Psi_0|T\left[\psi_N(x)\bar{\psi}_{N'}(x')\right]|\Psi_0\rangle \\ &= \langle\Psi_0|\psi_N(x)\bar{\psi}_{N'}(x')|\Psi_0\rangle\,\theta\left(t-t'\right) - \langle\Psi_0|\bar{\psi}_{N'}(x')\psi_N(x)|\Psi_0\rangle\,\theta\left(t'-t\right) \\ &= iG^{>}_{NN'}\left(x,x'\right)\theta\left(t-t'\right) + iG^{<}_{NN'}\left(x,x'\right)\theta\left(t'-t\right). \end{aligned}$$

[1] We adopt the manner given in Refs. P. C. Martin and J. S. Schwinger, Phys. Rev. **115**, 1342 (1959), B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).

The nucleon propagator which is applied by $(i\gamma_\mu \partial_x^\mu - M_N)$ is given by

$$\begin{aligned}(i\gamma_\mu \partial_x^\mu - M_N) G_{NN'}(x, x') &= \delta_{NN'} \delta^4 (x - x') - i\mathcal{F}_{NN'}(x, x') \\ &= (i\gamma_\mu \partial_x^\mu - M_N) G^0_{NN'}(x, x') - i\mathcal{F}_{NN'}(x, x'),\end{aligned}$$

with

$$\begin{aligned}\mathcal{F}_{NN'}(x, x') = &-g_{\sigma N} \langle \Psi_0 | T \left[\sigma(x) \psi_N(x) \bar{\psi}_{N'}(x') \right] | \Psi_0 \rangle \\ &+ g_{\omega N} \langle \Psi_0 | T \left[\gamma_\mu \omega^\mu(x) \psi_N(x) \bar{\psi}_{N'}(x') \right] | \Psi_0 \rangle \\ &\qquad - \frac{f_{\omega N}}{2\mathcal{M}} \langle \Psi_0 | T \left[\sigma_{\mu\nu} \partial_x^\nu \omega^\mu(x) \psi_N(x) \bar{\psi}_{N'}(x') \right] | \Psi_0 \rangle \\ &+ g_{\rho N} \langle \Psi_0 | T \left[\gamma_\mu \boldsymbol{\rho}^\mu(x) \psi_N(x) \bar{\psi}_{N'}(x') \right] | \Psi_0 \rangle \cdot \boldsymbol{\tau}_N \\ &\qquad - \frac{f_{\rho N}}{2\mathcal{M}} \langle \Psi_0 | T \left[\sigma_{\mu\nu} \partial_x^\nu \boldsymbol{\rho}^\mu(x) \psi_N(x) \bar{\psi}_{N'}(x') \right] | \Psi_0 \rangle \cdot \boldsymbol{\tau}_N \\ &+ \frac{f_{\pi N}}{m_\pi} \langle \Psi_0 | T \left[\gamma_5 \gamma_\mu \partial_x^\mu \boldsymbol{\pi}(x) \psi_N(x) \bar{\psi}_{N'}(x') \right] | \Psi_0 \rangle \cdot \boldsymbol{\tau}_N .\end{aligned}$$

Replacement of the meson fields:

e.g.)

$$
\begin{aligned}
& -g_{\sigma N} \langle \Psi_0 | T \left[\sigma(x) \psi_N(x) \bar{\psi}_{N'}(x') \right] | \Psi_0 \rangle \\
&= g_{\sigma N} \sum_{N''=p,n} g_{\sigma N''} \int d^4y \, \langle \Psi_0 | T \left[\bar{\psi}_{N''}(y^+) \psi_{N''}(y) \psi_N(x) \bar{\psi}_{N'}(x') \right] | \Psi_0 \rangle \, \Delta_\sigma^0(x,y) \\
&= g_{\sigma N} \sum_{N''=p,n} g_{\sigma N''} \int d^4y \, \langle \Psi_0 | T \left[\bar{\psi}_{N''}(y^+) \psi_{N''}(y) \psi_N(x) \bar{\psi}_{N'}(x') \right] | \Psi_0 \rangle \, \Delta_\sigma^0(x,y).
\end{aligned}
$$

We introduce **the four-point Green's function**:

$$
\begin{aligned}
& \langle \Psi_0 | T \left[\bar{\psi}_{N''}(y^+) \psi_{N''}(y) \psi_N(x) \bar{\psi}_{N'}(x') \right] | \Psi_0 \rangle \\
&= \langle \Psi_0 | T \left[\psi_N(x) \bar{\psi}_{N''}(y^+) \psi_{N''}(y) \bar{\psi}_{N'}(x') \right] | \Psi_0 \rangle \\
&= - \langle \Psi_0 | T \left[\psi_N(x) \psi_{N''}(y) \bar{\psi}_{N''}(y^+) \bar{\psi}_{N'}(x') \right] | \Psi_0 \rangle \\
&= G^2_{N N'' N' N''}(x, y; x', \mathbf{y}^+).
\end{aligned}
$$

Relativistic many-body calculations II

Nucleon self-energy

Nucleon self-energy

$$
\begin{aligned}
-i\mathcal{F}_{NN'}(x,x') &:= \sum_{N''=p,n} \int d^4y\, \Sigma_{NN''}(x,y) G_{N''N'}(y,x') \\
&= \sum_{N''=p,n} \int d^4y \left[\sum_{M=\sigma,\omega,\rho,\pi} \boldsymbol{\Gamma}^{+}_{MN\mu}(x) \cdot \boldsymbol{\Gamma}^{-}_{MN''\nu}(y) \left\{ i\Delta^{0\mu\nu}_{M}(x,y;r,r'') \right) \right. \\
&\quad \times G^2_{NN''N'N''}(x,y;x',y^+),
\end{aligned}
$$

where

$$
\boldsymbol{\Gamma}^{+}_{\sigma N\mu}(x) = \boldsymbol{\Gamma}^{-}_{\sigma N\mu}(x) = ig_{\sigma N}\left(\mathbf{1}_{(4\times4)}\right)_{\mu}\mathbf{1}_{(2\times2)}, \quad \boldsymbol{\Gamma}^{\pm}_{\omega N\mu}(x) = \left(-ig_{\omega N}\gamma_{\mu} \pm i\frac{f_{\omega N}}{2\mathcal{M}}\sigma_{\mu\lambda}\partial
$$

$$
\boldsymbol{\Gamma}^{\pm}_{\rho N\mu}(x) = \left(-ig_{\rho N}\gamma_{\mu} \pm i\frac{f_{\rho N}}{2\mathcal{M}}\sigma_{\mu\lambda}\partial^{\lambda}_{x} \right)\boldsymbol{\tau}_{N}, \quad \boldsymbol{\Gamma}^{\pm}_{\pi N\mu}(x) = \pm i\frac{f_{\pi N}}{m_{\pi}}\gamma_5\gamma_{\lambda}\partial^{\lambda}_{x}\left(\mathbf{1}_{(4\times4)}\right)_{\mu}\boldsymbol{\tau}_{N}
$$

with

$$
\Delta^{0\mu\nu}_{M}(x,y,r,r'') = \begin{cases} \Delta^{0}_{M}(x,y)\left(\mathbf{1}_{(4\times4)}\right)^{\mu}\left(\mathbf{1}_{(4\times4)}\right)^{\nu}\delta_{rr''} & (M=\sigma,\pi), \\ \Delta^{0\mu\nu}_{M}(x,y;r,r'') & (M=\omega,\rho). \end{cases}
$$

