

# トポロジカル輸送現象： 物性から宇宙物理まで

山本 直希

慶應義塾大学 理工学部 物理学科

@千葉工業大学 **核物理×物性**セミナー

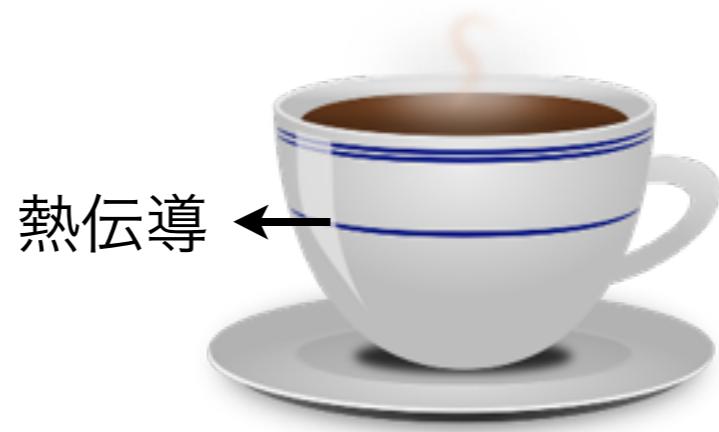
2015年2月7日

# 内容

- ・ 日常的な輸送現象
- ・ カイラルな輸送現象
- ・ カイラルプラズマ不安定性
- ・ マグネターの問題

# 輸送現象

- 古典的で身近な例:
  - Ohmの法則:  $j_e = \sigma E$
  - Fourierの法則:  $j_Q = \kappa(-\nabla T)$



# 色々な輸送現象

- 19世紀に既に色々発見

電流	1826 Ohm	1879 Hall	1886 Nernst
$j_e = \sigma E + RE \times B + \alpha(-\nabla T) + N(-\nabla T) \times B$			
熱流		Ettingshausen 1886	Fourier 1807
			Leduc-Righi 1887

- $E \rightarrow -\nabla \mu$  もOK.
- これで全てなのか？

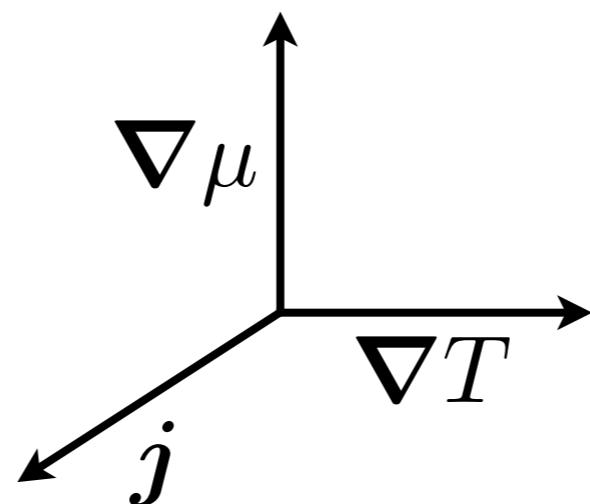
# クイズ

- 局所的に温度や密度の勾配があると、

$$j \sim \nabla T \quad j \sim \nabla \mu$$

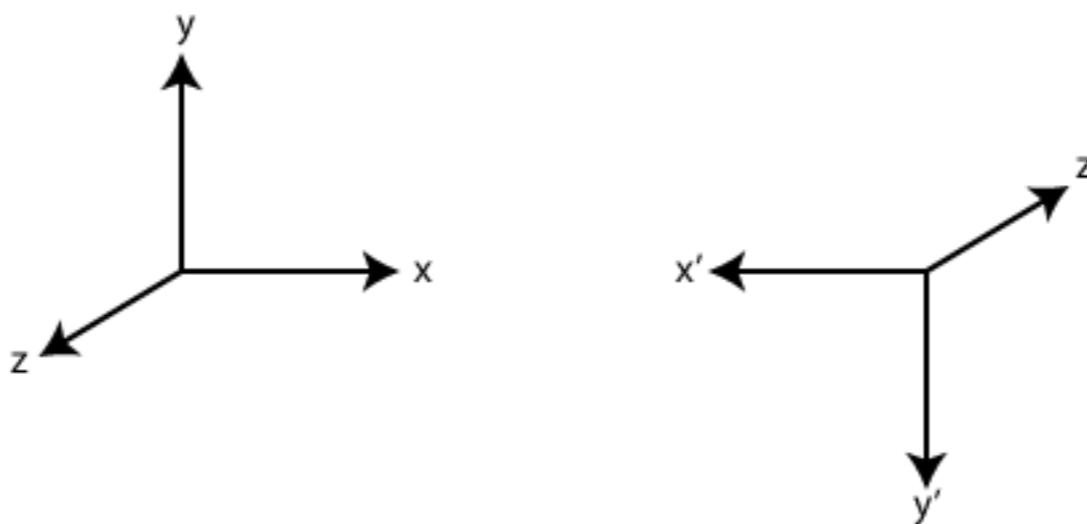
- ではこういう流れは？  $j \sim \nabla T \times \nabla \mu$

電磁場のない最も単純な2次輸送



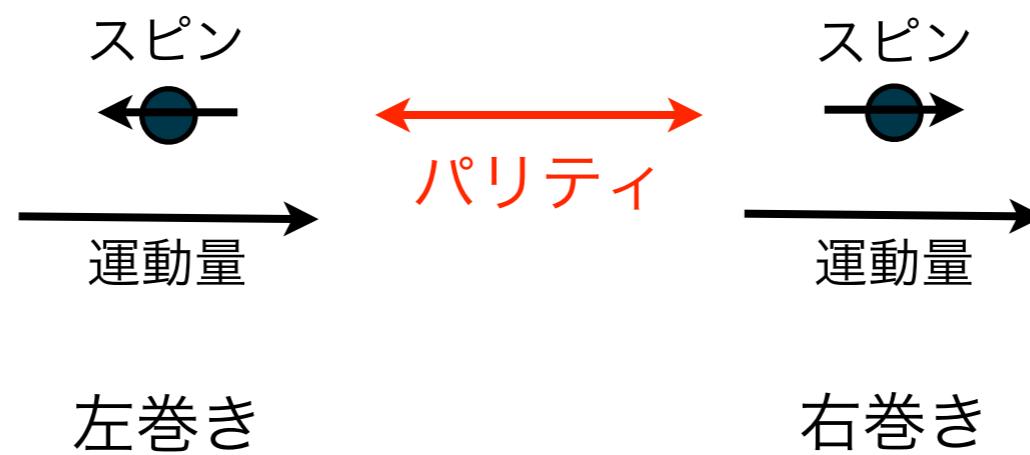
# パリティ

- この関係式を仮定：  $j = \kappa \nabla T \times \nabla \mu$
- パリティ変換のもとで  $-j = \kappa(-\nabla T) \times (-\nabla \mu)$
- パリティに矛盾するので普通は（空気や水では）起きない。