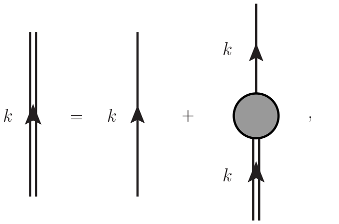
Dyson's equation

$$G_{NN'}(x,x') = G^0_{NN'}(x,x') + \int d^4y \int d^4z\, G^0_{NN}(x,z)\Sigma_{NN'}(z,y)G_{N'N'}(y,x')$$

$$= G^0_{NN'}(x,x') + \sum_{N''=p,n}\sum_{N'''=p,n}\int d^4y \int d^4z\, G^0_{NN'''}(x,z)\Sigma_{N'''N''}(z,y)G_{N''N'}(y,x'$$

$$= G^0_{NN'}(x,x') + \sum_{N''=p,n}\sum_{N'''=p,n}\int d^4y \int d^4z\, G^0_{NN'''}(x,z)$$

$$\times \left[\sum_{M=\sigma,\omega,\rho,\pi} \mathbf{\Gamma}^+_{MN'''\mu}(z)\cdot\mathbf{\Gamma}^-_{MN''\nu}(y)\left\{i\Delta^{0\mu\nu}_M(z,y;r''',r'')\right\}\right] G^2_{N'''N''N'N''}(z,y;x',$$



In momentum space,

$$G_{NN'}(k) = G^0_{NN'}(k) + G^0_{NN}(k)\Sigma_{NN'}(k)G_{N'N'}(k$$

Considering $\sum_{N'=p,n}$,

$$G_N(k) = G^0_N(k) + G^0_N(k)\Sigma_N(k)G_N(k).$$

Hartree-Fock approximation

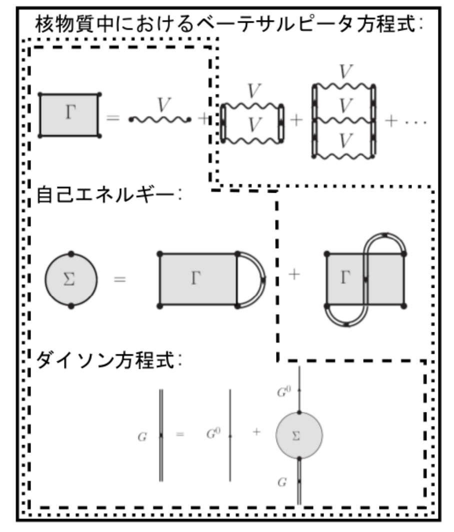
Using the Martin-Schwinger hierarchy **the four-point Green's function for fermion** is generally given by

$$G^2(1,2;1',2') := G(1,1')G(2,2') - G(1,2')G(2,1') + G(1,3)G(2,4)\langle T[(3,4;5,6)]\rangle G(5,1')G(6,2').$$

Within Hartree-Fock approximation, the four-point nucleon Green's function is factorized by the two-point Green's function:

$$G^2_{NN''N'N''}(x,y;x',y^+) \simeq G_{NN'}(x,x')G_{N''N''}(y,y^+) - G_{NN''}(x,y^+)G_{N''N'}(y,x').$$

Hartree-Fock approximation



our-point Green's function for

$$G(1,2')G(2,1')$$

$$[(3,4;5,6)]\rangle\, G(5,1')G(6,2').$$

r-point nucleon Green's function is

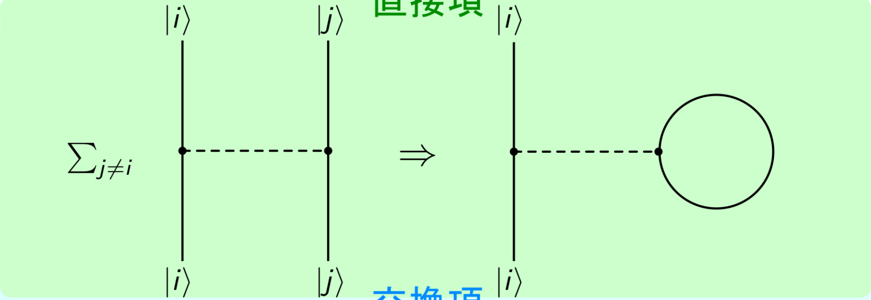
:

$$_{N''}(y,y^{+}) - G_{NN''}(x,y^{+})G_{N''N'}(y,x').$$

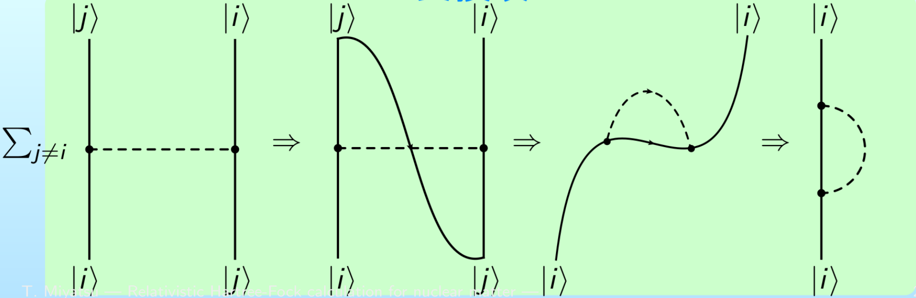
Given by Dr. T. Katayama.