# カイラリティー

- 相対論的に運動するフェルミオン：カイラリティー



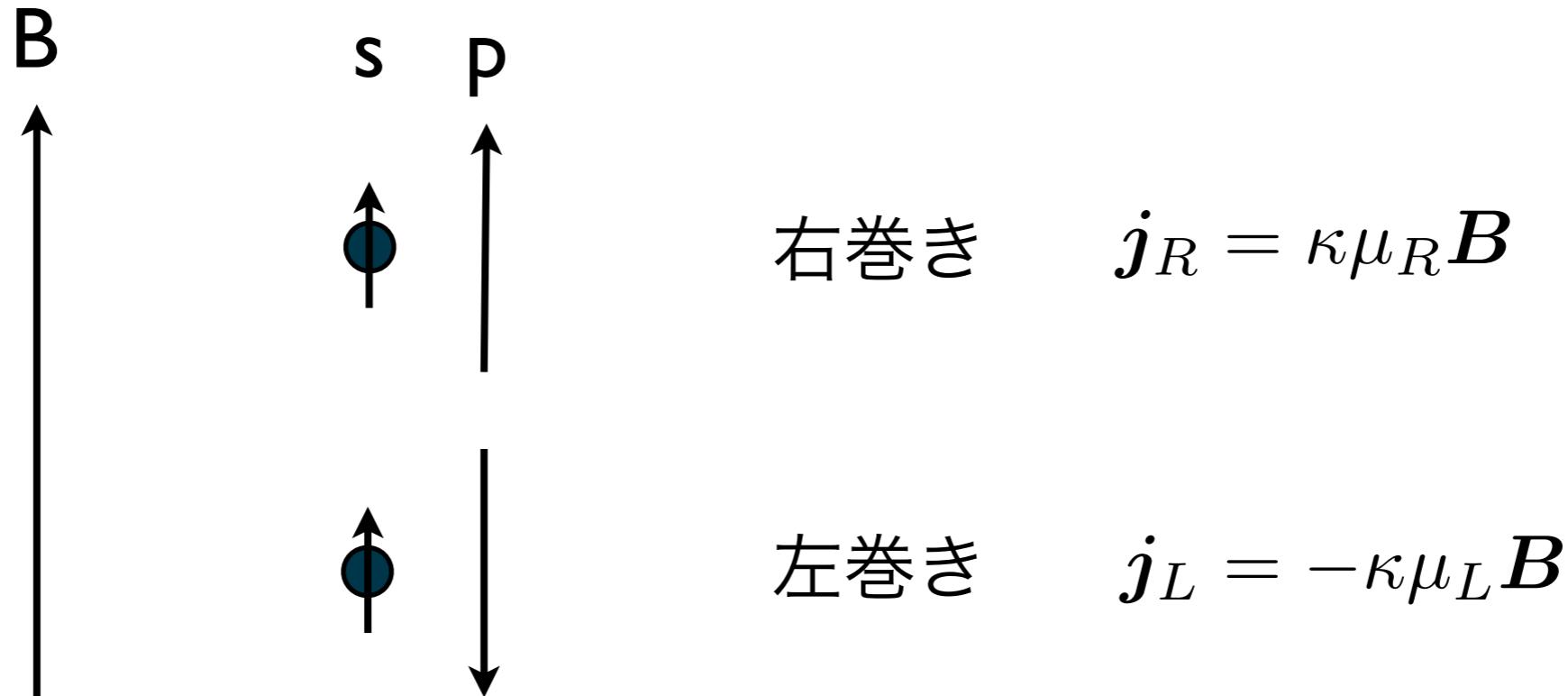
- $j = \tilde{\kappa}(\mu_R - \mu_L) \nabla T \times \nabla \mu$  : パリティと無矛盾
- 輸送係数は原理的に計算できるのか？

# Chiral Magnetic Effect

- Ohmの法則:  $j_e = \sigma E$
- $j_e \sim B$  はパリティに一見反する ( $B = \nabla \times A$ )
- $j_e \sim (\mu_R - \mu_L)B$  ならOK.

Vilenkin (1980); Nielsen, Ninomiya (1983); Kharzeev, Warringa, Fukushima (2008)

# 定性的な議論



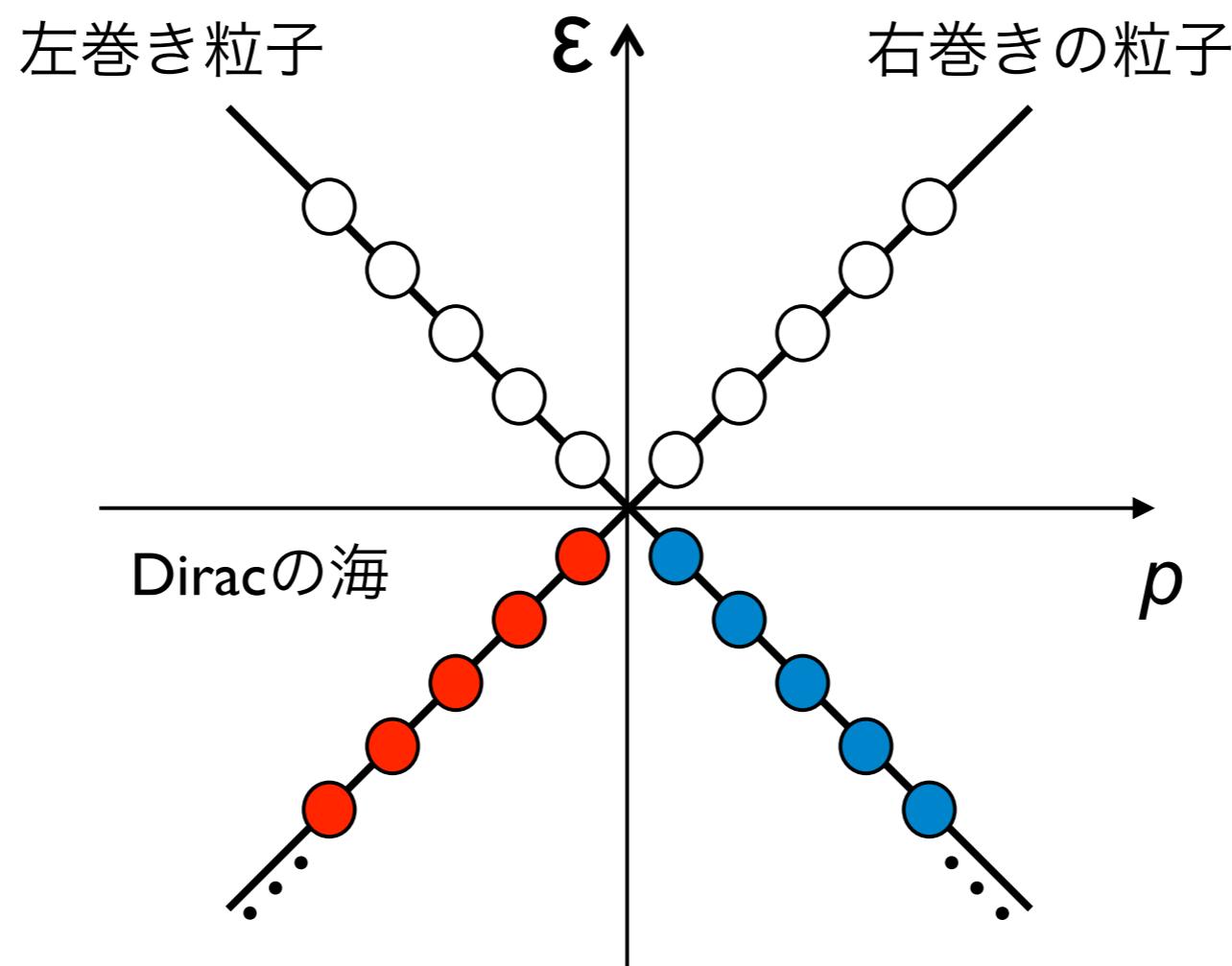
- 左右の差があると、磁場方向の電流： $j_e \sim \mu_5 B$
- 「量子異常」と密接に関係

Vilenkin (1980); Kharzeev, Warringa, Fukushima (2008)

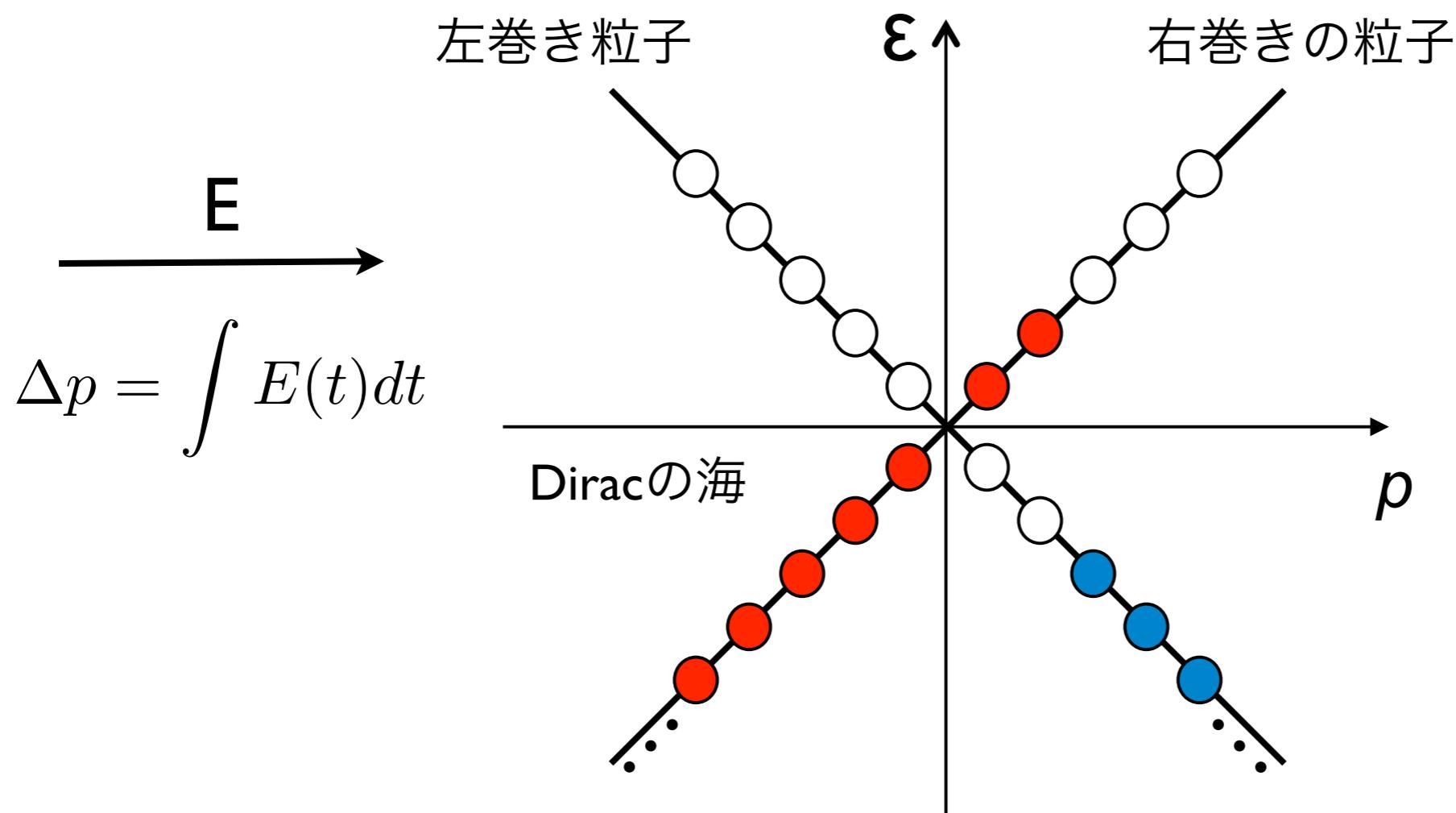
# 量子異常（アノマリー）

- 古典的な対称性が量子効果によって破れる現象
- トポロジーと深い関係
- 低エネルギーの物理に重要な帰結
  - 湯川中間子の寿命 Adler-Bell-Jackiw (1969)
  - 輸送現象：量子ホール効果、カイラル磁気効果 etc.