ポテンシャルとの対応

直接項



交換項



Thus, **the nucleon self-energy in momentum space** is given by

$$\Sigma_N(k) = \sum_{M=\sigma,\omega,\rho,\pi}$$

$$\times \left[I_{MN}\Gamma^{+}_{MN\mu}(0)\left\{i\Delta^{0\mu\nu}_{M}(0)\right\} \sum_{N''=p,n} I_{MN''} \int \frac{d^4q}{(2\pi)^4} e^{i\eta q^0} \mathrm{tr}\left[G_{N''N''}(q)\Gamma^{-}_{MN''\nu}(0)\right] \right.$$

$$\left. - \sum_{N''=p,n} (I_{MNN''})^2 \int \frac{d^4q}{(2\pi)^4} e^{i\eta q^0} \Gamma^{+}_{MN\mu}(k-q)\left\{i\Delta^{0\mu\nu}_{M}(k-q)\right\} G_{NN''}(q)\Gamma^{-}_{MN''\nu}(\right.$$

with

$$\Gamma^{+}_{\sigma N\mu}(k) = \Gamma^{-}_{\sigma N\mu}(k) = ig_{\sigma N}\left(\mathbf{1}_{(4\times4)}\right)_{\mu}, \quad \Gamma^{\pm}_{\omega N\mu}(k) = \left(-ig_{\omega N}\gamma_\mu \pm \frac{f_{\omega N}}{2\mathcal{M}}\sigma_{\mu\lambda}k^{\lambda}\right),$$

$$\Gamma^{\pm}_{\rho N\mu}(k) = \left(-ig_{\rho N}\gamma_\mu \pm \frac{f_{\rho N}}{2\mathcal{M}}\sigma_{\mu\lambda}k^{\lambda}\right), \quad \Gamma^{\pm}_{\pi N\mu}(k) = \pm\frac{f_{\pi N}}{m_\pi}\gamma_5\gamma_\lambda k^{\lambda}\left(\mathbf{1}_{(4\times4)}\right)_{\mu}.$$

In general, the nucleon self-energy is expressed as

$$\Sigma_N(k) := \sum_{M=\sigma,\omega,\rho,\pi} \Sigma_{MN}(k) = \sum_{M=\sigma,\omega,\rho,\pi}\left[\Sigma^{\mathrm{dir}}_{MN} + \Sigma^{\mathrm{ex}}_{MN}(k)\right] = \Sigma^{\mathrm{dir}}_{N} + \Sigma^{\mathrm{ex}}_{N}(k).$$

Feynmann diagram

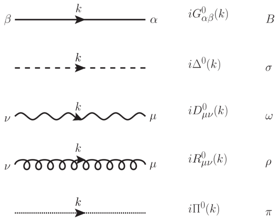


図 B.1　ファイマン図 (伝播関数)．バリオン (B) と各中間子 (σ, ω, $\vec{\rho}$, $\vec{\pi}$) に対応する伝播関数を示す．

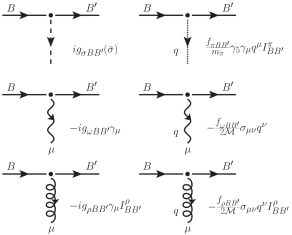


図 B.2　ファイマン図 (頂点)．上図左 (右) は σ 中間子のスカラー ($\vec{\pi}$ 中間子の擬ベクトル) 結合を，中図左 (右) は ω 中間子のベクター (テンソル) 結合を，下図左 (右) は $\vec{\rho}$ 中間子のベクター (テンソル) 結合を示す．

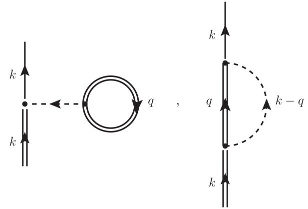


図 B.3　バリオンの自己エネルギー．左図のタドポール型ループは直接項による相互作用であり，右図は交換項による相互作用である．ここでは，例として σ 中間子によるバリオンの自己エネルギーを表している．

Relativistic many-body calculations II

Meson propagator

Meson propagator

$$\Delta_M^{\mu\nu}(k) = \Delta_M^{0,\mu\nu}(k) + \Delta_M^{\mathrm{dir},\mu\nu}(k) + \Delta_M^{\mathrm{ex},\mu\nu}(k)$$

$$= i\left\{i\Delta_M^{0\mu\lambda}(k)\right\}\left[\left(\sum_{N=p,n} I_{MN}\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}\left[G_{NN}(p)\Gamma^{-}_{MN\lambda}(k)\right]\right)\right.$$

$$\times(2\pi)^4\,\delta^4(k)\left(\sum_{N'=p,n} I_{MN'}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}\left[G_{N'N'}(q)\Gamma^{+}_{MN'\kappa}(k)\right]\right)$$

$$\left.-\sum_{N=p,n}\sum_{N'=p,n}(I_{MNN'})^2\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}\left[G_{NN'}(k+q)\Gamma^{-}_{MN\lambda}(k)G_{N'N}(q)\Gamma^{+}_{MN'\kappa}(k)\right]\right]\{i\Delta$$

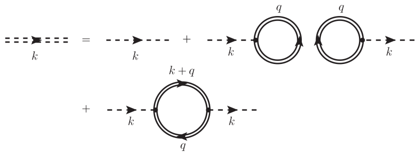


図 B.4　中間子の伝搬関数．第 1 項目は真空における中間子の伝搬関数であり，第 2 項目は直接項に対応するタドポール型ループの相互作用を含んだ伝搬関数である．第 3 項目は交換項に対応する中間子の伝搬関数である．この図では σ 中間子の伝搬関数を表しており，他の中間子に関しても同様に表される．

Relativistic many-body calculations III

Nucleon propagator in matter

Nucleon propagator in matter

The properties of dressed nucleons in nuclear matter are expressed by the nucleon self-energy which enters the in-medium nucleon propagator as the formal solution of the Dyson's equation:

$$G_N(k) = \frac{1}{[G_N^0(k)]^{-1} - \Sigma_N(k)} = \frac{1}{\gamma_\mu k^\mu - M_N - \Sigma_N(k) + i\varepsilon}.$$

Due to translational and rotational invariance, parity conservation and time reversal invariance the self-energy in isospin saturated nuclear matter has the general form:

$$\begin{aligned}\Sigma_N(k) &= \Sigma_N^s(k) - \gamma_\mu \Sigma_N^\mu(k) \\ &= \Sigma_N^s(k) - \gamma_0 \Sigma_N^0(k) + (\boldsymbol{\gamma} \cdot \hat{k}) \Sigma_N^v(k),\end{aligned}$$

with $\hat{k}$ being the unit vector along the (three) momentum $\boldsymbol{k}$ and $\Sigma_B^{s(0)[v]}$ being **the scalar part** (**the time component of the vector part**) [**the space component of the vector part**] of the self-energy.

Therefore, the effective baryon mass, momentum, and energy in matter are respectively defined by including the self-energy in matter as follows

$$M_N^*(k) = M_N + \Sigma_N^s(k),$$
$$k_N^{*\mu}(k) = (k_N^{*0}(k), \boldsymbol{k}_N^*(k)) = (k^0 + \Sigma_N^0(k), \boldsymbol{k} + \hat{k}\Sigma_N^v(k)),$$
$$E_N^*(k) = \left[\boldsymbol{k}_N^{*2} + M_N^{*2}(k)\right]^{1/2}.$$

In addition, the nucleon propagator **in matter** reads

$$G_N(k) = G_N^F(k) + G_N^D(k),$$
$$G_N^F(k) = \frac{1}{\gamma_\mu k_N^{*\mu}(k) - M_N^*(k) + i\varepsilon}$$
$$= [\gamma_\mu k_N^{*\mu}(k) + M_N^*(k)] \frac{1}{k_{N\nu}^*(k)k_N^{*\nu}(k) - M_N^{*2}(k) + i\varepsilon},$$
$$G_N^D(k) = [\gamma_\mu k_N^{*\mu}(k) + M_N^*(k)] \frac{i\pi}{E_N^*(k)} \delta[k^0 - E_N(k)]\theta(k_{F_N} - |\boldsymbol{k}|).$$

We note that the nucleon energy in vacuum, E_N, is satisfied with the transcendental equation,

$$E_N(k) = \left[E_N^*(k) - \Sigma_N^0(k)\right]_{k^0=E_N(k)} = \left[\boldsymbol{k}_N^{*2}(k) + M_N^{*2}(k)\right]^{1/2} - \Sigma_N^0(k)\Big|_{k^0=E_N(k)}.$$

Summary table for self-energy

The nucleon self energy is expressed as

$$\Sigma_N(k) = \Sigma_N^s(k) - \gamma_0 \Sigma_N^0(k) + (\boldsymbol{\gamma}\cdot\hat{\boldsymbol{k}})\Sigma_N^v(k),$$

with

$$\Sigma_N^s(k) = \Sigma_{\sigma N}^s + \sum_{N'=p,n}\sum_i \frac{(I_{iNN'})^2}{(4\pi)^2 k}\int_0^{k_{F_{N'}}} dq\, q \left[\frac{M_{N'}^*(q)}{E_{N'}^*(q)} B_i(k,q) + \frac{q_{N'}^*(q)}{E_{N'}^*(q)} D_i(q,k)\right]$$

$$\Sigma_N^0(k) = -\left(\Sigma_{\omega N}^0 + \Sigma_{\rho N}^0\right) - \sum_{N'=p,n}\sum_i \frac{(I_{iNN'})^2}{(4\pi)^2 k}\int_0^{k_{F_{N'}}} dq\, q A_i(k,q),$$

$$\Sigma_N^v(k) = \sum_{N'=p,n}\sum_i \frac{(I_{iNN'})^2}{(4\pi)^2 k}\int_0^{k_{F_{N'}}} dq\, q \left[\frac{q_{N'}^*(q)}{E_{N'}^*(q)} C_i(k,q) + \frac{M_{N'}^*(q)}{E_{N'}^*(q)} D_i(k,q)\right],$$

and

$$\Sigma_{\sigma N}^s = -g_{\sigma N}\bar{\sigma}, \quad \Sigma_{\omega N}^0 = g_{\omega N}\bar{\omega}, \quad \Sigma_{\rho N}^0 = g_{\rho N} I_{\rho N}\bar{\rho}, \quad \Sigma_{\pi N}^0 = g_{\pi N} I_{\pi N}\bar{\pi} = 0.$$

These terms correspond to the contribution of the mean-field values, namely **the mean-field or Hartree approximation (tadpole diagram)**.

TABLE I. Functions A_i, B_i, C_i, and D_i. The index i is specified in the left column, where $V(T)$ stands for the vector (tensor) coupling at each meson-NN' vertex. The last row is for the (pseudovector) pion contribution.

i	A_i	B_i	C_i	D_i
σ	$g_{\sigma N}^2(\bar{\sigma})\Theta_\sigma$	$g_{\sigma N}^2(\bar{\sigma})\Theta_\sigma$	$-2g_{\sigma N}^2(\bar{\sigma})\Phi_\sigma$	–
ω_{VV}	$2g_{\omega N}^2\Theta_\omega$	$-4g_{\omega N}^2\Theta_\omega$	$-4g_{\omega N}^2\Phi_\omega$	–
ω_{TT}	$-\left(\frac{f_{\omega N}}{2\mathcal{M}}\right)^2 m_\omega^2\Theta_\omega$	$-3\left(\frac{f_{\omega N}}{2\mathcal{M}}\right)^2 m_\omega^2\Theta_\omega$	$4\left(\frac{f_{\omega N}}{2\mathcal{M}}\right)^2 \Psi_\omega$	–
ω_{VT}	–	–	–	$6\frac{f_{\omega N}g_{\omega N}}{2\mathcal{M}}\Gamma_\omega$
ρ_{VV}	$2g_{\rho N}^2\Theta_\rho$	$-4g_{\rho N}^2\Theta_\rho$	$-4g_{\rho N}^2\Phi_\rho$	–
ρ_{TT}	$-\left(\frac{f_{\rho N}}{2\mathcal{M}}\right)^2 m_\rho^2\Theta_\rho$	$-3\left(\frac{f_{\rho N}}{2\mathcal{M}}\right)^2 m_\rho^2\Theta_\rho$	$4\left(\frac{f_{\rho N}}{2\mathcal{M}}\right)^2 \Psi_\rho$	–
ρ_{VT}	–	–	–	$6\frac{f_{\rho N}g_{\rho N}}{2\mathcal{M}}\Gamma_\rho$
π_{pv}	$-f_{\pi N}^2\Theta_\pi$	$-f_{\pi N}^2\Theta_\pi$	$2\left(\frac{f_{\pi N}}{m_\pi}\right)^2 \Pi_\pi$	–

$$\Theta_i(k,q) = \ln\left[\frac{m_i^2+(k+q)^2}{m_i^2+(k-q)^2}\right], \quad \Phi_i(k,q) = \frac{1}{4kq}\left(k^2+q^2+m_i^2\right)\Theta_i(k,q) - 1,$$

$$\Psi_i(k,q) = \left(k^2+q^2-\frac{m_i^2}{2}\right)\Phi_i(k,q) - kq\Theta_i(k,q),$$

$$\Pi_i(k,q) = \left(k^2+q^2\right)\Phi_i(k,q) - kq\Theta_i(k,q), \Gamma_i(k,q) = k\Theta_i(k,q) - 2q\Phi_i(k,q).$$