# $|+|$ 次元の量子異常

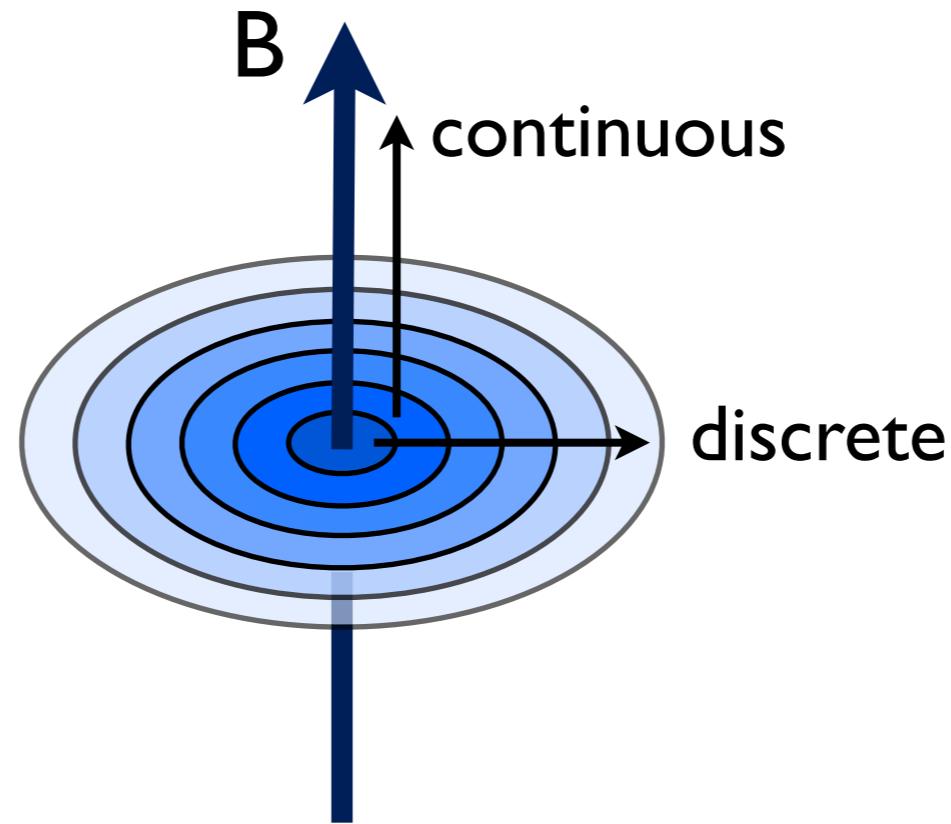


# |+|次元の量子異常



$$\Delta Q = 0, \quad \Delta Q_A = \frac{1}{\pi} \int d^2x E(t)$$

# 3+1次元の量子異常



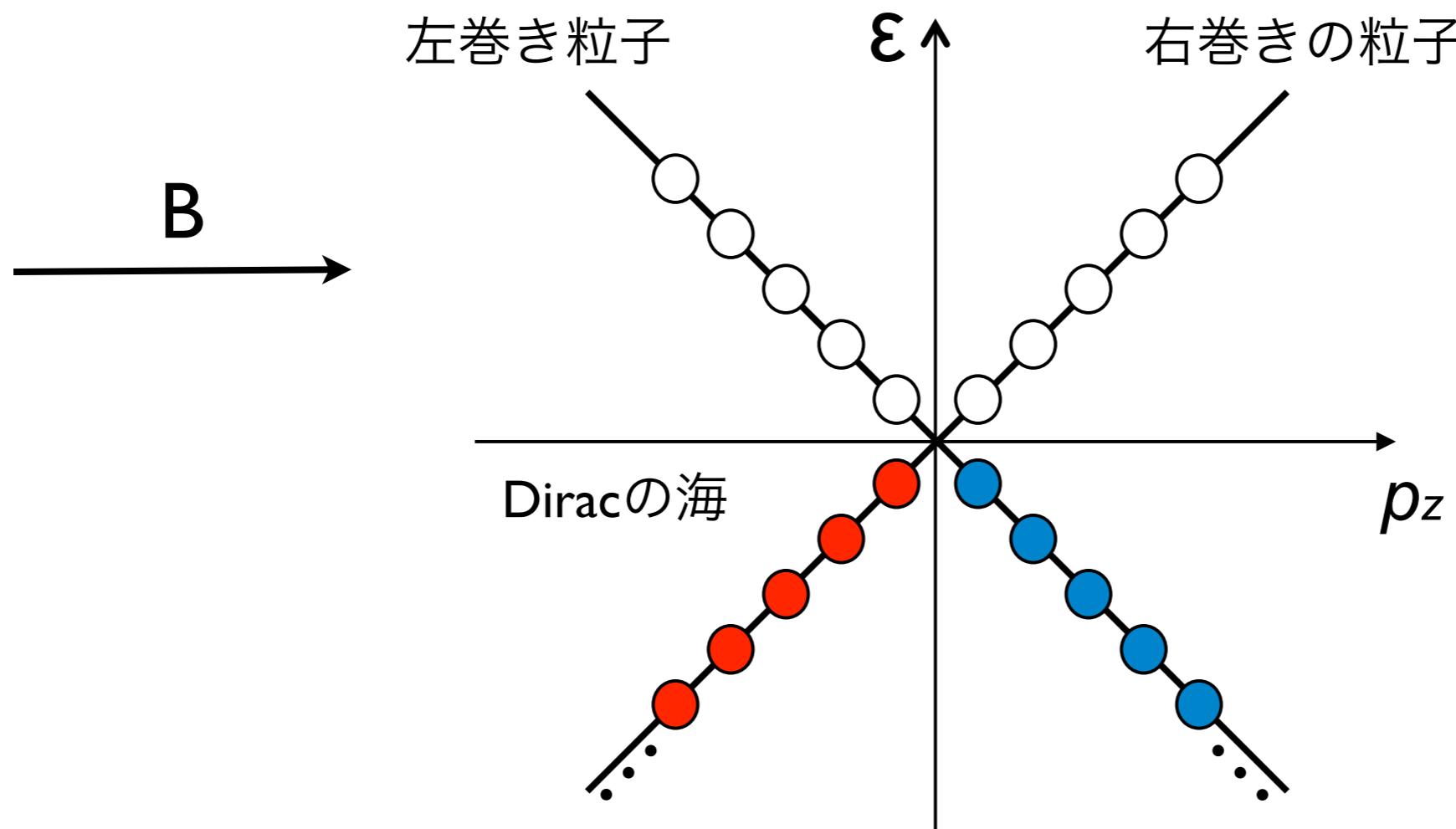
- 磁場によってLandau準位ができる

$$E_n^2 = p_z^2 + (2n + 1)B \pm B$$

Zeeman効果

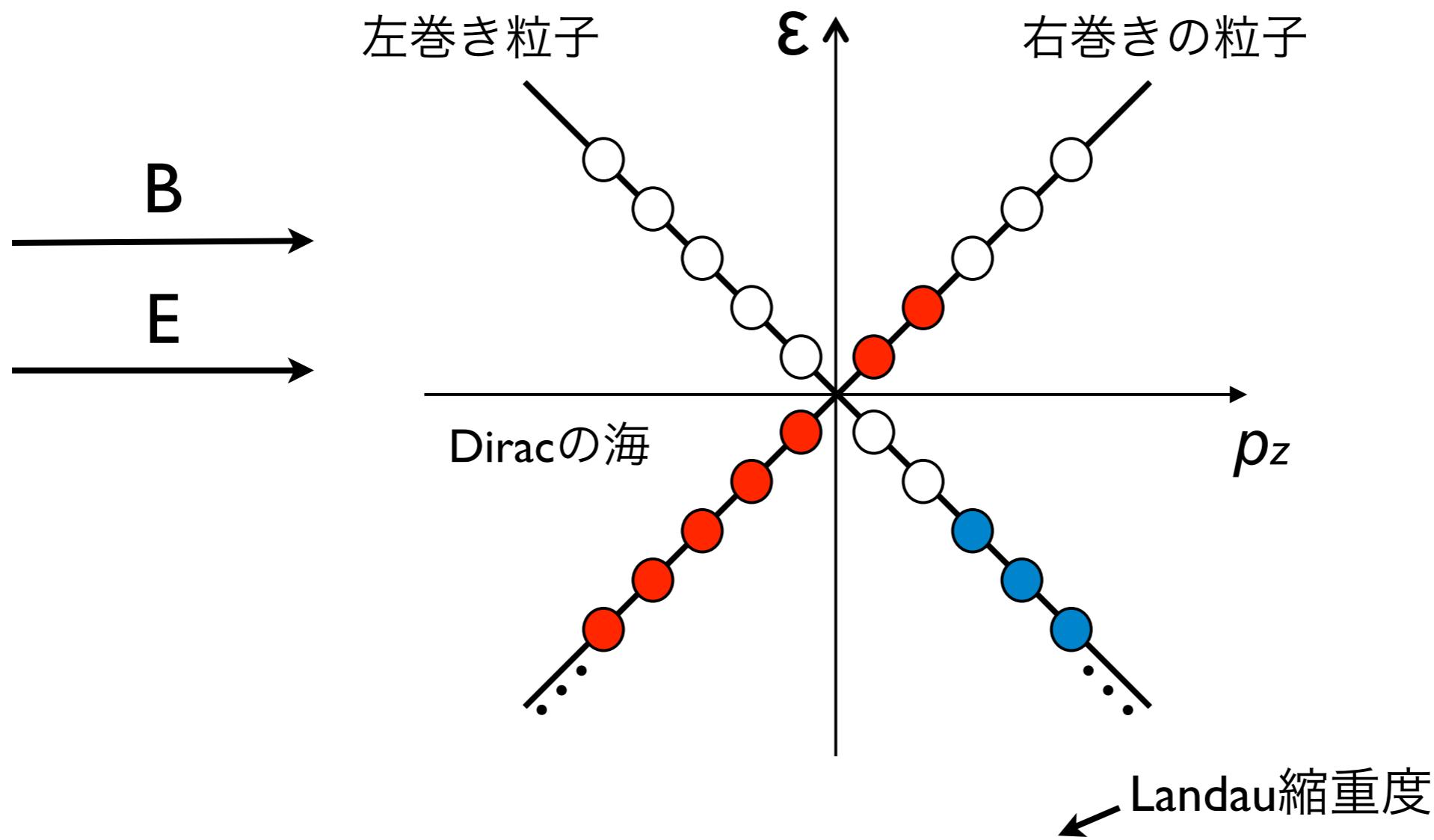
- 最低Landau準位: 1+1次元のカイラル粒子

# 3+1次元の量子異常



最低Landau準位のカイラル粒子

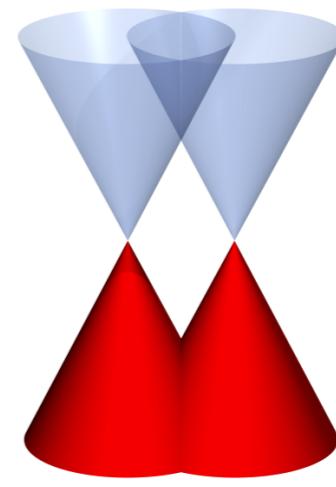
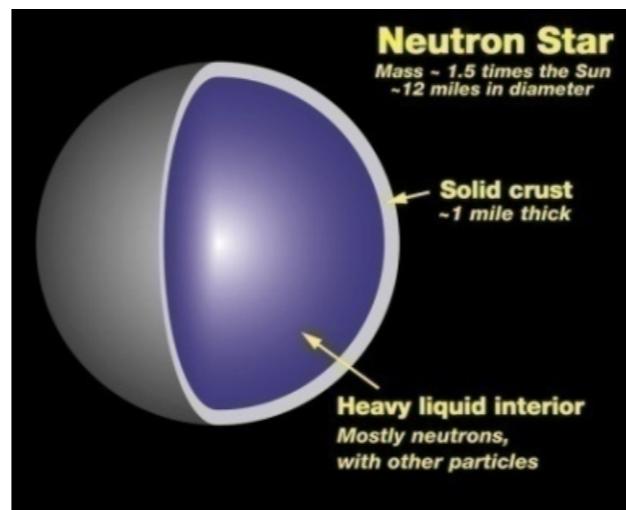
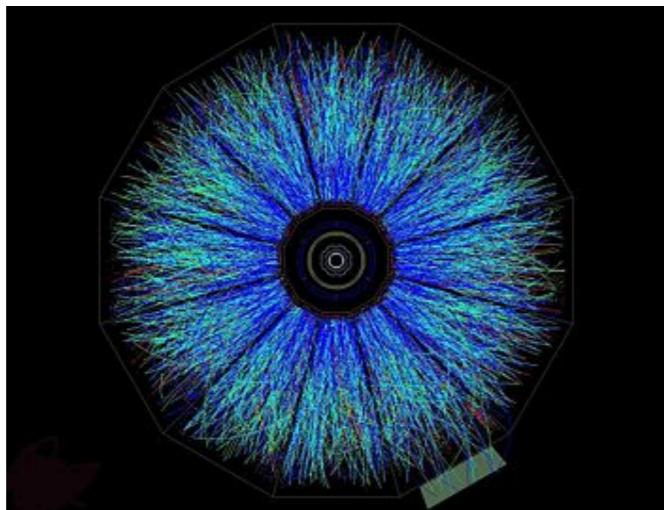
# 3+1次元の量子異常



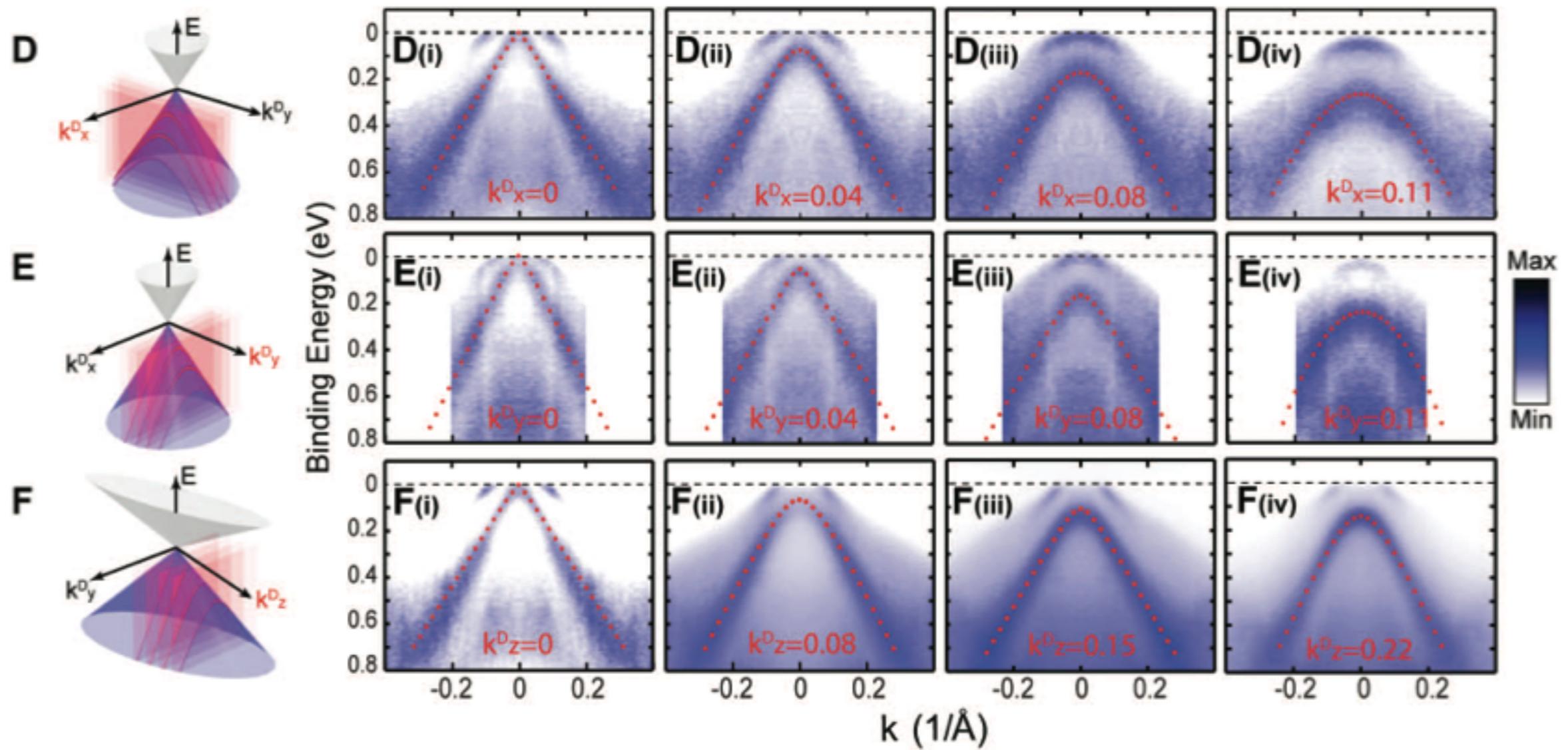
$$\Delta Q = 0, \quad \Delta Q_A = \frac{E}{\pi} \cdot \frac{B}{2\pi} = \frac{1}{2\pi^2} EB$$