Relativistic many-body calculations III

Energy momentum tensor

$$T^{\mu\nu} = \sum_{\alpha} \frac{\partial \mathcal{L}}{\partial \left(\partial_{\mu} \Phi_{\alpha}\right)} \partial^{\nu} \Phi_{\alpha} - g^{\mu\nu} \mathcal{L} \quad \left(\alpha = N, \sigma, \omega, \boldsymbol{\rho}, \boldsymbol{\pi}\right).$$

Energy density and pressure

The energy-momentum tensor is given by

$$\begin{aligned}\langle\Psi_0|T^{\mu\nu}(x)|\Psi_0\rangle = &-i\sum_{N=p,n}\int\frac{d^4k}{(2\pi)^4}e^{i\eta k^0}\,\mathrm{tr}\left[\gamma^\mu G_N(k)\right]k^\nu \\ &+\frac{i}{2}g^{\mu\nu}\sum_{N=p,n}\int\frac{d^4k}{(2\pi)^4}e^{i\eta k^0}\,\mathrm{tr}\left[\Sigma_N(k)G_N(k)\right] \\ &-i\int\frac{d^4k}{(2\pi)^4}e^{i\eta k^0}\left[\Delta_\sigma(k)-\Delta^\lambda_{\omega\lambda}(k)-\Delta^\lambda_{\rho\lambda}(k;r,r')+\Delta_\pi(k;r,r')\right]k^\mu k^\nu \\ &-\frac{1}{2}\left[\frac{1}{3}g_2\bar{\sigma}^3(x)+\frac{1}{2}g_3\bar{\sigma}^4(x)-\frac{1}{2}c_3\bar{\omega}^4(x)\right].\end{aligned}$$

Therefore

$$\begin{aligned}\epsilon &= \langle\Psi_0|T^{00}(x)|\Psi_0\rangle \\ &= \sum_{N=p,n}\frac{2J_N+1}{(2\pi)^3}\int_0^{k_{F_N}}d\boldsymbol{k}\left[T_N(k)+\frac{1}{2}V_N(k)\right]-\frac{1}{2}\left[\frac{1}{3}g_2\bar{\sigma}^3(x)+\frac{1}{2}g_3\bar{\sigma}^4(x)-\frac{1}{2}c_3\bar{\omega}^4(x)\right]\end{aligned}$$

$$P=\rho_B^2\frac{\partial}{\partial\rho_B}\left(\frac{\epsilon}{\rho_B}\right)=\rho_B\frac{\partial\epsilon}{\partial\rho_B}-\epsilon \quad \text{with} \quad \rho_B=\sum_{N=p,n}\rho_N=\rho_p+\rho_n.$$

Application I

Nuclear-matter properties

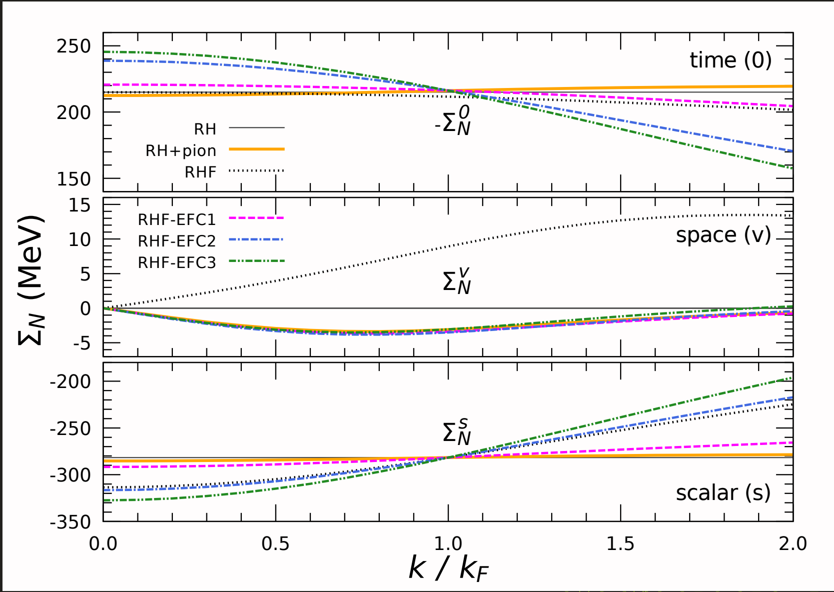
Nucleon self-energy at ρ_0 and k_{F_0}

		RH			RHF+pion		
		Σ_N^s	Σ_N^0	Σ_N^v	Σ_N^s	Σ_N^0	Σ_N^v
Direct	σ	-282	0	0	-277	0	0
	ω	0	-215	0	0	-220	0
Exchange	π	0	0	0	-4	4	-3
Total		-282	-215	0	-282	-216	-3

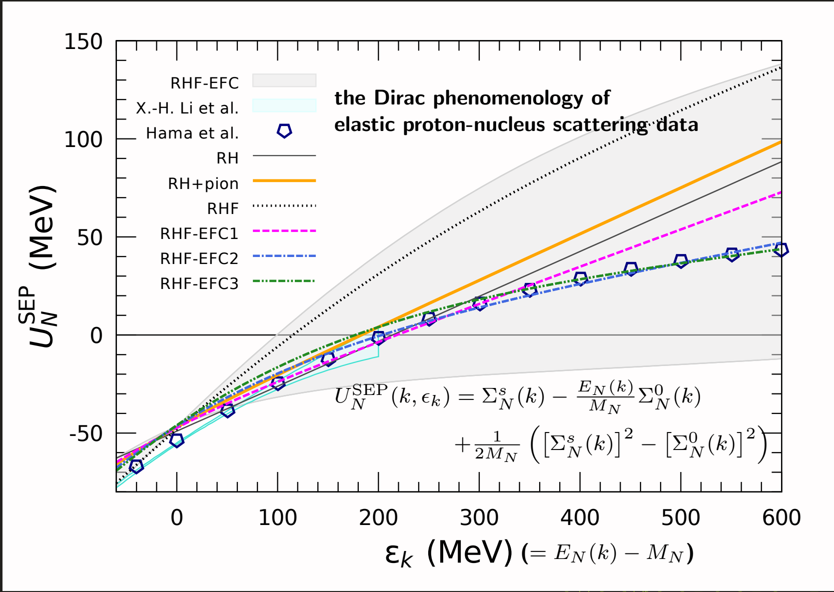
		RHF			RHF-EFC1		
		Σ_N^s	Σ_N^0	Σ_N^v	Σ_N^s	Σ_N^0	Σ_N^v
Direct	σ	-156	0	0	-247	0	0
	ω	0	-183	0	0	-185	0
Exchange	σ	14	-15	-1	15	-15	-1
	ω	-62	-33	-1	-41	-21	-1
	π	-4	4	-3	-4	4	-3
	ρ	-73	15	14	-5	1	1
		$(-14,-63,4)$	$(-7,22,0)$	$(0,-2,16)$	$(-1,-4,0)$	$(0,1,0)$	$(0,0,1)$
Total		-282	-212	9	-282	-216	-3

		RHF-EFC2			RHF-EFC3		
		Σ_N^s	Σ_N^0	Σ_N^v	Σ_N^s	Σ_N^0	Σ_N^v
Direct	σ	-133	0	0	-83	0	0
	ω	0	-104	0	0	-74	0
Exchange	σ	34	-36	-1	41	-43	-2
	ω	-160	-84	-3	-208	-109	-3
	π	-4	4	-3	-4	4	-3
	ρ	-18	4	3	-26	5	5
		$(-3,-16,1)$	$(-2,6,0)$	$(0,0,4)$	$(-5,-23,1)$	$(-3,8,0)$	$(0,-1,6)$
Total		-282	-216	-4	-282	-216	-3

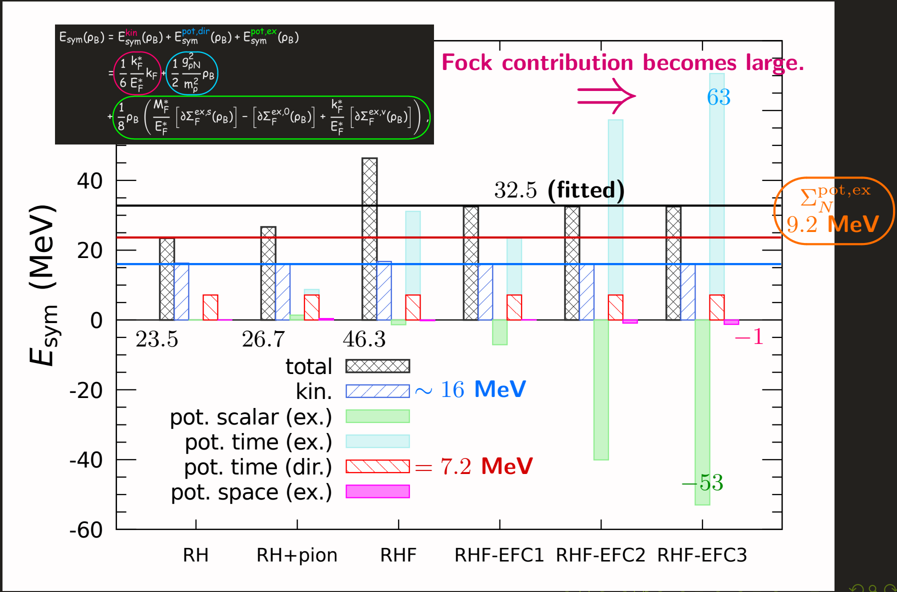
Momentum dependence of Σ_N at ρ_0



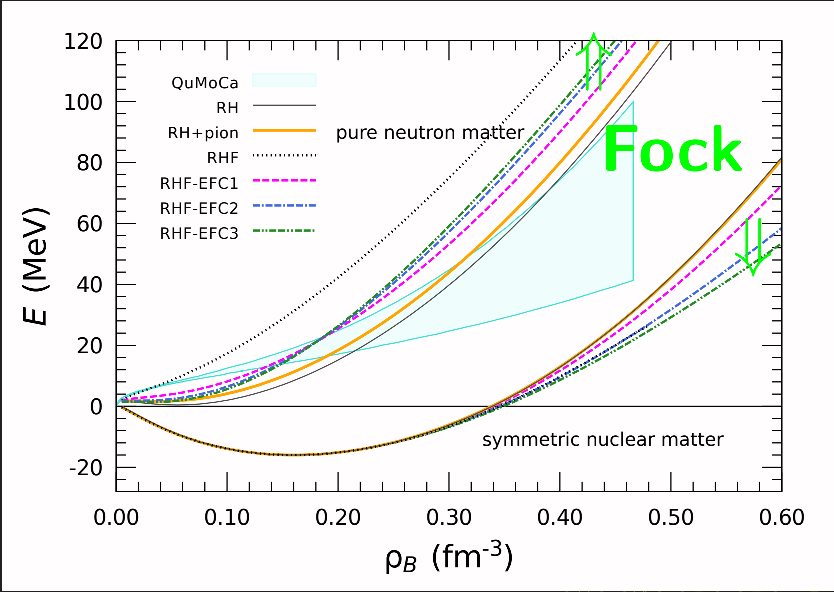
Optical potential at ρ_0



Nuclear symmetry energy



Nuclear binding energy



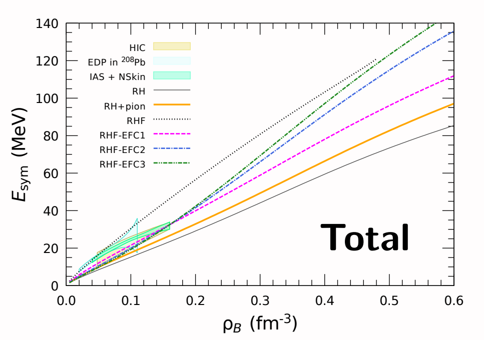
Symmetry energy

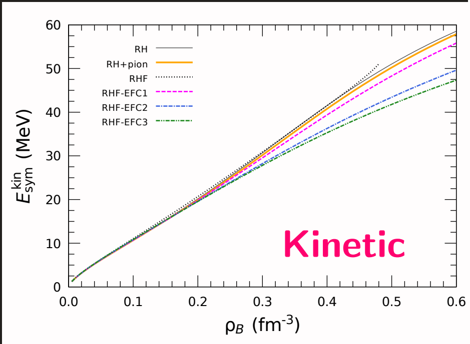
Nuclear symmetry energy can be divided into the **kinetic** and **potential** parts.