# 考えられる物理的な系

- RHIC/LHCにおける quark-gluon plasma
- 中性子星内部の電磁プラズマ
- 物性系の新物質：Weyl semimetal (“3D graphene”)



# Dirac semimetal ( $\text{Na}_3\text{Bi}$ )



# カイラルプラズマ不安定性 (CPI)

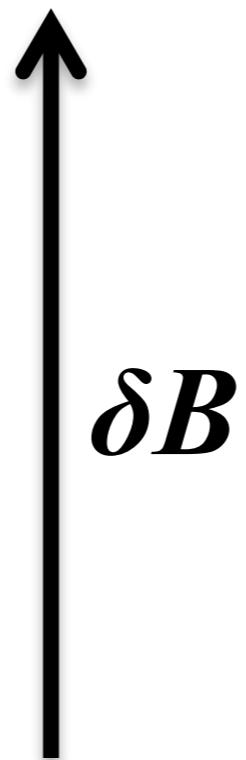
- 左右の差のもとでは不安定 (カイラルプラズマ不安定性)
- この不安定性は左右の差を減らそうとする

電弱理論 Redlich-Wijewardhana (1985); Rubakov (1986)

初期宇宙 Joyce-Shaposhnikov (1997)

QGP Akamatsu-NY (2013, 2014) 中性子星 Ohnishi-NY (2014)

# カイラルプラズマ不安定性 (CPI)



最初に一様な  $\mu_5$  があると仮定

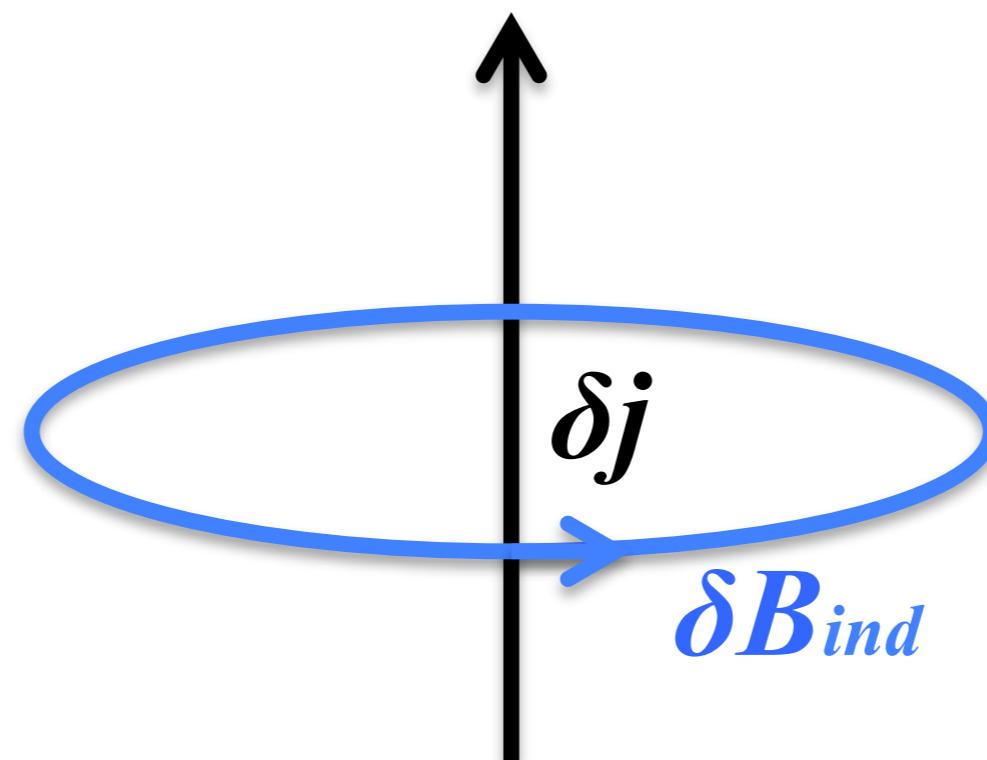
# カイラルプラズマ不安定性 (CPI)



カイラル磁気効果

$$\delta j \sim \mu_5 \delta B$$

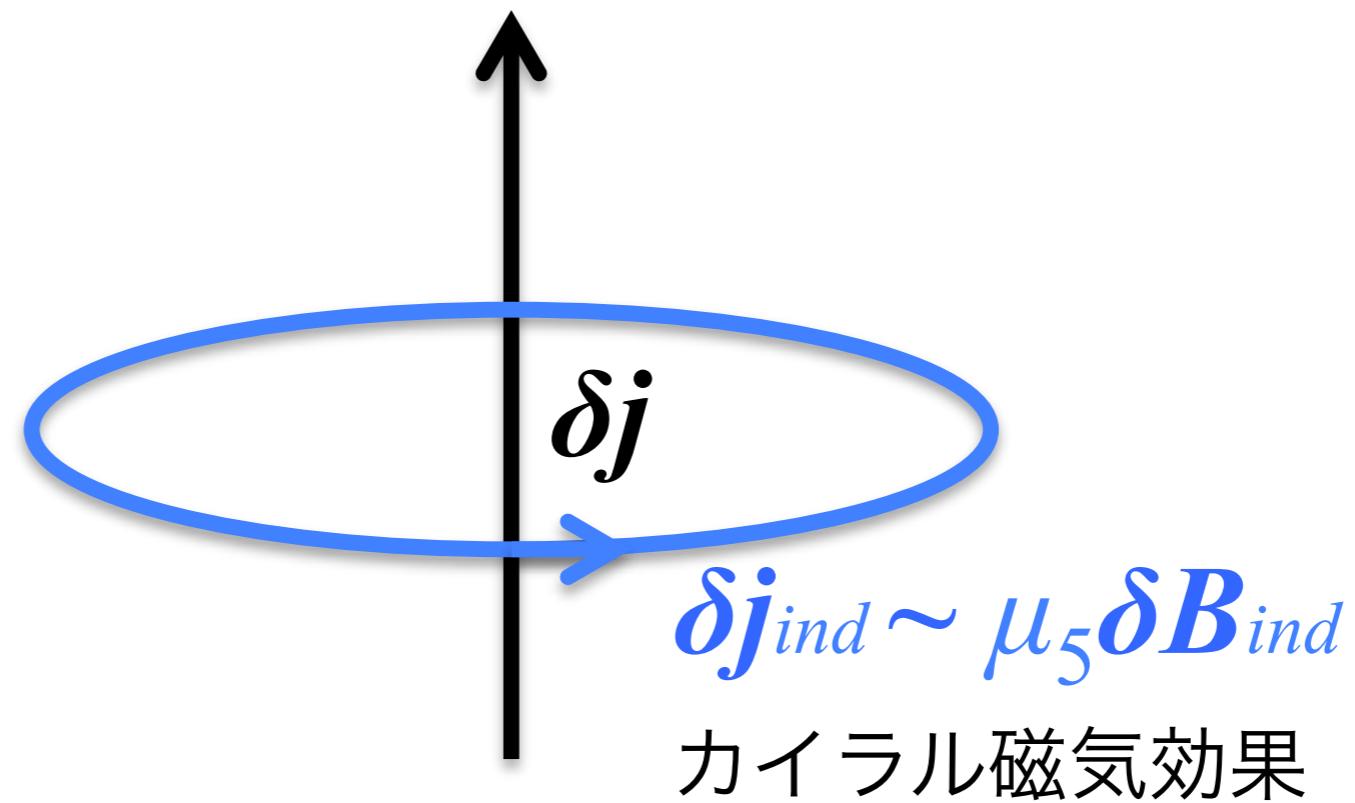
# カイラルプラズマ不安定性 (CPI)



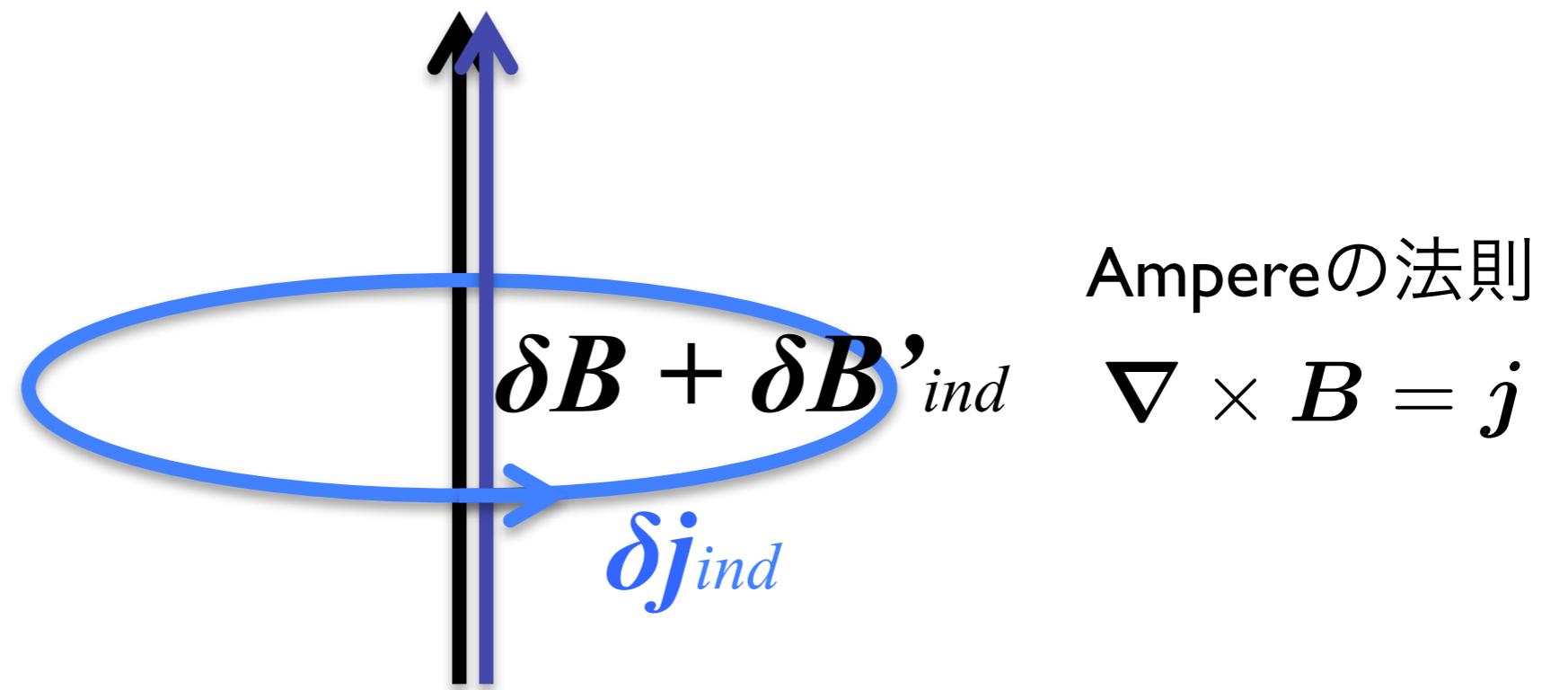
Ampereの法則

$$\nabla \times B = j$$

# カイラルプラズマ不安定性 (CPI)

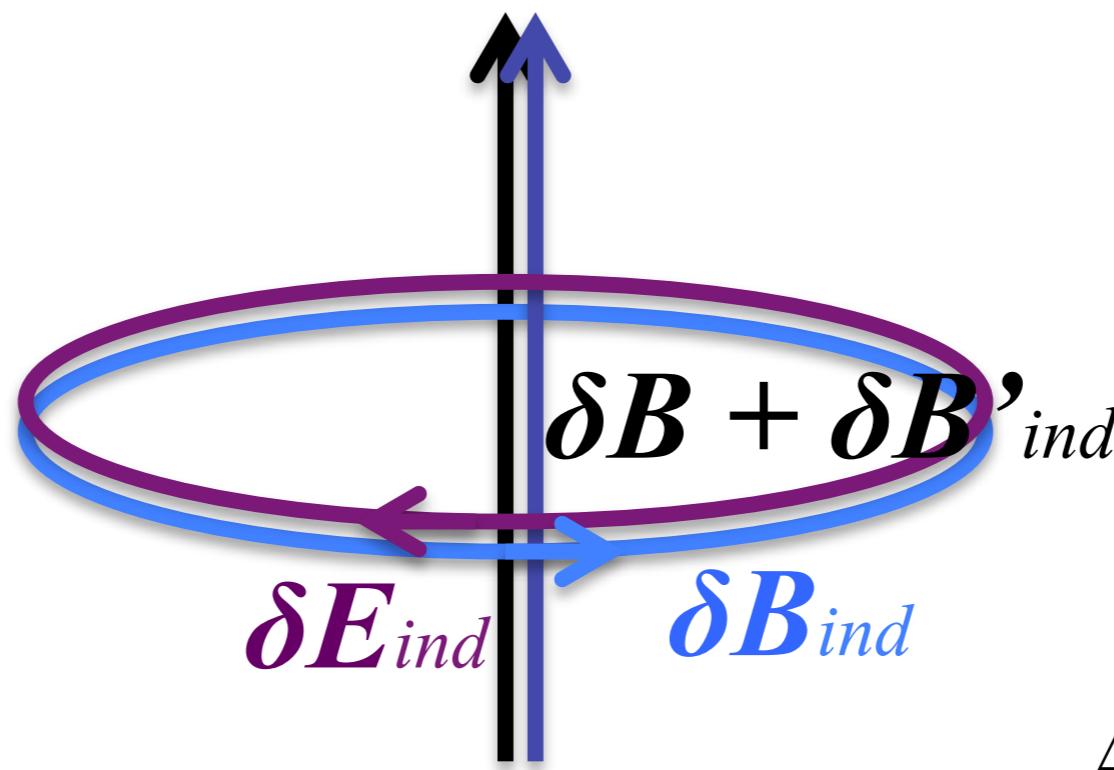


# カイラルプラズマ不安定性 (CPI)



正のフィードバック：不安定性

# カイラルプラズマ不安定性 (CPI)



Faradayの法則

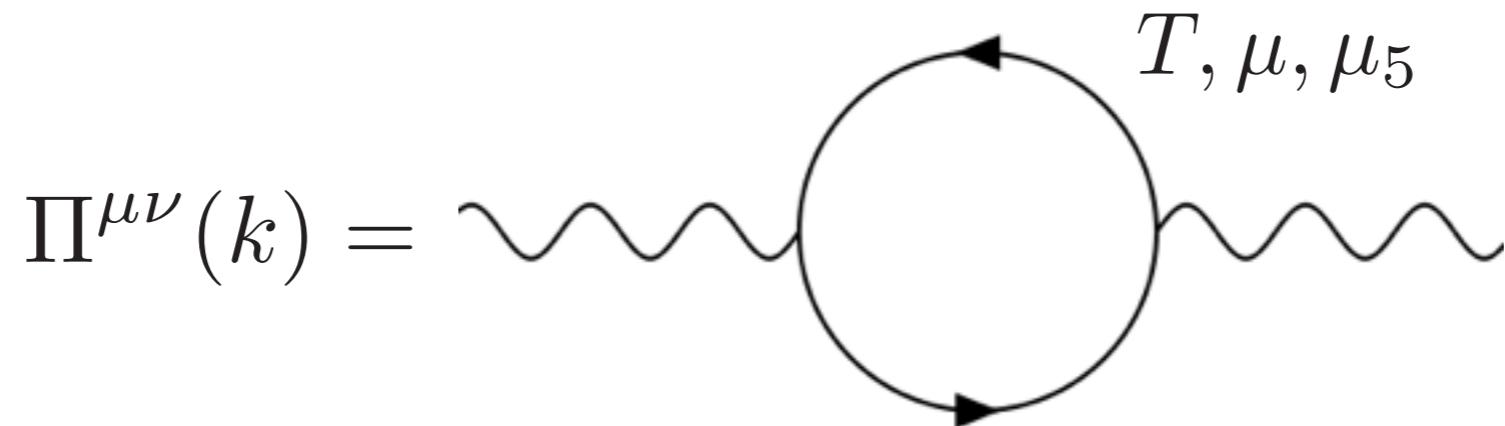
$$\nabla \times E = -\partial_t B$$

量子異常の式

$$\Delta Q_5 = \#E \cdot B < 0$$

自然は左右を等しくしようとする

# Polarization tensor with $\mu_5$



- $\Pi^{ij}(k)$  のテンソル分解 ( $A_0 = 0$ )