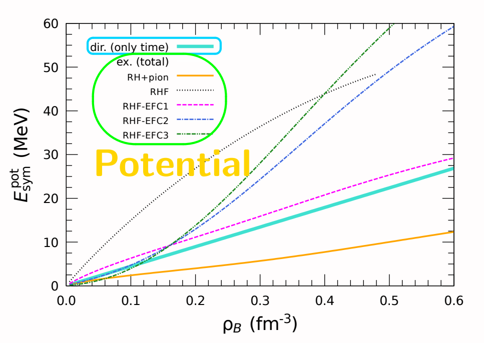
$$E_{\rm sym}(\rho_B) = E_{\rm sym}^{\rm kin}(\rho_B) + E_{\rm sym}^{\rm pot,dir}(\rho_B) + E_{\rm sym}^{\rm pot}$$

$$\frac{1}{6}\frac{k_F^*}{E_F^*}k_F \qquad \frac{1}{2}\frac{g_{\rho N}^2}{m_\rho^2}\rho_B \qquad \text{Fock}$$

Fock contribution is composed of the scalar (s), time(0), and space(v) components.

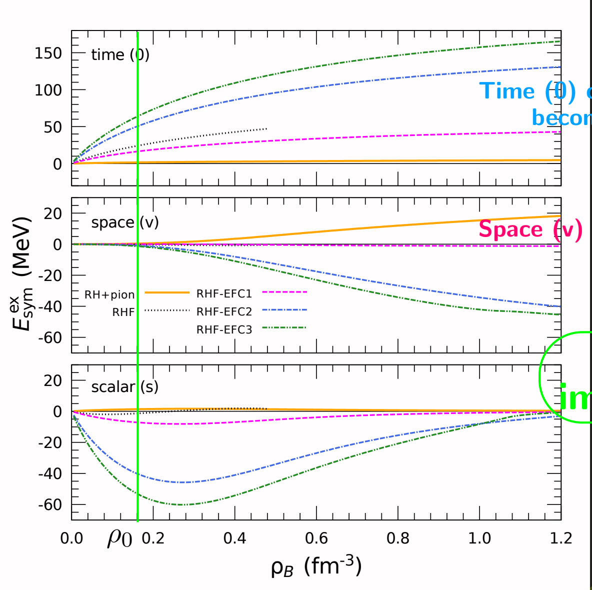






Fock contribution of symmetry energy

$E_{\rm sym}^{\rm pot,ex}(\rho_B)$ $\begin{cases} \text{scalar (s)} \\ \text{time (0)} \\ \text{space (v)} \end{cases}$



Time (0) component becomes dominant.

Space (v) component is not small.

$\Downarrow$

Σ_N^v is important.

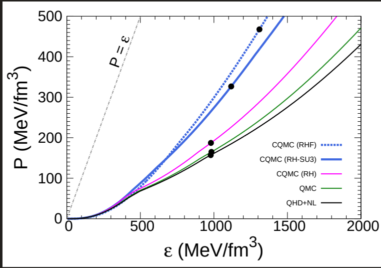
Application II

Neutron-star properties

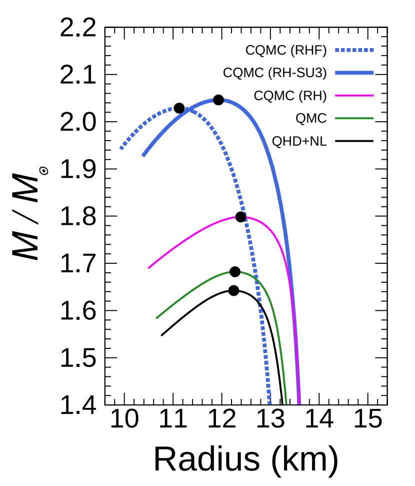
EoS for neutron stars

1. **Baryon-structure variation** in matter using **the CQMC model**
2. **SU(3) flavor symmetry**
3. Relativistic many-body calculation
   - ✓ **Hartree** approximation
   - ✓ **Hartree-Fock** approximation

These effects make the EoS stiff and the maximum mass can reach the $2M_{\odot}$ constraint.



K_0 (MeV)
269
309
309
286
286



∗Hyperons are taken into account.

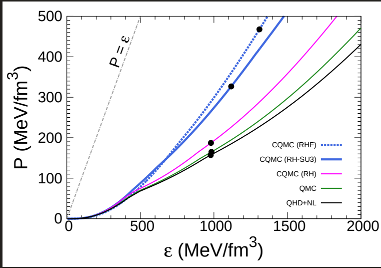
EoS for neutron stars

1 **Baryon-structure variation** in matter using **the CQMC model**

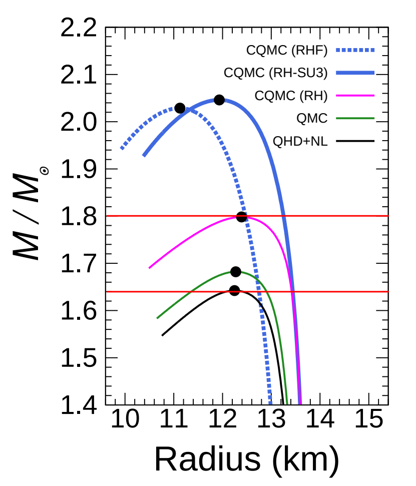
2 **SU(3) flavor symmetry**

3 Relativistic many-body calculation
 ✓ **Hartree** approximation
 ✓ **Hartree-Fock** approximation

These effects make the EoS stiff and the maximum mass can reach the $2M_{\odot}$ constraint.



K_0 (MeV)
269
309
309
286
286

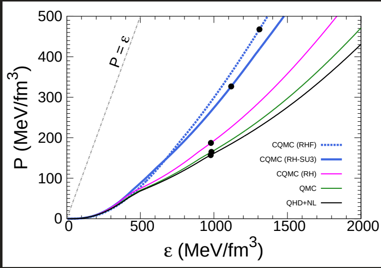


∗Hyperons are taken into account.

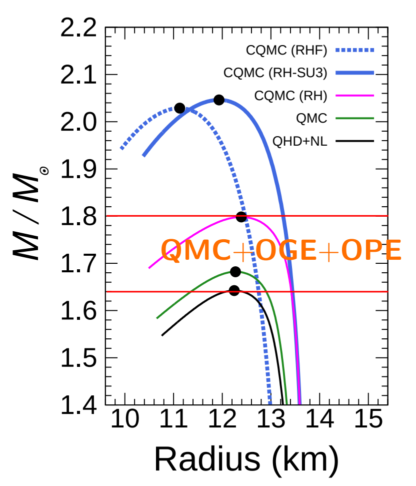
EoS for neutron stars

1. **Baryon-structure variation** in matter using **the CQMC model**
2. **SU(3) flavor symmetry**
3. Relativistic many-body calculation
   - ✓ **Hartree** approximation
   - ✓ **Hartree-Fock** approximation

These effects make the EoS stiff and the maximum mass can reach the $2M_{\odot}$ constraint.



K_0 (MeV)
269
309
309
286
286



∗Hyperons are taken into account.

EoS for neutron stars

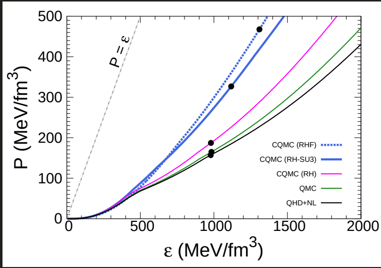
1 **Baryon-structure variation** in matter using **the CQMC model**

2 **SU(3) flavor symmetry**

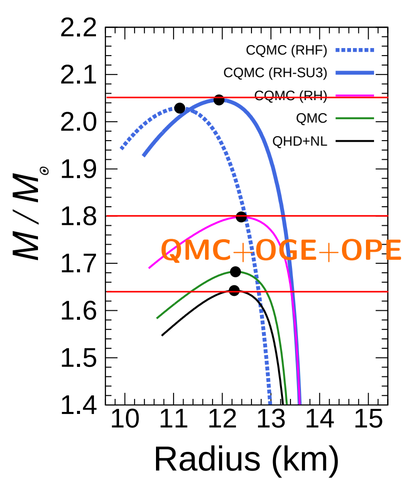
3 Relativistic many-body calculation

- ✓ **Hartree** approximation
- ✓ **Hartree-Fock** approximation

These effects make the EoS stiff and the maximum mass can reach the $2M_\odot$ constraint.



K_0 (MeV)
269
309
309
286
286

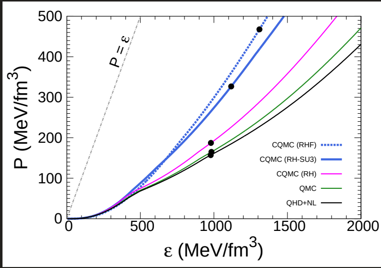


∗Hyperons are taken into account.

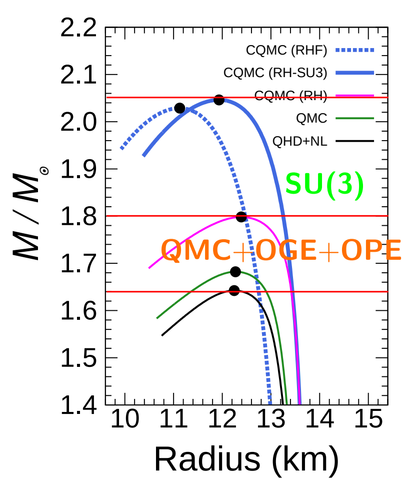
EoS for neutron stars

1. **Baryon-structure variation** in matter using **the CQMC model**
2. **SU(3) flavor symmetry**
3. Relativistic many-body calculation
   - ✓ **Hartree** approximation
   - ✓ **Hartree-Fock** approximation

These effects make the EoS stiff and the maximum mass can reach the $2M_{\odot}$ constraint.



K_0 (MeV)
269
309
309
286
286



∗Hyperons are taken into account.

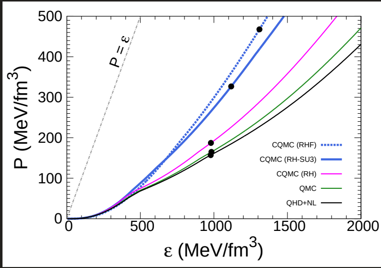
EoS for neutron stars

1 **Baryon-structure variation** in matter using **the CQMC model**

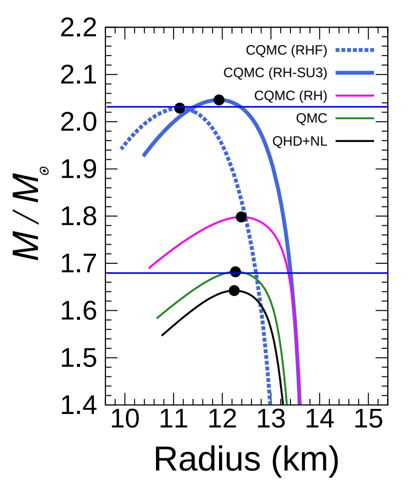
2 **SU(3) flavor symmetry**

3 Relativistic many-body calculation
- ✓ **Hartree** approximation
- ✓ **Hartree-Fock** approximation

These effects make the EoS stiff and the maximum mass can reach the $2M_{\odot}$ constraint.



K_0 (MeV)
269
309
309
286
286



∗Hyperons are taken into account.

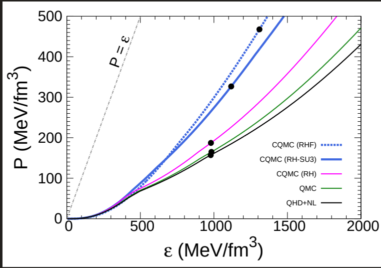
EoS for neutron stars

1 **Baryon-structure variation** in matter using **the CQMC model**

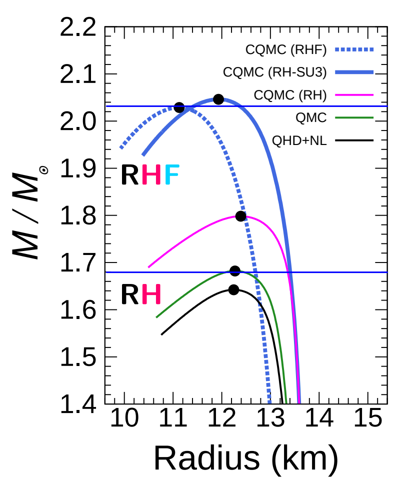
2 **SU(3) flavor symmetry**

3 Relativistic many-body calculation
- ✓ **Hartree** approximation
- ✓ **Hartree-Fock** approximation

These effects make the EoS stiff and the maximum mass can reach the $2M_\odot$ constraint.



K_0 (MeV)
269
309
309
286
286

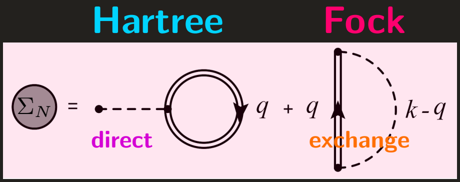


∗Hyperons are taken into account.

Summary

Summary (today's talk)

1. Introduction
2. Relativistic many-body calculations I
   - Lagrangian density
   - Eular-Lagrange equation
3. Relativistic many-body calculations II
   - Green's function
   - Nucleon self-energy (Hartree and Hartree-Fock approximation)
   - Meson propagator
4. Relativistic many-body calculations III
   - Nucleon propagator in matter
   - Energy momentum tensor
5. Application
6. Summary



Thank You for Your Attention.