$$\hat{k}^i \hat{k}^j, \delta^{ij} - \hat{k}^i \hat{k}^j, i\epsilon^{ijk} \hat{k}^k$$

$$\Pi_-^{ij}(k) = \frac{e^2 \mu_5}{4\pi^2} i \epsilon^{ijk} \left( 1 - \frac{\omega^2}{|\mathbf{k}|^2} \right) \left( 1 - \frac{\omega}{2|\mathbf{k}|} \ln \frac{\omega + |\mathbf{k}|}{\omega - |\mathbf{k}|} \right) k^k$$

T=0 [Son-NY (2013)] T≠0 [Manuel, Torres-Rincon (2014)]

# Collective modes

- Maxwell方程式:  $\partial_\nu F^{\nu\mu} = j^\mu$
- 線形応答:  $j^\mu(k) = \Pi^{\mu\nu}(k)A_\nu(k)$

$$[k^2 \delta^{ij} - k^i k^j + \Pi^{ij}(k)] A^j = 0$$

(Transverse part,  $\omega \ll k$ )

- $\mu_5=0$  では  $\omega = -\frac{4ik^3}{\pi m_D^2}$ ;  $e^{-i\omega t} \sim e^{-\gamma(k)t}$  Landau減衰
- $\mu_5 \neq 0$  では  $\omega = \frac{4ik^2}{\pi m_D^2} \left( \frac{e^2 |\mu_5|}{4\pi^2} - k \right)$ ;  $e^{-i\omega t} \sim e^{\gamma(k)t}$  for small  $k$  プラズマ不安定性

Akamatsu-NY (2013)

# 典型的なスケール

Ampere's law + CME

$$\nabla \times B = j \sim e^2 \mu_5 B$$

$$\therefore k_{\text{inst}} \sim e^2 \mu_5$$

# Non-Abelian Chiral Instability

- QGPはカラーをもつ → non-Abelian chiral plasma instability

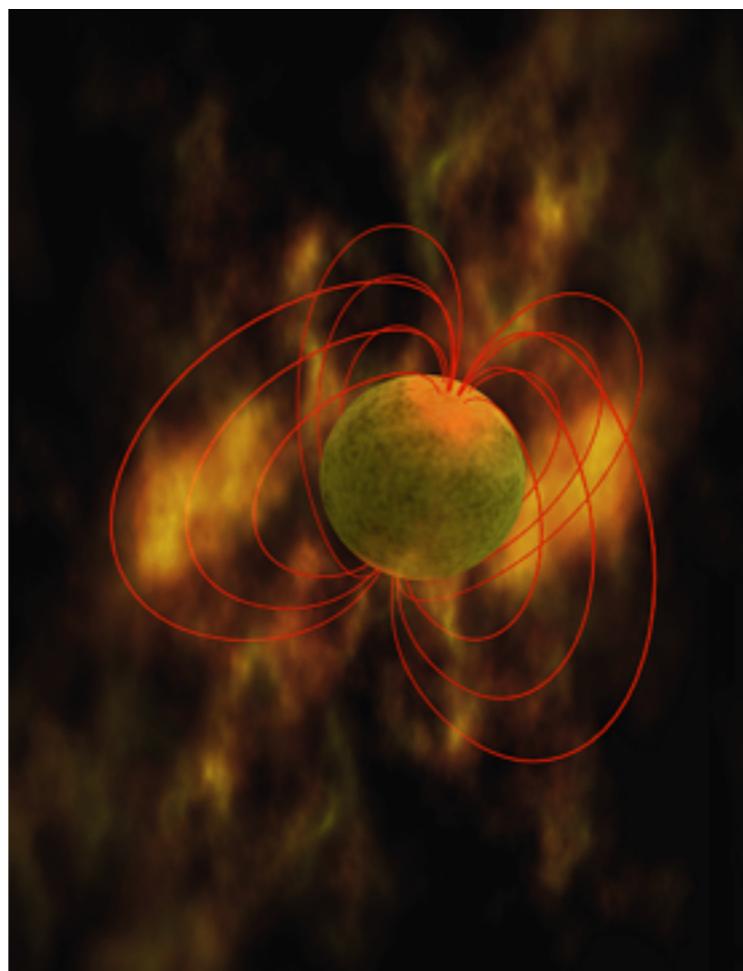
$$l_{\text{inst}} \sim (g^2 \mu_5)^{-1} \ll (g^4 T)^{-1} \sim l_{\text{mfp}}$$

$$\tau_{\text{inst}} \sim [g^4 T \ln(1/g)]^{-1} \sim \tau_{\text{mft}}$$

- $\mu_5 \sim T$ においてnon-Abelian CPIは流体力学で見えない
- もっと微視的な理論 (BoltzmannやLangevin方程式)では記述可能

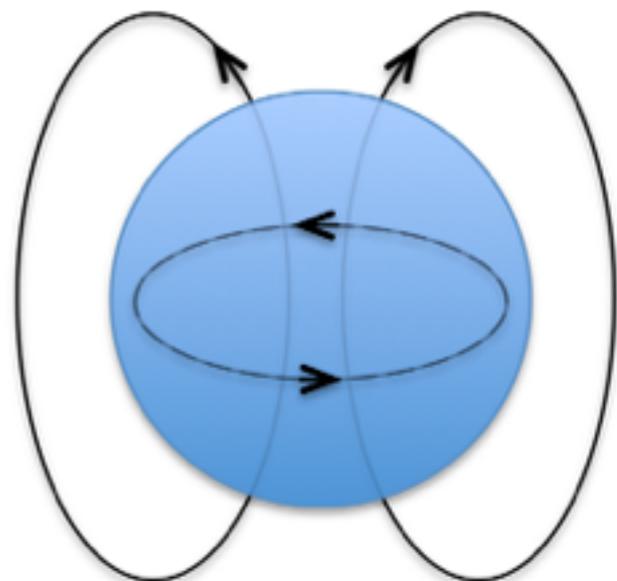
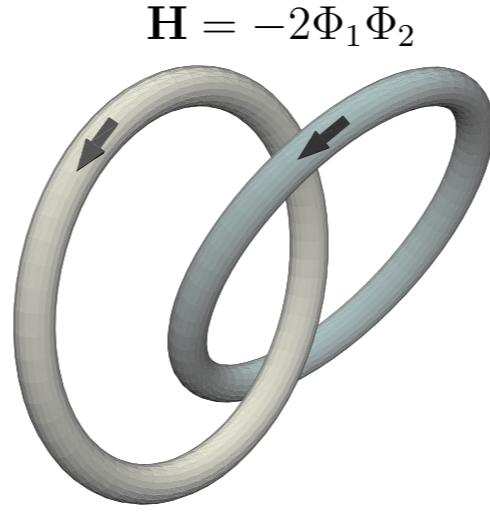
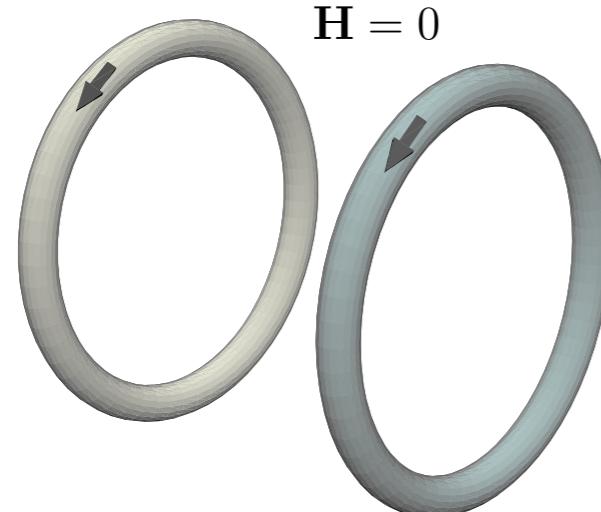
# マグネター

- マグネター：「宇宙最強の磁石」
- 表面磁場は最大  $10^{15}$  G 程度
- このような強くて安定な磁場はどのように作られるのか？



# 磁気ヘリシティ

- 磁気ヘリシティ:  $\mathcal{H} = \int d^3x \mathbf{A} \cdot \mathbf{B}$
- Gaussの絡み目数に比例: トポロジカルな安定性
- これまで電磁流体力学(MHD)の初期条件として存在が仮定
- しかし、そもそもその起源は非自明



poloidal/toroidal 磁場

# トロイダル磁場の証拠？

## Synopsis: Internal Magnetic Field Causes Neutron Star to Go Wobbly



L. Calçada/ESO

Possible Evidence for Free Precession of a Strongly Magnetized Neutron Star in the Magnetar 4U 0142+61

K. Makishima, T. Enoto, J. S. Hiraga, T. Nakano, K. Nakazawa, S. Sakurai, M. Sasano, and H. Murakami  
Phys. Rev. Lett. 112, 171102 (2014)

Published April 30, 2014

## Possible Evidence for Free Precession of a Strongly Magnetized Neutron Star in the Magnetar 4U 0142+61

K. Makishima,<sup>1, 2, 3</sup> T. Enoto,<sup>4, 5</sup> J. S. Hiraga,<sup>2</sup> T. Nakano,<sup>1</sup> K. Nakazawa,<sup>1</sup> S. Sakurai,<sup>1</sup> M. Sasano,<sup>1</sup> and H. Murakami<sup>1</sup>

<sup>1</sup>Department of Physics, Graduate School of Science,  
the University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

<sup>2</sup>Research Center for the Early Universe, Graduate School of Science,  
the University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

<sup>3</sup>MAXI team, RIKEN, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan

<sup>4</sup>High Energy Astrophysics Laboratory, RIKEN Nishina Center, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan

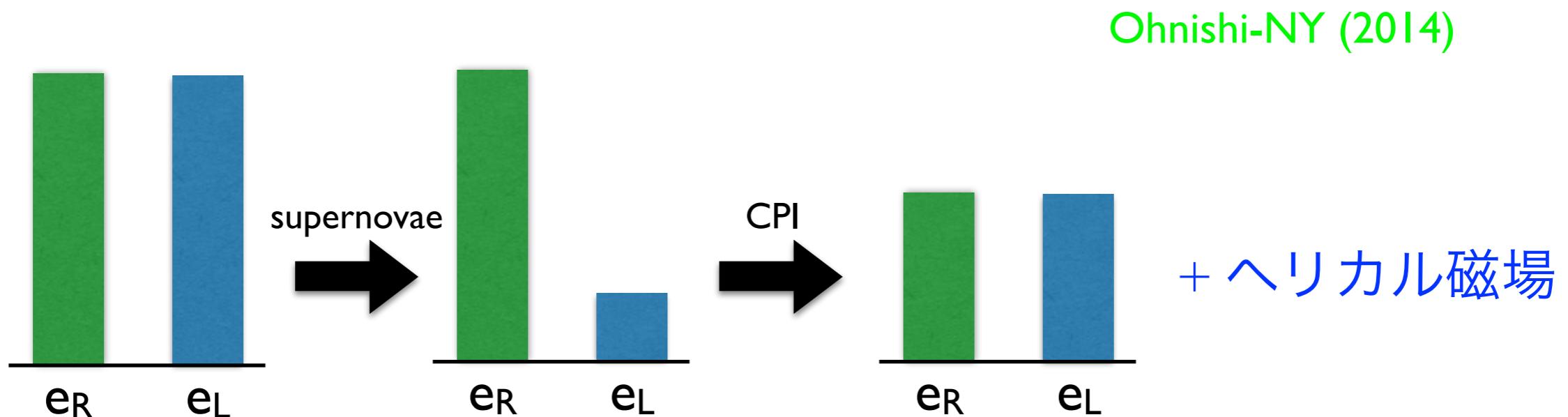
<sup>5</sup>NASA Goddard Space Flight Center, Astrophysics Science Division, Code 662, Greenbelt, MD 20771, USA

(Dated: April 22, 2014)

Magnetars are a special type of neutron stars, considered to have extreme *dipole* magnetic fields reaching  $\sim 10^{11}$  T. The magnetar 4U 0142+61, one of prototypes of this class, was studied in broadband X-rays (0.5–70 keV) with the *Suzaku* observatory. In hard X-rays (15–40 keV), its 8.69 sec pulsations suffered slow phase modulations by  $\pm 0.7$  sec, with a period of  $\sim 15$  hours. When this effect is interpreted as free precession of the neutron star, the object is inferred to deviate from spherical symmetry by  $\sim 1.6 \times 10^{-4}$  in its moments of inertia. This deformation, when ascribed to magnetic pressure, suggests a strong *toroidal* magnetic field,  $\sim 10^{12}$  T, residing inside the object. This provides one of the first observational approaches towards toroidal magnetic fields of magnetars.

# CPIによるマグネター磁場

- 超新星爆発におけるニュートリノ放出:  $p + e_L^- \rightarrow n + \nu_e^L$
- 右巻き電子の方が多く残る → この状態はCPIにより不安定
- 生成磁場の大きさ：中心コアで最大  $\sim 10^{18}$  G ガウス
- ヘリシティの保存：電子のヘリシティ → 磁気ヘリシティ



# Grabowska-Kaplan-Reddy

## The Role of the Electron Mass in Damping Chiral Magnetic Instability in Supernova and Neutron Stars

Dorota Grabowska,<sup>1,\*</sup> David B. Kaplan,<sup>1,†</sup> and Sanjay Reddy<sup>1,‡</sup>

<sup>1</sup>*Institute for Nuclear Theory, University of Washington, Seattle, WA*

We show that the nonzero electron mass plays a critical role in determining the magnetic properties of neutron stars, making it impossible to generate the chiral charge density needed to trigger a strong chiral magnetic instability during the core collapse of supernovae. This instability has been proposed as a plausible mechanism for generating extremely large helical magnetic fields in neutron stars at their birth; the mechanism relies on the generation of a large non-equilibrium chiral charge density via electron capture reactions that selectively deplete left-handed electrons during core-collapse and the early evolution of the protoneutron star. Our calculation shows that the electron chirality violation rate induced by Rutherford scattering, despite being suppressed by the smallness of the electron mass relative to the electron chemical potential, is still fast compared to the weak interaction electron capture rate. The resulting asymmetry between right and left-handed electron densities is therefore never able to attain an astrophysically relevant magnitude.

- 磁場の典型的な距離が短すぎる？

- $\Gamma_B(0) = \frac{2\alpha^2}{\sigma\mu_e^2} B_\star(0)^2 \lesssim \frac{8\alpha^3}{\mu_e^3} \left[ \ln \frac{4}{x} - 1 \right] B_\star(0)^2 \sim \Gamma_m \times \left( \frac{B_\star(0)}{5 \times 10^{14} \text{ G}} \right)^2$

# Grabowska-Kaplan-Reddy

## The Role of the Electron Mass in Damping Chiral Magnetic Instability in Supernova and Neutron Stars

Dorota Grabowska,<sup>1,\*</sup> David B. Kaplan,<sup>1,†</sup> and Sanjay Reddy<sup>1,‡</sup>

<sup>1</sup>*Institute for Nuclear Theory, University of Washington, Seattle, WA*

We show that the nonzero electron mass plays a critical role in determining the magnetic properties of neutron stars, making it impossible to generate the chiral charge density needed to trigger a strong chiral magnetic instability during the core collapse of supernovae. This instability has been proposed as a plausible mechanism for generating extremely large helical magnetic fields in neutron stars at their birth; the mechanism relies on the generation of a large non-equilibrium chiral charge density via electron capture reactions that selectively deplete left-handed electrons during core-collapse and the early evolution of the protoneutron star. Our calculation shows that the electron chirality violation rate induced by Rutherford scattering, despite being suppressed by the smallness of the electron mass relative to the electron chemical potential, is still fast compared to the weak interaction electron capture rate. The resulting asymmetry between right and left-handed electron densities is therefore never able to attain an astrophysically relevant magnitude.

- 磁場の典型的な距離が短すぎる?  
→ MHDでのinverse cascadeで長くなる Pouquet et al. (1976), ...
- $\Gamma_B(0) = \frac{2\alpha^2}{\sigma\mu_e^2} B_\star(0)^2 \lesssim \frac{8\alpha^3}{\mu_e^3} \left[ \ln \frac{4}{x} - 1 \right] B_\star(0)^2 \sim \Gamma_m \times \left( \frac{B_\star(0)}{5 \times 10^{14} \text{ G}} \right)^2$

# Conclusion

- カイラリティによる新奇な輸送現象
- マグネターの可能なメカニズム
- Chiral MHDの宇宙・原子核物理への応用？